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# A Formulation of Poincaré's Intuitive Concept of Linear Continuum *

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**Abstract:** Henri Poincaré's intuitive concept of the linear continuum was described by his famous remark which was noted by Bertrand Russell, and in which the notion of "intimate bond" first appeared. This semi-expository paper gives an exposition of Poincaré's remark, and also aims at a formulation of the intimate bond with the aid of introducing a kind of "leap structure" into Robinson's $^*R$ as a supplemental construction. As a result, we obtain a kind of hyperstandard model that may be called "Poincaré continuum".

**Key words:** Poincaré's intimate bond; leap-structure; Poincaré continuum; semi-infinitesimal.

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## 1. Introduction–an exposition of Poincaré's remark

Poincaré's remark on Cantor's continuum was translated into English by Russell and appeared in Russell's book[8]. It reads as follows: "The continuum thus conceived is nothing but a collection of individuals arranged in a certain order, infinite in number, it is true, but external to each other. This is not the ordinary concept, in which there is supposed to be, between the elements of the continuum, a sort of intimate bond which makes a whole of them, in which the point is not prior to the line, but the line to the point. Of the famous formula $2^{\aleph_0} = c$, the continuum is unity in multiplicity, the multiplicity alone subsists, the unity has disappeared."

Clearly, Poincaré's remark has rendered a deep insight into the paradoxical nature of Cantor's "point-constituted continuum", of which the concept has never been accepted by intuitionists'school. In fact, Cantor-Dedekind's point set theoretical continuum $R$ is merely a kind of single-phase abstraction of the native continuum, in which the characteristic-phase "continuity" has been entirely dropped or disregarded.

Before giving an exposition of Poincaré's remark, let us mention some related terminologics to be used. We agree with the following equivalences between several technical terms:

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Linear continuum≡ Line continuum ≡ Real line ≡ Real continuum (Idealization of the native continuum);

Cantor's continuum ≡ Point-constituted continuum ≡ Real number continuum ≡ Real number-axis ≡ Set of positional points on a line.

Note that the main distinction between the real continuum and Cantor's continuum is that the former concept consists of the double character, namely the continuity (characterizing the unity) plas the infinite divisibility (producing the multiplicity), and the latter one is merely a single-phase abstraction, taking multiplicity as the sole fundamental characteristic of the continuum.

The first paragraph of Poincar'e's remark points out that Cantor's continuum is an ordered set of infinitely many elements (points) having all its elements external to each other. In other words, all the points in Cantor's continuum are disconnected. Evidently, this agrees with Aristotle's viewpoint: "Real numbers (positional points of the number-axis) don't touch each other, so that they cannot yield a real continuum." In fact, Aristotle, Poincaré, Brouwer, Weyl[10] and their followers all shared the same view that the real continuum can by no means be regarded as merely a collection (set) of distinct elements (points).

The unusual concept mentioned in the second paragraph of the remark is the so-called "intimate bond" that actually implies the continuity of the line continuum and also makes whole of the points of Cantor's continuum into the line. Certainly, Poincaré had a picture of the line continuum in mind, so very naturally he could suppose that a sort of intimate bond should exist between the points of Cantor's continuum and could also connect all the points into the line.

Note that the above exposition should be understood to be a re-construction of the line continuum. Actually, both in reality and in the conceptual reasoning, line is prior to the point, inasmuch as "points" just mean "positions" which are a derived concept from the line.

The third paragraph of Poincaré's remark involves a dialectical comprehension of Cantor's cardinality formula $2^{\aleph_0} = c$ which certainly holds only for the point-constituted continuum. Clearly the LHS of the formula displays the multiplicity of Cantor's continuum (say, all the possibilities of infinite bisections of a line-segment), and the cardinality $c$ on the RHS of the formula just represents a common characteristic (continuity) of every real line that possesses the posibility of infinite bisections. So the formula just means that the unity (continuity) involves the multiplicity and also lies within the multiplicity. However, the sole "multiplicity" as a concept is a negation of "continuity", and any single-phase abstraction of the multiplicity naturally has to leave the continuity aside. So the remark has such a conclusion: "the multiplicity alone subsists, the unity has disappeared."

