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Some Strengthed and Reversed Pachpatte Inequalities *

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Abstract: The main purpose of the present article is to establish some new strengthed and reversed Pachpatte's type inequalities. As applications, some new type Hilbert's inequlities are generalized and strengthened.

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1. Introduction

In recent years several authors[1−5] have given considerable attention to Hilbert inequalities, Hilbert's type inequalities and their various generalizations

Very recently, B.G.Pachpatte[6] proved some new inequalities similar to Hilbert inequality and gave two basic theorems as follows.

Theorem A *Let $p \geq 1, q \geq 1$, and let $\{a_m\}$ and $\{b_n\}$ be two nonnegative sequences of real numbers defined for $m = 1, 2, \ldots, k$ and $n = 1, 2, \ldots, r$, where k, r are natural numbers. Define $A_m = \sum_{s=1}^{m} a_s$ and $B_n = \sum_{t=1}^{n} b_t$. Then*

$$\sum_{m=1}^{k}\sum_{n=1}^{r}\frac{A_m^p B_n^q}{m+n} \leq C(p,q,k,r)\left(\sum_{m=1}^{k}(k-m+1)\left(A_m^{p-1}a_m\right)^2\right)^{1/2} \times \left(\sum_{n=1}^{r}(r-n+1)\left(B_n^{q-1}b_n\right)^2\right)^{1/2}, \tag{1}$$

unless $\{a_m\}$ or $\{b_n\}$ is null, where

$$C(p,q,k,r) = \frac{1}{2}pq\sqrt{kr}. \tag{2}$$

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Theorem B *Let $p \geq 1, q \geq 1$ and $f(\sigma) \geq 0, g(\tau) \geq 0$ for $\sigma \in (0,x), \tau \in (0,y)$, where x, y are positive real numbers and define $F(s) = \int_0^s f(\sigma)\mathrm{d}\sigma$ and $G(t) = \int_0^t g(\tau)\mathrm{d}\tau$ for $s \in (0,x), t \in (0,y)$. Then*

$$\int_0^x \int_0^y \frac{F^p(s)G^q(t)}{s+t}\mathrm{d}s\mathrm{d}t \leq D(p,q,x,y)\left(\int_0^x (x-s)(F^{p-1}(s)f(s))^2\mathrm{d}s\right)^{1/2} \times \left(\int_0^y (y-t)(G^{q-1}(t)g(t))^2\mathrm{d}t\right)^{1/2}, \tag{3}$$

unless $f \equiv 0$ or $g \equiv 0$, where

$$D(p,q,x,y) = \frac{1}{2}pq\sqrt{xy}. \tag{4}$$

In the present article we establish some new strengthed and reversed inequalities of the above inequalities. As applications, we generalize and strengthen some new Hilbert type inequalities.

2. Lemmas

Lemma 1[7,P.39] *If x and y are positive and unequal, then*

$$rx^{r-1}(x-y) < x^r - y^r < ry^{r-1}(x-y) \ \ (0 < r < 1), \tag{5}$$

$$rx^{r-1}(x-y) > x^r - y^r > ry^{r-1}(x-y) \ \ (r > 1), \tag{6}$$

Obviously, the inequalities (5) and (6) become itentities when $r = 0, r = 1$, or $x = y$.

Lemma 2 *Let $\frac{1}{p} + \frac{1}{q} = 1, \alpha > 0, a > 0$ and $b > 0$. Then we have the following estimates.*

(1) for $0 < p < 1$,

$$\sup_{\alpha>0}[\frac{1}{p}\alpha^{-1/q}a + \frac{1}{q}\alpha^{1/p}b] = a^{1/p}b^{1/q}; \tag{7}$$

(2) for $p > 1$,

$$\inf_{\alpha>0}[\frac{1}{p}\alpha^{-1/q}a + \frac{1}{q}\alpha^{1/p}b] = a^{1/p}b^{1/q}. \tag{8}$$

Lemma 3 *If $f(x) > 0, g(x) > 0, f(x) \in L^p[a,b], g(x) \in L^q[a,b]$ and $0 < p < 1, \frac{1}{p} + \frac{1}{q} = 1$, then for $\beta > 0$,*

$$\frac{1}{C(p,\beta)}\int_a^b f(x)g(x)\mathrm{d}x \geq \left(\int_a^b f^p(x)\mathrm{d}x\right)^{1/p}\left(\int_a^b g^q(x)\mathrm{d}x\right)^{1/q}, \tag{9}$$

where $C(p,\beta) = \frac{1}{p}\beta^{\frac{1}{p}-1} + (1-\frac{1}{p})\beta^{1/p}$. The inequality is reversed if $p > 1$.

