Journal of Mathematical Research & Exposition
Vol.24, No.1, 29-36, February, 2004

Optimal Boundary Control for a Parabolic and Hyperbolic Coupled System *

KE Yuan-yuan[1], *YANG Wei*[2]
(1. Dept. of Math., Jilin University, Changchun 130012, China;
2. Inst. of Chemistry Defense, P.L.A., 102205, China)

Abstract: We discuss the third boundary-value optimal control problem governed by a parabolic and hyperbolic coupled system. We establish the existence of the optimal control and prove that the optimal control is bang-bang.

Key words: third boundary value; parablic-hyperbolic system; optimal control; bang-bang.

Classification: AMS(2000) 35K99, 35M20, 93A30/CLC number: O175.28

Document code: A **Article ID:** 1000-341X(2004)01-0029-08

1. Introduction

In this paper, we are concerned with the third boundary-value optimal control problem governed by the following parabolic and hyperbolic coupled system

$$\partial u_\varepsilon/\partial t - \Delta u_\varepsilon = 0, \quad x = (x_1, x_2) \in \Omega_\varepsilon(t), \tag{1.1}$$

$$\partial f_\varepsilon/\partial t = p u_\varepsilon/\{1 + [\partial(\varepsilon f_\varepsilon)/\partial x_1]^2\}^{1/2}, \quad \text{on } \Gamma_\varepsilon(t), \tag{1.2}$$

subject to the initial value conditions

$$u_\varepsilon(x, 0) = 0, \quad f_\varepsilon(x_1, 0) = f_0(x_1, x_1/\varepsilon), \tag{1.3}$$

and the boundary value conditions

$$\partial u_\varepsilon/\partial \nu + \alpha u_\varepsilon = g, \quad \text{on } \Gamma_b = \{(x_1, b); -a < x_1 < a\}, \tag{1.4}$$

$$\partial u_\varepsilon/\partial x_1 = 0, \quad \text{on } x_1 = \pm a, \tag{1.5}$$

$$\partial u_\varepsilon/\partial \nu + p(x_1, x_1/\varepsilon) u_\varepsilon = 0, \quad \text{on } \Gamma_\varepsilon(t), \tag{1.6}$$

*Received date: 2002-06-17
Foundation item: Supported by the Doctoral Program Foundation of the Education Ministry of China (20030183010).
Biography: KE Yuan-yuan (1978-), female, Ph.D. student.

© 1995-2005 Tsinghua Tongfang Optical Disc Co., Ltd. All rights reserved.

where $\Omega_\varepsilon(t)=\{(x_1,x_2)\in\Omega_0;\varepsilon f_\varepsilon(x_1,t)<x_2<b\}$, $\Omega_0=\{(x_1,x_2);-a<x_1<a,0<x_2<b\}$, $\Gamma_\varepsilon(t)=\{x_2=\varepsilon f_\varepsilon(x_1,t),-a<x_1<a\}$, ε is a small positive parameter of the same order of magnitude as the feature size, ν denotes the outward normal to the boundary $\Gamma_\varepsilon(t)$, α is a coefficient. The basic assumptions on p and f_0 are

(H1) $p(x_1,\xi_1)$ and $f_0(x_1,\xi_1)$ are smooth functions in $(x_1,\xi_1)\in[-a,a]\times\mathbf{R}$, 1-periodic in ξ_1;

(H2) $p_{x_1}(\pm a,\xi_1)=p_{\xi_1}(\pm a,\xi_1)\equiv 0$, $(f_0)_{x_1}(\pm a,\xi_1)=(f_0)_{\xi_1}(\pm a,\xi_1)\equiv 0$;

(H3) $f_0(x_1,\xi_1)\geq 0$, there exists a $p_0>0$, such that $p(x_1,\xi_1)\geq p_0$.