Evidently, the above conclusion is also a severe comment upon Cantor-Dedekind's concept of the point-constituted continuum, in which the continuity as the basic feature of a continuum has been entirely ignored.

But, how to define Poincaré's intimate bond mathematically within Cantor's continuum? This is really an unthinkable problem of difficulty, and its impossibility will be mentioned briefly in the next section.

## 2. Methodological considerations

First, it should be not difficult to see that Poincaré's intimate bond (PIB) finds no place to be defined within the real number continuum $R$, since $R$ has been made both complete and closed (known as first by Dedekind, Cantor, Weierstrass, et. al.) so that nothing could be defined between the elements of it. Consequently, our first supposition is that Robinson's hyper-real number field $^*R$ (cf.[7]) may possibly provide a ground space on which PIB may be defined as an extra-structure.

It is known that a certain kind of hyperstandard measure (i.e., a simpler case of Loeb measure) could even be used to show that $R$ is of "measure zero" within $^*R$ (cf.[3]). This obviously suggests that the structure of monads is the real source of positive measure, and the same conclusion certainly applies to PIB whenever it could get formally defined. Accordingly we think that PIB may be formulated via monads.

However, monads have no boundaries and are external to each other. In other words, there are "gaps" between monads, so that $^*R$ is still not an ideal model for the real continuum. This brings to us with a second consideration that we have to define the "gaps or leaps" before introducing the formal definition of PIB.

Recently we got learned that S.Kamo has built up a theory on "gaps" which are defined within monads (cf. [4],[5]). However, our intention is to formulate the concept of "leaps" that lie outside of monads and could bridge the gaps between monads.

In order to reach the goal mentioned above, we have to develop a kind of generalized Dedekind cut on the set of monads. This our third consideration.

Finally, we think that once the "leaps" have got well defined between monads of $^*R$, we should be able to formulate an extended ordering structure including $^*R$ as a main part of it. Meanwhile, every monad plus its "right-leap" and "left-leap" will be called "Poincaré's element" or "line quantum" which actually represents a sort of non-punctiform element having the similar meaning as that expounded recently by John Bell in his text-book[1].

More precisely, what we wish to construct is a sort of hyperstandard model for what we may call "Poincaré continuum" that consists of all the non-punctiform elements so-called Poincaré's elements.

## 3. Leap structure and Poincaré continuum

In this section we will give constructive definitions for the so-called "leaps" and Poincaré continuum via the concept of monads. For the sake of simplicity, we will be concerned chiefly with $^*R_0 \equiv G(0)$, galaxy of 0, instead of Robinson's $^*R$. In parallel we take $R_0 = \{st(x) | x \in {}^*R_0\}$, and we always assumed that $R_0$ is embeded in $^*R_0$.

In order to introduce the unusual concept "leap", we may make use of the original idea of Dedekind's cut. Recall that every Dedekind's cut of rational numbers, say $(Q_1|Q_2)$ with non-empty $Q_1$ and $Q_2$ so that $Q_1 \cup Q_2 = Q$ (set of rational numbers) and $\forall a \in Q_1, \forall b \in Q_2 \Rightarrow a < b$, just defines a real number of $R$.

Similarly, we may think that a kind of generalized Dedekind cut could be formulated on the ordered set of monads of $^*R_0$, namely $^*R_0' := \{m(a) | a \in R_0\}$.

Note that any two different monads $m(a)$ and $m(b)$ of $^*R_0'$ can be ordered as $m(a) <$

$m(b)$ or $m(a) > m(b)$ according as $a < b$ or $a > b$. Thus, if ${}^*R_0'$ is divided into two disjoint non-empty subsets of monads, say ${}^*R_{01}'$ and ${}^*R_{02}'$, such that

$$\forall m(a) \in {}^*R_{01}', \forall m(b) \in {}^*R_{02}' \Rightarrow m(a) < m(b),$$

then the bisection denoted by $({}^*R_{01}'|{}^*R_{02}')$ may be called a hyperstandard Dedekind section.

For any given section $({}^*R_{01}'|{}^*R_{02}')$ on the set of monads, let $R_{01}$ and $R_{02}$ be standard real number sets defined by

$$R_{01} = \{x|m(x) \in {}^*R_{01}'\} \quad \text{and} \quad R_{02} = \{y|m(y) \in {}^*R_{02}'\}$$

Clearly $(R_{01}|R_{02})$ just yields a classical Dedekind cut on $R_0$, which may be called a companion cut of $({}^*R_{01}'|{}^*R_{02}')$.