Proof Let us consider the following function

$$\Phi(\beta) = \frac{1}{p}\beta^{1/(p-1)}\left(\frac{f^p(x)}{F}\right) + (1-\frac{1}{p})\beta^{1/p}\left(\frac{g^q(x)}{G}\right), \tag{10}$$

where $F = \int_a^b f^p(x)\mathrm{d}x$, $G = \int_a^b g^q(x)\mathrm{d}x$.

Only consider the case of $0 < p < 1$. We obtain that

$$\max[\Phi(\beta)] = \Phi\left(\frac{Gf^p(x)}{Fg^q(x)}\right) = \frac{f(x)g(x)}{\left(\int_a^b f^p(x)\mathrm{d}x\right)^{1/p}\left(\int_a^b g^q(x)\mathrm{d}x\right)^{1/q}}, \tag{11}$$

and because

$$\max[\Phi(\beta)] \geq \Phi(\beta), \tag{12}$$

integrating both sides of (12) over x form a to b and in view of (10) and (11), we can get (9).

Similarly, we can also get the following

Lemma 4 *If $x_i > 0, y_i > 0$ and $0 < p < 1, \frac{1}{p} + \frac{1}{q} = 1$, then for $\beta > 0$,*

$$\frac{1}{C(p,\beta)}\sum_{i=1}^{n} x_i y_i \geq \left(\sum_{i=1}^{n} x_i^p\right)^{1/p}\left(\sum_{i=1}^{n} y_i^q\right)^{1/q}, \tag{13}$$

where $C(p,\beta)$ is as in Lemma 3. The inequality is reversed if $p > 1$.

3. Main results

Our main results are given in the following theorems.

Theorem 1 *Under the hypotheses of Theorem A, assume $0 < p \leq 1$, $0 < q \leq 1$ and $\frac{1}{h} + \frac{1}{l} = 1$, $0 < h < 1$. Let Φ, Ψ be two real-valued nonnegative, concave and monotonely increasing functions defined on $R^+ = [0,\infty)$. We denote integrated functions $\underbrace{\Phi(\cdots\Phi}_{M}(x))$ and $\underbrace{\Psi(\cdots\Psi}_{N}(y))$ by $\Phi^M(x)$ and $\Psi^N(y)$, respectively, where M, N are natural numbers. Then for $\alpha > 0, \beta_i > 0$ $(i = 1,2,3,4)$,*

$$\sum_{m=1}^{k}\sum_{n=1}^{r}\frac{hlA_mB_n\cdot\Phi^M(\frac{A_m^{p-1}}{p})\cdot\Psi^N(\frac{B_n^{q-1}}{q})}{\inf_{\alpha>0}(ln\cdot\alpha^{-1/l} + hm\cdot\alpha^{1/h})} \geq k^{1/l}r^{1/h}\prod_{i=1}^{4}C(h,\beta_i)\times$$
$$\left(\sum_{m=1}^{k}\left(a_m\cdot\Phi^M(A_m^{p-1})\right)^h(k-m+1)\right)^{1/h}\left(\sum_{n=1}^{r}\left(b_n\cdot\Psi^N(B_n^{q-1})\right)^l(r-n+1)\right)^{1/l} \tag{14}$$

where $C(h,\beta_i) = \frac{1}{h}\beta_i^{\frac{1}{h}-1} + (1-\frac{1}{h})\beta_i^{1/h}$.