The problem arises from a simple model of chemical vapor deposition on a silicon wafer. In this model, u_ε and f_ε, which represent the concentration of one active species and the surface of the moving boundary respectirely, are all the condition variables. The control variable $g=g(x_1,t)$, which can control both the concentration and the speed of the flux, is defined in the rectangle $Q=\{(x_1,t);-a\leq x_1\leq a,0\leq t\leq T\}$. We want to choose the control variable g in the admissible control set $\mathcal{U}=\{g\in L^\infty(Q);0\leq g\leq M\}$, so that the surface of the film will approximate a prescribed profile. For this reason, the cost functional is chosen as

$$J(g)=\int_0^T\int_{-a}^{a}\psi(t)(f_\varepsilon(x_1,t)-c_\varepsilon(x_1,t))^2\mathrm{d}x_1\mathrm{d}t,$$

where $c_\varepsilon(x_1,t)$ is the given profile. So the problem can be formulated as

$$(\mathrm{P})\quad \min\{J(g);g\in\mathcal{U}\},\quad \text{where}\quad (f_\varepsilon,u_\varepsilon)\text{ solves }(1.1)-(1.6).$$

Optimal control problems for chemical vapor deposition reactors were considered by several authors. The model in [6] is based on ODE and do not incorporate the moving surface of the film. Belyaev[4] investigated the model of stationary film surface $x_2=\varepsilon f(x_1,x_1/\varepsilon)$, by assuming that u satisfies the Poisson equation. More recently, Friedman, Hu and Liu[10] extend the results of Belyaev to a more general three scales stationary boundary, $x_2=f_0(x_1)+\varepsilon f_1(x_1,x_1/\varepsilon)+\varepsilon^2f_2(x_1,x_1/\varepsilon,x_1/\varepsilon^2)$. The matched asymptotic expansion was employed in [7] and [8] to study the same problem. In [8] and [9] the authors describe the physical aspects of the model. [11] and [12] provide the theoretical framework for this work, which is meant as the first step towards a practical simulator for chemical vapor deposition in the context of semiconductor manufacturing.

It was Friedman and Hu[1] who first considered the optimal control problem by PDE model with non-stationary moving boundary. They investigated the second boundary value control problem, namely, the control variable is the flux of the chemical vapor. Using a parabolic variational inequality they deduced that the optimal control is bang-bang. Our paper is concerned with the third boundary value optimal control problem. This work can be regarded as a natural continuation of [1]. From the practical point of view, it is natural to control both the flux and the concentration of the chemical vapor. Moreover, from the mathematical point of view, the third boundary value problem differs from the second boundary value problem in several aspects. So, for the treatment of the current problem, we need not only to utilize the known technique, but also to tackle the new aspect among the problem.

© 1995-2005 Tsinghua Tongfang Optical Disc Co., Ltd. All rights reserved.

The purpose of this paper is to prove the existence of the optimal control, and to prove that the optimal control is bang-bang. In order to get the existence of the optimal control, we should firstly consider the boundary value condition for fixed g and establish the existence and uniqueness of the corresponding problem. Due to the weakness of the smoothness of the control g, the classical theory for parabolic equation cannot be used directly. However, we may utilize the result about the second boundary value established by Friedman and Hu[2]. In other words, we get the existence of solutions of (1.1)–(1.6), based on the fixed-point theorem and the Schauder type estimates. In order to consider the properties of the minimizers, the problem is formulated in terms of its homogenized approximation. We establish the homogenized approximation of the system (1.1)–(1.6), and then obtain an L^∞ estimate on the error between the free boundary and the homogenized boundary. Based on the estimate we prove the existence of the optimal control. After that we use the method similar to Friedman and Hu[1] to discuss the properties of the optimal control g_0, and get that g_0 is bang-bang.

Our work is organized as follows. Section 2 collects the preliminaries and statements of results. The proofs of theorems will be given subsequently in Section 3.

