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Elimination in Weyl Algebra and q-Identities *

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For the algorithmic proof of q-proper-hypergeometric identities, H.Wilf and D.Zeiberg gave a theoretical frame work. In [1], they proved that q-proper-hypergeometric terms satisfy recurrence relations with polynomial coefficients and could obtain quite explicit bounds for the order of such a recurrence. But how can we find the recurrence relations? We consider single-variable q-proper-hypergeometric identities based on Zeilberg's basic idea. To find the recurrence relations, an elimination in the non-commutative Weyl algebra has been developed. Thereby we obtained the algorithm of proving single-variable q-proper-hypergeometric identities.

To prove this type identity, $\sum_k F(q^n, q^k) = rhs(q^n)$, the most significant work is to find the recurrence equation on q^n of $F(q^n, q^k)$. If partial linear recurrence operators $P(Q_n, Q_k, q^n, q^k)$ and $Q(Q_n, Q_k, q^n, q^k)$ with polynomial coefficients are given, where Q_n and Q_k are dilation operators and both annihilate $F(q^n, q^k)$, we would like to find two operators A and B such that $R = AP + BQ$ (R is independent of q^k). Therefore R can annihilate $F(q^n, q^k)$. Thus we can get the recurrence equation satisfied by $F(q^n, q^k)$. Under dilation operator Q_n which satisfies $Q_n q^n = q \cdot q^n \cdot Q_n, Q_n(q^n)^m = (q \cdot q^n)^m Q_n, Q_n^x q^n = (q^{n+x})Q_n^x$, to eliminating q^k, we get

Theorem *Let $A, B \in C\langle Q_n, Q_k, q^n\rangle$. Then there always exist $u, v \in C\langle Q_n, Q_k, q^n\rangle$ such that $uA = vB$ and corresponding Getuv algorithm.*

We have also tested the following identities by the algorithm in Maple language

(1) q-Vandermonde-Chu identity $\sum_k q^{k^2}\binom{n}{k}^2 = \binom{2n}{n}_q$.

(2) $\sum_k \frac{q^{k^2-m^2}(q)_{n+b+c-k}}{(q)_{n-k}(q)_{b-k}(q)_{c-k}(q)_{k+m}(q)_{k-m}} = \frac{(q)_{n+b}(q)_{n+c}(q)_{b+c}}{(q)_{n+m}(q)_{b-m}(q)_{c+m}(q)_{n-m}(q)_{b+m}(q)_{c-m}}$.

(3) q-Saalschutz identity $\sum_k q^{(k-n)(k+n)}\binom{a+b+c-k}{a-k,b-k,c-k,k-n,k+n}_q = \binom{a+b}{a+n}_q\binom{a+c}{c+n}_q\binom{b+c}{b+n}_q$.

The whole paper will be appear in some journal.

References:

[1] WILF H S, ZEILBERG D. *An algorithmic proof theory for hypergeomitric (ordinary and "q") multisum/integral identities* [J]. Invent. Math., 1992, **108**: 575–633.

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