Proof From (5), we obtain that

$$\sum_{m=0}^{k-1}(A_{m+1}^p - A_m^p) = A_k^p \geq p\sum_{m=0}^{k-1}a_{m+1}A_{m+1}^{p-1} = p\sum_{m=1}^{k}a_mA_m^{p-1},$$

where $0 < p \leq 1$.

Hence

$$\frac{A_m^{p-1}}{p} \geq \frac{\sum_{s=1}^m a_s A_s^{p-1}}{\sum_{s=1}^m a_s}. \tag{15}$$

From the hypotheses of Theorem 1 and in view of Jensen inequality[8] and the special case of Lemma 4, we have for $\beta_1 > 0$

$$\Phi^M\left(\frac{A_m^{p-1}}{p}\right) \geq \frac{1}{A_m} C(h, \beta_1) m^{1/l} \left(\sum_{s=1}^m \left(a_s \cdot \Phi^M(A_s^{p-1})\right)^h\right)^{1/h}. \tag{16}$$

Similarly, for $0 < q \leq 1$ and $\beta_2 > 0$

$$\Psi^N\left(\frac{B_n^{q-1}}{q}\right) \geq \frac{1}{B_n} C(h, \beta_2) n^{1/h} \left(\sum_{t=1}^n \left(b_t \cdot \Psi^N(B_t^{q-1})\right)^l\right)^{1/l}. \tag{17}$$

From (16),(17) and in view of (7), we obtain that

$$\begin{aligned} &\frac{hlA_m B_n \cdot \Phi^M(\frac{A_m^{p-1}}{p}) \cdot \Psi^N(\frac{B_n^{q-1}}{q})}{\inf_{\alpha>0}(ln \cdot \alpha^{-1/l} + hm \cdot \alpha^{1/h})} \\ &\geq \prod_{i=1}^2 C(h, \beta_i) \left(\sum_{s=1}^m \left(a_s \cdot \Phi^M(A_s^{p-1})\right)^h\right)^{1/h} \left(\sum_{t=1}^n \left(b_t \cdot \Psi^N(B_t^{q-1})\right)^l\right)^{1/l}. \end{aligned} \tag{18}$$

Taking sum in both sides of (18) over n from 1 to r first and then taking the sum for the resulting inequality over m from 1 to k, by using Lemma 4 and interchanging the order of the summations[5], we have for $\beta_3 > 0, \beta_4 > 0$

$$\begin{aligned} &\sum_{m=1}^k \sum_{n=1}^r \frac{hlA_m B_n \cdot \Phi^M(\frac{A_m^{p-1}}{p}) \cdot \Psi^N(\frac{B_n^{q-1}}{q})}{\inf_{\alpha>0}(ln \cdot \alpha^{-1/l} + hm \cdot \alpha^{1/h})} \\ &\geq \prod_{i=1}^2 C(h, \beta_i) \sum_{m=1}^k \left(\sum_{s=1}^m \left(a_s \cdot \Phi^M(A_s^{p-1})\right)^h\right)^{1/h} \cdot \sum_{n=1}^r \left(\sum_{t=1}^n \left(b_t \cdot \Psi^N(B_t^{q-1})\right)^l\right)^{1/l} \\ &\geq k^{1/l} r^{1/h} \prod_{i=1}^4 C(h, \beta_i) \left(\sum_{m=1}^k \sum_{s=1}^m \left(a_s \cdot \Phi^M(A_s^{p-1})\right)^h\right)^{1/h} \left(\sum_{n=1}^r \sum_{t=1}^n \left(b_t \cdot \Psi^N(B_t^{q-1})\right)^l\right)^{1/l} \\ &= k^{1/l} r^{1/h} \prod_{i=1}^4 C(h, \beta_i) \left(\sum_{m=1}^k \left(a_m \cdot \Phi^M(A_m^{p-1})\right)^h (k - m + 1)\right)^{1/h} \times \\ &\quad \left(\sum_{n=1}^r \left(b_n \cdot \Psi^N(B_n^{q-1})\right)^l (r - n + 1)\right)^{1/l}. \end{aligned}$$

This completes the proof.