2. Preliminaries and main results

As a preliminary, we first have

Theorem 2.1 *For any fixed $g \in \mathcal{U}$, there exists a unique solution (u_ϵ, f_ϵ) of the problem (1.1)–(1.6).*

Since the problem (1.1)–(1.6) relates a moving boundary, it is not so easy to show the existence of the optimal control, so we consider the homogenized approximation as follows:

$$\partial u/\partial t - \Delta u = 0, \quad x \in \Omega_0, \tag{2.1}$$

$$\partial u/\partial \nu + \alpha u = g(x_1, t), \quad x \in \Gamma_b = \{x_2 = b\}, \tag{2.2}$$

$$\partial u/\partial x_1 = 0, \quad x_1 = \pm a, \tag{2.3}$$

$$\partial u/\partial \nu + Pu = 0, \quad x \in \Gamma = \{x_2 = 0\}, \tag{2.4}$$

$$u(x, 0) = 0, \quad x \in \Omega_0, \tag{2.5}$$

$$\partial f(x_1, \xi_1, t)/\partial t = p(x_1, \xi_1, t)u(x_1, 0, t)\{1 + [\partial f(x_1, \xi_1, t)/\partial \xi_1]^2\}^{1/2}, \tag{2.6}$$

$$f(x_1, \xi_1, 0) = f_0(x_1, \xi_1), \tag{2.7}$$

where

$$P(x_1, t) = \int_0^1 p(x_1, \xi_1)\{1 + [\partial f(x_1, \xi_1, t)/\partial \xi_1]^2\}^{1/2} \mathrm{d}\xi_1. \tag{2.8}$$

The corresponding problem is then changed to

$$(\mathrm{P}') \quad \min\{\tilde{J}[g]; g \in \mathcal{U}\}, \quad \text{where} \quad (f, u) \quad \text{is the solution of } (2.1) - -(2.8),$$

$$\tilde{J}[g] = \int_0^T \int_{-a}^a \int_0^1 \psi(t)\Big(f(x_1, \xi_1, t) - c(x_1, \xi_1, t)\Big)^2 \mathrm{d}\xi_1 \mathrm{d}x_1 \mathrm{d}t,$$

© 1995-2005 Tsinghua Tongfang Optical Disc Co., Ltd. All rights reserved.

where $c(x_1,\xi_1,t)$ is a continuous positive function for $x_1 \in [-a,a]$, $\xi_1 \in \mathbf{R}$, $0 \le t \le T$, 1-periodic in ξ_1.

Similar to Theorem 2.1, we have

Theorem 2.2 *For any fixed $g \in \mathcal{U}$, there exists a unique solution (u,f) of the problem (2.1)–(2.8).*

For the error estimate between $(u_\varepsilon, f_\varepsilon)$ and (u,f), we have

Theorem 2.3 *Let $(u_\varepsilon, f_\varepsilon)$ be the solution of (1.1)–(1.6), and (u,f) be the solution of (2.1)–(2.8). Then*

$$\max_{0\le t\le T} \|u_\varepsilon - u\|_{L^2(\Omega_\varepsilon(t))} \le C\varepsilon,$$

$$\max_{0\le t\le T} \|f_\varepsilon(x_1,t) - f(x_1, x_1/\varepsilon, t)\|_{L^2(\Omega_\varepsilon(t))} \le C\varepsilon.$$

Theorem 2.4 *There exist at least one $g^* \in \mathcal{U}$, which minimizes $\tilde{J}(g)$.*

Theorem 2.5 *Let (g_0, u^0) be the solution of (P'), and assume*

$$\int_{-a}^{a} \sigma^2(x_1,t)\mathrm{d}x_1 \ne 0 \quad \text{for a.e. } t \in (0,T). \tag{2.9}$$

Then

$$g_0 = M\chi_A, \tag{2.10}$$

where A is the subset of Q, and

$$\sigma(x_1,t) = \psi(t)\int_0^1 [G(x_1,\xi_1,\int_0^t u^0(x_1,0,\tau)\mathrm{d}\tau) - c(x_1,\xi_1,t)] \times G_s(x_1,\xi_1,\int_0^t u^0(x_1,0,\tau)\mathrm{d}\tau)\mathrm{d}\xi_1,$$

$$\partial G/\partial s = p(x_1,\xi_1)[1+G_{\xi_1}^2]^{1/2}, \quad 0 \le s \le S,$$

$$G(x_1,\xi_1,0) = f_0(x_1,\xi_1).$$

Theorem 2.6 *Under the assumptions of Theorem 2.5, there is a $\lambda \le 0$, so that*

$$g_0 = M, \quad \text{a.e. on } A = \{(x_1,t) \in Q, W(x_1,b,t) < \lambda\}, \tag{2.11}$$

$$g_0 = 0, \quad \text{a.e. on } A = \{(x_1,t) \in Q, W(x_1,b,t) > \lambda\}; \tag{2.12}$$

and when $\lambda = 0$, (P') get the minimum.