As usual, for any given set $E \subset R_0$, we shall denote by $\sup E$ and $\inf E$ the supremum and the infimum of the set $E$, respectively. Now we are able to state a formal definition concerning the leaps between monads.

**Definition 3.1** *Let $a \in R_0 \subset G(0)$. Suppose that there is a hyperstandard Dedekind section $({}^*R_{01}'|{}^*R_{02}')$ such that $m(a) \in^* R_{01}'$ with $a = \sup R_{01} \in R_{01}$. Then we denote*

$$m(a)^+ := ({}^*R_{01}'|{}^*R_{02}')$$

*and call $m(a)^+$ the right-leap of $m(a)$. Similarly, the left-leap $m(a)^-$ may be defined by*

$$m(a)^- := ({}^*R_{01}'|{}^*R_{02}')$$

*in which $m(a) \in^* R_{02}'$ with $a = \inf R_{02} \in R_{02}$.*

**Definition 3.2** *The set-theoretic union given by*

$$\overline{m(a)} := m(a) \cup \{m(a)^-, m(a)^+\}$$

*is called "Poincaré's line-quantum" at position $a$, or in brief, "Poincaré's element" center at $a$. Moreover, the collection given by*

$$\Lambda := \{m(a)^+, m(a)^- | a \in R_0\}$$

*is called the leap-structure.*

**Remark 3.3** One may conceive of the two sets ${}^*R_{01}'$ and ${}^*R_{02}'$ as having a certain "gap" between them, so that the terminology "leaps" for $m(a)^+$ and $m(a)^-$ may be well understood as jumps over the gaps from either sides of $m(a)$.

Equivalently, Definition 3.1 can be re-stated by using the similar idea of Cantor's principle of nested intervals for defining irrational numbers.

**Definition 3.4** *Given $a \in R_0 \subset {}^*R_0 = G(0)$. Then we may define*

$$m(a)^+ := \{(a+\varepsilon, b-\varepsilon)|\forall \varepsilon \in {}^*R_0 : \varepsilon > 0, \mathrm{st}(\varepsilon) = 0; \forall b \in R_0 : b > a\},$$

$$m(a)^{-} := \{(c+\varepsilon, a-\varepsilon) | \forall \varepsilon \in {}^{*}R_0 : \varepsilon > 0, \mathrm{st}(\varepsilon) = 0; \forall c \in R_0 : c < a\},$$

As may be observed, the above two definitions for leaps are substantially equivalent. With Remark 3.3 in mind we are led to the following definition.

**Definition 3.5** *Poincaré continuum (PC) and Poincaré's intimate bond (PIB) can be defined and denoted, respectively, as follows*

$$(PC) := \bigcup_{a \in R_0} \overline{m(a)} =^{*} R_0 \cup \Lambda,$$

$$(PIB) := \bigcup_{a \in R_0} \overline{m(a)} \backslash R_0 = (\mathrm{PC}) \backslash R_0$$

*where for brevity (PC) may also be written as* $\overline{G(0)} \equiv \overline{{}^{*}R_0}$.

Clearly, the above definition just means that (PC) is a set-theoretic union of Poincaré's line-quanta with $R_0$ as the set of positions. The (PIB) is given by the complementary set of (PC) with respect to Cantor's point-constituted set $R_0$. As we have mentioned, $R_0$ is of measure zero in accordance with a certain kind of hyperstandard measure (implied by the general measure theory of Loeb[6]). Consequently we may assert that (PIB) is the only real source of positive measure for some sets (e.g., intervals, line-segments, etc.) of the real continuum (PC).