Remark 1 By the same steps as in the proof of Theorem 1 with suitable modification, and in view of inequalities (6) and (8) and the inverse inequality of (13), we can get the inverse inequality of (14) as follows:

Corollary 1 *Under the hypotheses of Theorem 1, let h,p,q change to $h>1, p\geq 1, q\geq 1$ and let Φ and Ψ be two real-valued nonnegative, convex and monotonely increasing functions defined on $R^+=[0,\infty)$. Then for $\alpha>0, \beta_i>0$*

$$\sum_{m=1}^{k}\sum_{n=1}^{r}\frac{hlA_mB_n\cdot\Phi^M(\frac{A_m^{p-1}}{p})\cdot\Psi^N(\frac{B_n^{q-1}}{q})}{\inf_{\alpha>0}(ln\cdot\alpha^{-1/l}+hm\cdot\alpha^{1/h})}\leq k^{1/l}r^{1/h}\prod_{i=1}^{4}C(h,\beta_i)\times$$
$$\left(\sum_{m=1}^{k}\left(a_m\cdot\Phi^M(A_m^{p-1})\right)^h(k-m+1)\right)^{1/h}\left(\sum_{n=1}^{r}\left(b_n\cdot\Psi^N(B_n^{q-1})\right)^l(r-n+1)\right)^{1/l} \quad (19)$$

In (19), when $M=N=1, h=l=2, \beta_i=1(i=1,2,3,4), \Phi(x)=x, \Psi(y)=y$, then the inequality (19) reduces to the following inequality

$$\sum_{m=1}^{k}\sum_{n=1}^{r}\frac{A_m^pB_n^q}{m^{1/2}n^{1/2}}$$
$$\leq pq\sqrt{kr}\left(\sum_{m=1}^{k}\left(a_mA_m^{p-1}\right)^2(k-m+1)\right)^{1/2}\left(\sum_{n=1}^{r}\left(b_nB_n^{q-1}\right)^2(r-n+1)\right)^{1/2}. \quad (20)$$

Since that $m+n\geq 2m^{1/2}n^{1/2}(m>0, n>0)$, inequality (20) is just a strengthened form of inequality (1).

Theorem 2 Let p,q,h and l be as in Theorem 1, $f(\sigma)>0, g(\tau)>0$ and $F(s)=\int_0^s f(\sigma)d\sigma, G(t)=\int_0^t g(\tau)d\tau$, *where $\sigma,s\in(0,x)$, $\tau,t\in(0,y)$ and x, y are two positive real numbers. Let Φ and Ψ are two real-valued nonnegative and concave functions defined on R_+. Then for $\alpha>0,\beta_i>0$*

$$\int_0^x\int_0^y\frac{hlF(s)G(t)\Phi\left(\frac{F^{p-1}(s)}{p}\right)\Psi\left(\frac{G^{q-1}(t)}{q}\right)}{\inf_{\alpha>0}(lt\cdot\alpha^{-1/l}+hs\cdot\alpha^{1/h})}\mathrm{d}s\mathrm{d}t$$
$$\geq x^{1/l}y^{1/h}\prod_{i=1}^{4}C(h,\beta_i)\cdot\left(\int_0^x(x-s)\Big(f(s)\cdot\Phi\big(F^{p-1}(s)\big)\Big)^h\mathrm{d}s\right)^{1/h}\times$$
$$\left(\int_0^y(y-t)\Big(g(t)\cdot\Psi\big(G^{q-1}(t)\big)\Big)^l\mathrm{d}t\right)^{1/l}, \quad (21)$$

where $C(h,\beta_i)=\frac{1}{h}\beta_i^{\frac{1}{h}-1}+(1-\frac{1}{h})\beta_i^{1/h}$.