3. Proofs of the main results

We are now in a position to prove our main results. Considering the limitation of length of the paper, we give the proofs only on Theorem 2.1 and 2.4. The ideas of the proofs Theorem 2.2, 2.3, 2.5 and 2.6 are similar to those in [1] and [2], and hence the details are omitted.

Proof of Theorem 2.1 Based on the results established by Friedman and Hu[2], the

© 1995-2005 Tsinghua Tongfang Optical Disc Co., Ltd. All rights reserved.

proof can be processed by applying the fixed point theorem. For this purpose, we need to construct an iterate sequence of solutions. Without loss of generality, we assume that $ab \le 1, \alpha < 1$, otherwise, we may use the rescalling technique (replacing x by x/ε and t by t/ε^2). Let $g \in \mathcal{U}$ be fixed. First, we choose $u_0 \in L^\infty(Q_{\varepsilon T})$. By deduction, if for some fixed n, $u_{n-1} \in L^\infty(Q_{\varepsilon T})$ is determined, then the result of Friedman and Hu [2] gives a pair (u_n, f_n) satisfying the following system

$$\partial u_n/\partial t - \Delta u_n = 0, \quad x \in \Omega_\varepsilon(t), \tag{3.1}$$

$$\partial u_n/\partial \nu = g - \alpha u_{n-1}, \quad x \in \Gamma_b = \{(x_1, b); -a < x_1 < a\}, \tag{3.2}$$

$$\partial u_n/\partial x_1 = 0, \quad x_1 = \pm a, \tag{3.3}$$

$$\partial u_n/\partial \nu + p u_n = 0, \quad x \in \Gamma_\varepsilon(t), \tag{3.4}$$

$$u_n(x, 0) = 0, \tag{3.5}$$

$$\partial f_n/\partial t = p u_n [1 + (\partial(\varepsilon f_n)/\partial x_1)^2]^{1/2}, \quad x \in \Gamma_\varepsilon(t), \tag{3.6}$$

$$f_n(x_1, 0) = f_0(x_1, x_1/\varepsilon), \tag{3.7}$$

and the following estimates hold

$$||f_n||_\infty \le C, \quad ||(\varepsilon f_n)_{x_1}||_\infty \le C, \quad ||(f_n)_t||_\infty \le C, \tag{3.8}$$

$$||u_n||_\infty \le C^*(||g||_\infty + ||\alpha u_{n-1}||_\infty), \tag{3.9}$$

where C and C^* are two constants independent of n. Since $ab < 1$ and $\alpha < 1$, we can obtain $C^* < 1$. Then

$$||u_n||_\infty \le \frac{C^*}{1 - \alpha C^*}||g||_\infty + ||u_0||_\infty \le C||g||_\infty + ||u_0||_\infty. \tag{3.10}$$

Now, we estimate the Hölder norm of u_n and f_n. Let $w_1 = \partial u_n/\partial x_1$ and $w_2 = \partial u_n/\partial x_2$. By the results above, we then find that $||w_1||_\infty \le C, \quad ||w_2||_\infty \le C$. It is clear that

$$||\nabla u_n||_\infty \le C. \tag{3.11}$$

Consequently, when n is large enough, we can derive that

$$|u_n(x_1, x_2, t) - u_n(y_1, y_2, t)| \le C(|x_1 - y_1| + |x_2 - y_2|). \tag{3.12}$$

For all $t, s \in [0, T]$, we define $|\Delta t| = |t - s|$, and let x_1, $x_1 + (\Delta t)^{1/2}$, x_2, $x_2 + (\Delta t)^{1/2} \in [-a, a]$. Integrating (3.1), we then have

$$\begin{aligned}
&\int_s^t \int_{x_1}^{x_1+(\Delta t)^{1/2}} \int_{x_2}^{x_2+(\Delta t)^{1/2}} \frac{\partial u_n}{\partial t} dy_1 dy_2 d\tau \\
&= \int_s^t \int_{x_1}^{x_1+(\Delta t)^{1/2}} \int_{x_2}^{x_2+(\Delta t)^{1/2}} [\frac{\partial^2 u_n}{\partial x_1^2} + \frac{\partial^2 u_n}{\partial x_2^2}] dy_1 dy_2 d\tau.
\end{aligned}$$

© 1995-2005 Tsinghua Tongfang Optical Disc Co., Ltd. All rights reserved.