## 4. (PC) as a hyper-ordered structure

Having defined (PC) $\equiv \overline{G(0)}$ as above (§3), the ordering relation "$\leq$" pertaining to ${}^{*}R_0 \equiv G(0)$ can be naturally extended to $\overline{{}^{*}R_0} \equiv \overline{G(0)}$. More precisely, we may define the ordering for leaps as follows

$$a+\varepsilon < m(a)^{+} < b-\varepsilon \ (\forall \varepsilon > 0, \mathrm{st}(\varepsilon) = 0; \forall b \in R_0, b > a),$$

$$c+\varepsilon < m(a)^{-} < a-\varepsilon \ (\forall \varepsilon > 0, \mathrm{st}(\varepsilon) = 0; \forall c \in R_0, c < a).$$

In this way we get a hyper-ordered set $(\overline{G(0)}, \leq)$.

Consequently, the singletons $\{m(a)^{+}\}$ and $\{m(a)^{-}\}$ may also be expressed as the results given by the hyperstandard set-theoretic intersection of intervals, namely

$$\{m(a)^{+}\} = \bigcap_{\varepsilon>0} \bigcap_{b>a} (a+\varepsilon, b-\varepsilon), \quad \{m(a)^{-}\} = \bigcap_{\varepsilon>0} \bigcap_{c<a} (c+\varepsilon, a-\varepsilon)$$

where the intersection operators are assumed to be taken over all $\varepsilon > 0$ with $\varepsilon \in {}^{*}R, \mathrm{st}(\varepsilon) = 0$, and all $b \in R_0, c \in R_0$ such that $b > a$ and $c < a$.

Note that in the above expressions we have actually already assumed that the domain for intersection operators has been extended to $\overline{G(0)}$ so that $\{m(a)^{\pm}\}$ are allowed to be the intersection results.

Suppose that the operation + (addition) adopted in ${}^{*}R_0$ is extended to (PC). Then in the ordering structure of (PC) we will find it useful to have a "Translation Formula" for $m(a)^{\pm}$ of the form

$$m(a)^{\pm} = m(0)^{\pm} + a = a + m(0)^{\pm} \quad (a \in R_0).$$

This implies that the right and left leaps of zero monad, $m(0)^+$ and $m(0)^-$, are most fundamental. For the leaps $m(0)^{\pm}$ we also have to postulate that

$$m(0)^{\pm} + m(0)^{\pm} = m(0)^{\pm}.$$

This means that $m(0)^{\pm}$ have the property-"idempotence".

In order to disclose certain unusual property of $m(0)^{\pm}$, let us introduce the following definition.

**Definition 4.1** *An element $\theta$ of (PC) is said to be a semi-infinitesimal if $\theta \neq 0, \theta \notin m(0)$ and one of the following conditions*

*(i)* $0 < \theta < \delta (\forall \delta : \delta > 0, \delta \in R_0)$;

*(ii)* $-\delta < \theta < 0 (\forall \delta : \delta > 0, \delta \in R_0)$

*is satisfied. Moreover, any element $\theta$ is called a nonpunctiform element ~~if it has no standard~~ part, zero or nonzero.*

**Proposition 4.2** *Both $m(0)^+$ and $m(0)^-$ are semi-infinitesimals. Also, they are non-punctiform elements in (PC).*

**Proof** It suffices to consider $m(0)^+$, since $m(0)^-$ can be treated entirely similarly. In accordance with the ordering relation in (PC) we have

$$\varepsilon < m(0)^+ < b - \varepsilon (\forall \varepsilon : \varepsilon > 0, \mathrm{st}(\varepsilon) = 0; \forall b : b \in R_0, b > 0)$$

Since the inequality holds for every $\varepsilon > 0$ with $\mathrm{st}(\varepsilon) = 0$, it is clear that $m(0)^+ \notin m(0)$. Otherwise there will appear a contradiction. Thus $m(0)^+$ is not an infinitesimal.

On the other hand, the ordering relations imply $0 < m(0)^+ < b$ for every $b > 0$ with $b \in R_0$. This involves that $m(0)^+$ cannot take any number of ${}^*R_0$ with positive standard part. Hence we may conclude that $m(0)^+$ is a semi-infinitesimal.