Proof From the hypotheses, it is easy to observe that

$$\frac{F^{p-1}(s)}{p}=\frac{\int_0^s F^{p-1}(\sigma)f(\sigma)\mathrm{d}\sigma}{\int_0^s f(\sigma)\mathrm{d}\sigma}, s\in(0,x).$$

On the other hand, in view of Jensen integral inequality and Lemma 3, it turns out that

$$\Phi\left(\frac{F^{p-1}(s)}{p}\right) \geq \frac{1}{F(s)}\int_0^s \Phi\left(F^{p-1}(\sigma)\right)\cdot f(\sigma)\mathrm{d}\sigma$$
$$\geq \frac{1}{F(s)}C(h,\beta_1)s^{1/l}\left(\int_0^s \left(f(\sigma)\cdot\Phi\left(F^{p-1}(\sigma)\right)\right)^h \mathrm{d}\sigma\right)^{1/h}, \qquad (22)$$

and similarly,

$$\Psi\left(\frac{G^{q-1}(t)}{q}\right) \geq \frac{1}{G(t)}C(h,\beta_2)t^{1/h}\left(\int_0^t \left(g(\tau)\cdot\Psi\left(G^{q-1}(\tau)\right)\right)^l \mathrm{d}\tau\right)^{1/l}. \qquad (23)$$

From (22),(23) and in view of (7), we have

$$\frac{hlF(s)G(t)\Phi\left(\frac{F^{p-1}(s)}{p}\right)\Psi\left(\frac{G^{q-1}(t)}{q}\right)}{\inf_{\alpha>0}(lt\cdot\alpha^{-1/l}+hs\cdot\alpha^{1/h})} \geq \prod_{i=1}^{2}C(h,\beta_i)\times$$
$$\left(\int_0^s \left(f(\sigma)\cdot\Phi\left(F^{p-1}(\sigma)\right)\right)^h \mathrm{d}\sigma\right)^{1/h}\left(\int_0^t \left(g(\tau)\cdot\Psi\left(G^{q-1}(\tau)\right)\right)^l \mathrm{d}\tau\right)^{1/l}. \qquad (24)$$

Integrating both sides of (24) over t from 0 to y and then integrating the resulting inequality over s from 0 to x, and using again Lemma 3, we observe that for $\beta_3>0,\beta_4>0$

$$\int_0^x\int_0^y \frac{hlF(s)G(t)\Phi\left(\frac{F^{p-1}(s)}{p}\right)\Psi\left(\frac{G^{q-1}(t)}{q}\right)}{\inf_{\alpha>0}(lt\cdot\alpha^{-1/l}+hs\cdot\alpha^{1/h})}\mathrm{d}s\mathrm{d}t$$
$$\geq \prod_{i=1}^{2}C(h,\beta_i)\int_0^x\left(\int_0^s \left(f(\sigma)\cdot\Phi\left(F^{p-1}(\sigma)\right)\right)^h \mathrm{d}\sigma\right)^{1/h}\mathrm{d}s\times$$
$$\int_0^y\left(\int_0^t \left(g(\tau)\cdot\Psi\left(G^{q-1}(\tau)\right)\right)^l \mathrm{d}\tau\right)^{1/l}\mathrm{d}t$$
$$\geq \prod_{i=1}^{4}C(h,\beta_i)\cdot x^{1/l}\left(\int_0^x\left(\int_0^s \left(f(\sigma)\cdot\Phi\left(F^{p-1}(\sigma)\right)\right)^h \mathrm{d}\sigma\right)\mathrm{d}s\right)^{1/h}\times$$
$$y^{1/h}\left(\int_0^y\left(\int_0^t \left(g(\tau)\cdot\Psi\left(G^{q-1}(\tau)\right)\right)^l \mathrm{d}\tau\right)\mathrm{d}t\right)^{1/l}$$
$$= x^{1/l}y^{1/h}\prod_{i=1}^{4}C(h,\beta_i)\left(\int_0^x (x-s)\left(f(s)\cdot\Phi\left(F^{p-1}(s)\right)\right)^h\mathrm{d}s\right)^{1/h}\times$$
$$\left(\int_0^y (y-t)\left(g(t)\cdot\Psi\left(G^{q-1}(t)\right)\right)^l\mathrm{d}t\right)^{1/l}.$$

This completes the proof.