By the estimate (3.11), it then follows that

$$
\begin{aligned}
&|\int_{x_1}^{x_1+(\Delta t)^{1/2}}\int_{x_2}^{x_2+(\Delta t)^{1/2}}(u_n(y_1,y_2,t)-u_n(y_1,y_2,s))\mathrm{d}y_1\mathrm{d}y_2|\\
&=|\int_s^t\int_{x_2}^{x_2+(\Delta t)^{1/2}}(\frac{\partial u_n}{\partial x_1}(x_1+(\Delta t)^{1/2},y_2,\tau)-\frac{\partial u_n}{\partial x_1}(x_1,y_2,\tau))\mathrm{d}y_2\mathrm{d}\tau+\\
&\quad\int_s^t\int_{x_1}^{x_1+(\Delta t)^{1/2}}(\frac{\partial u_n}{\partial x_2}(y_1,x_2+(\Delta t)^{1/2},\tau)-\frac{\partial u_n}{\partial x_2}(y_1,x_2,\tau))\mathrm{d}y_1\mathrm{d}\tau|\\
&\le 4C|\Delta t|^{3/2}.
\end{aligned}
$$

Manipulating the integral mean value theorem, we can show that there are $x_1^*\in[x_1,x_1+(\Delta t)^{1/2}]$ and $x_2^*\in[x_2,x_2+(\Delta t)^{1/2}]$, which satisfy

$$
|u_n(x_1^*,x_2^*,t)-u_n(x_1^*,x_2^*,s)|\le C|\Delta t|^{1/2}. \tag{3.13}
$$

We exploit (3.12) and (3.13), then know when n is large enough, there have

$$
\begin{aligned}
&|u_n(x_1,x_2,t)-u_n(y_1,y_2,s)|\\
&\quad\le|u_n(x_1,x_2,t)-u_n(x_1^*,x_2^*,t)|+|u_n(x_1^*,x_2^*,t)-u_n(x_1^*,x_2^*,s)|+\\
&\qquad|u_n(x_1^*,x_2^*,s)-u_n(y_1,y_2,s)|\\
&\quad\le C(|x_1-y_1|+|x_2-y_2|+|\Delta t|^{1/2}).
\end{aligned}
$$

So, there exists a subsequence of $\{u_n\}$, denoted also by $\{u_n\}$, and $u_n\to u_\varepsilon,\ \nabla u_n\overset{w^*}{\to}\nabla u_\varepsilon$. Similarly, we see that for some subsequence of $\{f_n\}$, denoted also by $\{f_n\}$,

$$
f_n\to f_\varepsilon,\quad \frac{\partial(\varepsilon f_n)}{\partial x_1}\overset{w^*}{\to}\frac{\partial(\varepsilon f_\varepsilon)}{\partial x_1}.
$$

By using the fact that (u_n,f_n) is the classical solution of (3.1)–(3.7), and taking $n\to\infty$, we then assert that $(u_\varepsilon,f_\varepsilon)$ is the weak solution of (1.1)–(1.6).