Moreover, form what we have explained above, it is obvious that $m(0)^+$ has neither a standard part 0, nor a standard part $> 0$. Actually no standard part could be given of $m(0)^+$. Hence $m(0)^+$ is a non-punctiform element. □

Note that ${}^*R_0$ does not contain non-punctiform elements so that $m(0)^{\pm}$ and $m(a)^{\pm} = a + m(0)^{\pm}$ are external of ${}^*R_0$. Consequently we have the following

**Corollary 4.3** *In the (PC) the leap-structure $\Lambda = \{m(a)^{\pm} | a \in R_0\}$ is an extra-structure of ${}^*R_0$.*

**Remark 4.4** Since the right-leap and left-leap $m(a)^{\pm}$ lie between monads, and $m(0)^{\pm}$ are semi-infinitesimals, we could deduce from $m(a)^{\pm} = a + m(a)^{\pm}$ that every monad $m(a)$ is a semi-infinitesimal distant from its neighboring monads. Accordingly, any two "neighboring (or successive) real numbers" (points of $R$) are different from each other by a semi-infinitesimal, nevertheless it is impossible to exhibit them pairwisely.

As usual, denote by $(a, b)$ and $[a, b]$ the open and closed intervals of $R_0$. Similarly, ${}^*(a, b)$ and ${}^*[a, b]$ denote the intervals of ${}^*R_0$. Then we may define the intervals of (PC) by the following set-theoretic unions

$$\overline{{}^*(a,b)} := \bigcup_{x \in (a,b)} \overline{m(x)}, \quad \overline{{}^*[a,b]} := \bigcup_{x \in [a,b]} \overline{m(x)}.$$

Surely the lengths of these intervals are given by the real number $(b-a)$. On the other hand, we have learned that $^*(a,b)$ and $^*[a,b]$ have gaps between the monads contained therein, yet they still have the same length (linear measure) $(b-a)$. Here, from the standpoint of (PC), it has actually made an implicit "assumption" that the linear measures of $^*(a,b)$ and $^*[a,b]$ have to be given precisely by that of their covering sets $\overline{^*(a,b)}$ and $\overline{^*[a,b]}$, respectively.

Finally, we could also define Poincaré's real line (PRL) as a set of line-quanta, namely

$$(\mathrm{PRL}) := \{\overline{m(x)} | x \in R_0\}.$$

This means that the real line (without $\pm\infty$) is built up by Poincaré's line-quanta having the position-set $R_0$. Certainly $\overline{m(x)}$ may also be called a non-punctiform element since it consists of non-punctiform elements $m(x)^{\pm}$.

## 5. Some expository remarks

In what follows we will give several expository and philosophical remarks.

**Remark 5.1** It may be found that a comparison of Hegel's remark on Zeno's paradoxes with Poincaré's remark on Cantor's continuum is really interesting. Note that the ancient Zeno paradoxes of motion are closely related to the double characteristics of the space-time continuum. In fact, G.W.F.Hegel had in his "Lectures on the History of Philosophy"[2] remarked that the concept of native continuum (time, line, motion) consists of two contradictory yet unseparable sub-concepts, namely the continuity (characteristic of motion) and the point-accumulativity or point-collectivity (implied by the divisibility of space-time continuum). Hegel stressed on the point that the sole abstraction of the concept of point-collectivity would deny the continuity-the existence of motion, since motion just means "connection" that is precisely the continuity (a negation of the distictness of point-positions). Clearly, this is the same point of view as given by Poincaré's remark that "multiplicity alone subsists, the unity (characterized by continuity) has disappeared." Thus what we have found is that both Hegel and Poincaré have got the same deep insight into the nature of line continuum, and both regarded the concept of point-constituted-continuum as a poor conceptual abstraction of the real continuum.

Of course, from the constructive pattern-view of mathematics, the point-constituted continuum $R$ is ever necessary and important for the classical analysis.

**Remark 5.2** With the notional picture of native continuum in mind, we may regard Euclidean straight line as a continuum concept with the 1st degree of abstraction. Accordingly, Cantor-Dedekind's real number continuum $R$ and Robinson's hyper-real field $^*R$ could be regarded as analytical models with the 2nd degree and the 3rd degree of abstraction, respectively. Reasoning in this way, both $(PC) - \overline{G(0)}$ and (PIB) could be considered as hyperstandard models having the 4th degree of abstraction. Moreover, it is apparent that Poincaré's line-quanta $\overline{m(x)}(x \in R_0)$ are different from Leibniz monads and Cavalieri's indivisibles, since each $\overline{m(x)}$ has itsown inner structure. However, if the inner structure is entirely disregarded, one still could imagine that $\overline{m(x)}(x \in R_0)$ may be used to represent Cavalieri's indivisibles or Leibniz monads. Higher dimensional cases are given by Cartesian products.