Remark 2 From the proof of Theorem 2 and in view of inequalities (6) and (8) and the inverse inequality of (9), we can get the inverse inequality of (21) as follows:

Corollary 2 *Under the hypotheses of Theorem 2, if* $0 < p \le 1, 0 < q \le 1$ *and* $0 < h < 1$ *change to* $p \ge 1, q \ge 1$ *and* $h > 1$, Φ *and* Ψ *are two real-valued nonnegative and convex functions. Then*

$$\int_0^x \int_0^y \frac{hlF(s)G(t)\Phi\left(\frac{F^{p-1}(s)}{p}\right)\Psi\left(\frac{G^{q-1}(t)}{q}\right)}{\inf_{\alpha>0}(lt\cdot\alpha^{-1/l}+hs\cdot\alpha^{1/h})}\mathrm{d}s\mathrm{d}t$$
$$\le x^{1/l}y^{1/h}\prod_{i=1}^{4}C(h,\beta_i)\cdot\left(\int_0^x(x-s)\Big(f(s)\cdot\Phi\big(F^{p-1}(s)\big)\Big)^h\mathrm{d}s\right)^{1/h}\times$$
$$\left(\int_0^y(y-t)\Big(g(t)\cdot\Psi\big(G^{q-1}(t)\big)\Big)^l\mathrm{d}t\right)^{1/l}. \tag{25}$$

In (25), if $h = l = 2, \alpha = 1, \beta_i = 1$ $(i = 1,2,3,4), \Phi(x) = x$ and $\Psi(y) = y$, then we obtain that the inequality (25) reduces to a strengthened form of the inequality (3).

Similarly, by the same method, we can also establish strengthed and reversed inequalities for other inequalities given by Pachpatte [6].

Finally, we give an example as follows:

Estimate the following double integration

$$H = \int_0^1\int_0^1 \frac{(e^s-1)(e^t-1)}{2t-s}\mathrm{d}s\mathrm{d}t, \quad t > s/2, \tag{26}$$

where $s \in (0,1), t \in (0,1)$.

In fact, we can change (26) to

$$H = \frac{1}{\Phi(1)\cdot\Psi(1)}\times$$
$$\int_0^1\int_0^1\frac{1/2\cdot(-1)(e^s-1)(e^t-1)\cdot\Phi\left(\frac{(e^s-1)^{1-1}}{1}\right)\cdot\Psi\left(\frac{(e^t-1)^{1-1}}{1}\right)}{(-1)\cdot t\cdot 1^{(-1)/(-1)}+1/2\cdot s\cdot 1^2}\mathrm{d}s\mathrm{d}t. \tag{27}$$

Let $f(\sigma) = e^\sigma, g(\tau) = e^\tau, h = 1/2, p = q = 1, \alpha = 1$, and $x = y = 1$, Then

$$H = \frac{1}{\Phi(1)\cdot\Psi(1)}\int_0^x\int_0^y\frac{hlF(s)G(t)\cdot\Phi\left(\frac{F^{p-1}(s)}{p}\right)\cdot\Psi\left(\frac{G^{q-1}(t)}{q}\right)}{\inf_{\alpha>0}(lt\cdot\alpha^{-1/l}+hs\cdot\alpha^{1/h})}\mathrm{d}s\mathrm{d}t.$$

By using the inequality (21), we have for $\beta_i = 1 (i = 1,2,3,4)$

$$H \ge \frac{1}{\Phi(1)\cdot\Psi(1)}\left(\int_0^1(1-s)(e^s\cdot\Phi(1))^{1/2}\mathrm{d}s\right)^2\times\left(\int_0^1(1-t)(e^t\cdot\Psi(1))^{-1}\mathrm{d}t\right)^{-1}$$
$$= \left(\int_0^1(1-s)e^{s/2}\mathrm{d}s\right)^2\times\left(\int_0^1(1-t)e^{-t}\right)^{-1}\mathrm{d}s = 4e(2e^{1/2}-3)^2.$$

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一些 Pachpatte 不等式的加强与反向形式

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摘 要: 本文建立了一些新的加强和反向 Pachpatte 不等式. 作为应用，推广和加强了一些新型 Hilbert 不等式.

关键词: 反向不等式；积分不等式； Jensen 不等式.