Now we prove the uniqueness. Let u_1,u_2 be two solutions of the problem. Then

$$
\begin{aligned}
&\iint_{Q_{\varepsilon T}}(\nabla u_1-\nabla u_2)\nabla\varphi\mathrm{d}x\mathrm{d}t-\iint_{Q_{\varepsilon T}}(u_1-u_2)\varphi_t\mathrm{d}x\mathrm{d}t+\\
&\quad\int_{\Gamma_\varepsilon\times(0,T)}p(u_1-u_2)\varphi\mathrm{d}s\mathrm{d}t+\int_{\Gamma_b\times(0,T)}\alpha(u_1-u_2)\varphi\mathrm{d}s\mathrm{d}t\\
&=\int_{\Omega_\varepsilon}(u_2(x,T)-u_1(x,T))\varphi(x,T)\mathrm{d}x.
\end{aligned}
$$

Choosing $\varphi=u_1-u_2$, we obtain

$$
\begin{aligned}
&\iint_{Q_{\varepsilon T}}|\nabla(u_1-u_2)|^2\mathrm{d}x\mathrm{d}t+\int_{\Gamma_\varepsilon\times(0,T)}p(u_1-u_2)^2\mathrm{d}s\mathrm{d}t+\int_{\Gamma_b\times(0,T)}\alpha(u_1-u_2)^2\mathrm{d}s\mathrm{d}t\\
&=\iint_{Q_{\varepsilon T}}(u_1-u_2)(u_1-u_2)_t\mathrm{d}x\mathrm{d}t-\int_{\Omega_\varepsilon}(u_1(x,T)-u_2(x,T))^2\mathrm{d}x,
\end{aligned}
$$

© 1995-2005 Tsinghua Tongfang Optical Disc Co., Ltd. All rights reserved.

and hence

$$\int_{\Gamma_\varepsilon\times(0,T)} p(u_1-u_2)^2\mathrm{d}s\mathrm{d}t+\int_{\Gamma_b\times(0,T)} g(u_1-u_2)^2\mathrm{d}s\mathrm{d}t$$
$$\le -\frac{1}{2}\int_{\Omega_\varepsilon}(u_1(x,T)-u_2(x,T))^2\mathrm{d}x\le 0, \tag{3.14}$$

which implies that $u_1(x,t)=u_2(x,t)$, a.e. on $\bar{Q}_{\varepsilon T}$.

The proof of uniqueness of f_ε is similar to Friedman and Hu[2]. The existence and uniqueness of solutions of (2.1)–(2.8) for fixed g can be discussed similarly.

Proof of Theorem 2.4 Let $\{g_n\}$ be a sequence in $\mathcal{U}$, for which

$$\lim_{n\to\infty}\tilde{J}(g_n)=\inf_{g\in\mathcal{U}}\tilde{J}(g).$$

By the uniqueness result, for each n, we can define (u_n,f_n) as the solution of the problem (2.1)–(2.8), with $g=g_n$. Then the following equality holds

$$\frac{1}{2}\int_{\Omega_0}u_n^2(x,T)\mathrm{d}x+\iint_{Q_T}|\nabla u_n|^2\mathrm{d}x\mathrm{d}t+\int_{\Gamma\times(0,T)}Pu_n^2\mathrm{d}s\mathrm{d}t+$$
$$\int_{\Gamma_b\times(0,T)}\alpha u_n^2\mathrm{d}s\mathrm{d}t-\int_{\Gamma_b\times(0,T)}g_nu_n\mathrm{d}s\mathrm{d}t=0. \tag{3.15}$$

Noticing that the first three terms in (3.15) are nonnegative, we can get

$$\int_{\Gamma_b\times(0,T)}u_n\mathrm{d}s\mathrm{d}t\le C,$$

where C is a positive constant independent of n. Since $g_n\in\mathcal{U}$, we have

$$\int_{\Omega_0}u_n^2(x,T)\mathrm{d}x\le C,\quad \iint_{Q_T}|\nabla u_n(x,t)|^2\mathrm{d}x\mathrm{d}t\le C,$$

where C is a positive constant independent of n. From theorem 2.1, there exists a subsequence of $\{u_n\}$, denoted also by $\{u_n\}$, and u^*, which satisfy

$$u_n\to u^*,\ \text{ a.e. on } Q_T,;\quad \nabla u_n\xrightarrow{w^*}\nabla u^*,\ \text{ on } L^2(Q_T).$$

Using Theorem 2.1, we make sure that $\|f_n\|_\infty\le C$. So $f_n\xrightarrow{w^*}f^*$. In view of the compactness result in [3], we conclude that

$$g_n\xrightarrow{*}g^*,\ \text{ on } L^\infty((0,T)\times\partial\Omega_0). \tag{3.16}$$

For $\mathcal{U}$ is closed in this topology, $g^*\in\mathcal{U}$. Letting $n\to\infty$, we see that (u^*,f^*) is the weak solution of (2.1)–(2.8), corresponding to g^*.