**Remark 5.3** We have not gone into any details (in §4) about the simple algebraic aspect of the leaps. Here we just mention a simple instance. Let us denote

$$\Lambda^+ := \{m(x)^+ | x \in R_0\}, \quad \Lambda^- := \{m(x)^- | x \in R_0\}$$

and define $(PC)^+ := G(0) \cup \Lambda^+, (PC)^- := G(0) \cup \Lambda^-$. Then by assuming the associative law for addition in $(PC)^\pm$, one will be able to prove that both $\langle (PC)^+, +, 0\rangle$ and $\langle (PC)^-, +, 0\rangle$ are commutative semi-groups with zero element 0 and idempotent elements $m(0)^+$ and $m(0)^-$, respectively. The details is too simple to be presented here.

Moreover, according to the definition for leaps, we see that each right leap $m(x)^+$ could bridge the "gap" (between monads) on the right-side of $m(x)$. Thus, as $x$ ranges over $R_0$, we should get $(PC)^+ \equiv (PC)$. Similarly, we have $(PC) \equiv (PC)^-$. All of this may suggest that any algebraic or analytical consideration of (PC) could be confined to the case of $G(0)$ plus the structure $\Lambda^+$ (or $\Lambda^-$).

**Remark 5.4** By and large, this is a concept-expository paper with the main objective for getting a formulation of Poincaré's "intimate bond" on the structure of monads in $^*R$. The somewhat strange concept "semi-infinitesimal" seems to be not very difficult. Actually, the fact that successive monads cannot be distant from each other by an infinitesimal or by a non-infinitesimal could follow by "reductio ad absurdum proof". We think that John Bell's treatment of smooth analysis[1] using non-punctiform elements is highly innovative and useful. Accordingly we guess that the Poincaré continuum (PC) with its non-punctiform elements $\overline{m(x)}(x \in R_0)$ may also possibly find certain applications to the smooth world. Surely much remains to be researched.

**Appendix** On the Relativity of Measure Concepts

For brevity, let $|a, b| = \overline{{}^*(ab)}$ with $a < b$ denote an interval of the Poincaré continuum (PC). For a given linear set $X$ we write

$$|a, b|_X \equiv X \cap |a, b| := \{x | a < x < b, x \in X \cap \overline{G(0)}\}.$$

Denote by $\mu_J(\cdot), \mu_L(\cdot)$, and $\mu_h(\cdot)$ the Jordan content, Lebesgue measure, and the hyper-standard measure (a kind of Loeb measure, cf.[3],[6]), respectively. Also we use $\mu_P(\cdot)$ to denote a kind of extended Loeb measure defined on (PC) and empolying covering sets belonging to (PC). Then we have

$$\mu_J(|a, b|_Q) = b - a, \quad \mu_L(|a, b|_Q) = 0;$$

$$\mu_L(|a, b|_R) = b - a, \quad \mu_h(|a, b|_R) = 0;$$

$$\mu_h(|a, b|_{^*R}) = b - a, \quad \mu_P(|a, b|_{^*R}) = 0.$$

However, for $|a, b| \in \overline{G(0)}$ we have $\mu_P(|a, b|_{\overline{G(0)}}) = b - a$, as may be justified by Poposition 4.2. Apparently the above facts just show the relativity of various measure concepts. In fact, for instance $\mu_J(|a, b|_Q) = b - a$ just means that $(b - a)$ is a measure (length) of the extent embedding $|a, b|_Q$ in it, and not the measure ($L$-measure) for $|a, b|_Q$ itself. The other cases could be understood similarly. More in details will be given elsewhere.

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# Poincaré 线性连续统直觉概念的公式化


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**摘 要**: 本文研究了 Poincaré 著名注记中“内束”观念的数学表述法问题，通过构建 Poincaré 连续统模型，得到了这一问题的一种解答. 文中还论述了有关数理哲学及方法论问题；文末特别指出了须继续研究的数学问题.



**关键词**: 内束； Poincaré 连续统；跃距结构；半无限小.