Now we prove g^* is the optimal control. For (u_n,f_n) is the solution, which is corresponding g_n, we have

$$\tilde{J}(g_n)=\int_0^T\int_{-a}^a\int_0^1\psi(t)(f_n(x_1,\xi_1,t)-c(x_1,\xi_1,t))^2\mathrm{d}\xi_1\mathrm{d}x_1\mathrm{d}t.$$

© 1995-2005 Tsinghua Tongfang Optical Disc Co., Ltd. All rights reserved.

Recalling (3.16), we can assert that

$$g_n \to g^*, \quad \text{on} \quad L^2((0,T) \times \partial\Omega_0).$$

At last, by the lower semicontinuity of the cost functional and using the weak convergences derived above, we see that g^* is an optimal control. The proof is completed.

Acknowledgement The author is very grateful for the useful discussion and continuous encouragement of Prof. Yin Jing-xue.

References:

[1] FRIEDMAN A, HU B. *Optimal control of a chemical vapor deposition reactor* [J]. Journal of Optimization Theory and Applications, 1998, **97**(3): 623–644.

[2] FRIEDMAN A, HU B. *A nonstationary multi-scale oscillating free boundary for the Laplace equation* [J]. Journal of Differential Equations, 1997, **137**(1): 119–165.

[3] AUBIN J P. *Un theoreme de compacité* [J]. Comptes Rendus de l'Academy de Sciences de Paris, 1963, **265**: 5042–5045.

[4] BELYAEV A G. *On Singular Perturbations of Boundary Problems* [D]. Ph. D. Thesis, Moscow State University, 1990. (In Russian)

[5] WU Z Q, ZHAO J N, YIN J X. et al. *Nonlinear Diffusion Equation* [M]. World Scientific, Singapore, 2001.

[6] MEZA J C, PLANTENGA T D. *Optimal Control of a CVD Reactor for Prescribed Temperature Behavior* [R]. SANDIA Report 95-8224, UC-406, 1995.

[7] GOBBERT M K, RINGHOFER C. *An asymptotic analysis for a model of chemical vapor deposition on a micro- structure surface* [J]. SIAM Journal on Applied Mathematics, 1998, **58**(3): 737–752.

[8] GOBBERT M K, CALE T S, RINGHOFER C. *One Approach to Combining Equipment Scale and Feature Scale Models* [C]. Proceedings of the 187th Meeting of the Electrochemical Society, Reno, Nevada, 1995, 553–563.

[9] FRIEDMAN A. *Mathematics in Industrial Problem* [M]. Part 9, Institute for Mathematics and Its Applications, Springer Verlag, New York, 1997, 88.

[10] FRIEDMAN A, HU B, LIU Y. *A boundary value problem for the Poisson equation with multi-scale ocillating boundary* [J]. J. Differential Equations, 1997, **137**(1): 54–93.

[11] DAMLAMIAN A, LI Ta-Tsien, *Boundary homogenization for elliptic problems* [J]. J. Math. Pure Appl., 1987, **66**: 351–362.

[12] KOHN R, VOGELIUS M. *A new model for thin plates with rapidly varying thickness* [J]. Internat. J. Solids Structures, 1984, **20**: 333-350.

一类由抛物 - 双曲耦合组支配的边界最优控制问题

柯 媛 元 [1]， 杨 伟 [2]

(1. 吉林大学数学学院， 吉林 长春 130012;
2. 中国人民解放军防化指挥工程学院， 北京 102205)

摘 要: 本文讨论了一类由抛物 - 双曲耦合组支配的第三边值最优控制问题. 证明了该问题最优控制的存在性及其 bang-bang 性质.

关键词: 第三边值; 抛物 - 双曲耦合组; 最优控制; bang-bang.

© 1995-2005 Tsinghua Tongfang Optical Disc Co., Ltd. All rights reserved.