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Some Results on Completly Restricted Lie Superalgebras

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Abstract: This paper is devoted to the study of completely restricted Lie superalgebras. We give some sufficient and necessary conditions for both completely restricted Lie superalgebras and strongly completely restricted Lie superalgebras. Some other important results on completely restricted Lie superalgebras are also obtained.

Key words: completely restricted Lie superalgebras; strongly completely restricted Lie superalgebras; nondegenerate Killing form.
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1. Introduction

In the last ten years, the study of modular Lie superalgebras has obtained many important results [1,8–11]. But the complete classification of the finite-dimensional modular simple Lie superalgebras remains an open problem. The definition and the necessary and sufficient conditions for the restricted envelope of a restricted Lie superalgebra were given by V. M. Petrogradskiv in 1992[1]. If L is a modular Lie superalgebra, such that $L_{\bar{0}}$ is a restricted Lie algebra and $L_{\bar{0}}$-module $L_{\bar{1}}$ is restricted, then L is called a restricted Lie superalgebra. Zhang Yong-zheng and Wang Ying gave an equivalent definition of restricted Lie superalgebras[10]. In [2], it is shown that the classification of the simple restricted Lie algebras is equivalent to the classification of the modular simple Lie algebras. Analogously, the study of restricted Lie superalgebras will play an important role in the classification of the finite-dimensional modular simple Lie superalgebras.

The definition of a complete Lie algebra was given by N.Jacobson in 1962[6]. In recent years some important theories of complete Lie algebras over a field of positive characteristic zero have been developing by Prof. Meng Dao-ji with his students Jiang Cui-bo, Zhu Lin-sheng, Ren Bin and Wang Li-yun[12–18]. Moreover, some results on complete Lie superalgebras over a field of positive characteristic zero have been obtained in[19,20]. In this paper, we announce and prove some results on completely restricted Lie superalgebras and strongly completely restricted Lie superalgebras. We give some sufficient and necessary conditions for both completely restricted Lie superalgebras and strongly completely restricted Lie superalgebras.

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Definition 1.1[10] *Let $L = L_{\bar{0}} \bigoplus L_{\bar{1}}$ be a Lie superalgebra over* $\mathbf{F}$. *A mapping $[p] : L \to L, a \to a^{[p]}$ is called a p-mapping if*

(1) $\mathrm{ad}(a+b)^{[p]} = (\mathrm{ad}a)^p + (\mathrm{ad}b)^{2p}$ *for all $a \in L_{\bar{0}}, b \in L_{\bar{1}}$;*

(2) $(sa+tb)^{[p]} = s^p a^{[p]} + t^{2p} b^{[p]}$ *for all $a \in L_{\bar{0}}, b \in L_{\bar{1}}, s, t \in \mathbf{F}$;*

(3) $(a+b)^{[p]} = a^{[p]} + b^{[p]} + \sum_{i=1}^{p-1} s_i(a,b)$ *for all $a, b \in L_{\bar{0}}$,*

where $s_i(a,b)$ satisfies $(\mathrm{ad}(a\otimes x + b\otimes 1))^{p-1}(a\otimes 1) = \sum_{i=1}^{p-1} i s_i(a,b)\otimes x^{i-1}$ in the Lie superalgebra $L \otimes_{\mathbf{F}} \mathbf{F}[x]$. The pair $(L, [p])$ is called a restricted Lie superalgebra.

Obviously, all restricted Lie algebras are contained in restricted Lie superalgebras.

Definition 1.2 *Let $(L, [p])$ be a restricted Lie superalgebra over* $\mathbf{F}$. *A subalgebra (respectively, ideal) I of L is called a restricted subalgebra (respectively, restricted ideal) or p-subalgebra (respectively, p-ideal) of L if $x^{[p]} \in I$ for all x of I.*

Throughout this paper, let all algebras be finite-dimensional over a field $\mathbf{F}$ of positive characteristic $p \geq 3$. All ideals of Lie superalgebras are assumed to be graded. Our notation and terminology are standard as may be found in [1], [10], [12], [13].

2. Main results

Definition 2.1 *Let $(L, [p])$ be a restricted Lie superalgebra over* $\mathbf{F}$. *L is called a completely restricted Lie superalgebra if $C(L) = \{0\}$ and $\mathrm{Der}L = \mathrm{ad}L$.*

Theorem 2.1 *Let $(L, [p])$ be a restricted Lie superalgebra over* $\mathbf{F}$. *If A is a graded ideal of L, then $C_L(A)$ is graded and restricted. In particular, $C(L)$ and $C(A)$ are graded and restricted ideals of L.*

Proof If $x \in C_L(A)$ and $a \in A = A_{\bar{0}} \bigoplus A_{\bar{1}}$, then $[x, a] = 0$, where $x = x_0 + x_1, a = a_0 + a_1, x_0 \in L_{\bar{0}}, x_1 \in L_{\bar{1}}, a_0 \in A_{\bar{0}}$ and $a_1 \in A_{\bar{1}}$. Since A is graded, we have $a_0, a_1 \in A$. Then $[x_0 + x_1, a_0] = [x_0 + x_1, a_1] = 0$ and $[x_0, a_0] + [x_1, a_0] = [x_0, a_1] + [x_1, a_1] = 0$. So $[x_0, a_0], [x_1, a_1], [x_0, a_1], [x_1, a_0] \in A_{\bar{0}} \cap A_{\bar{1}}$ and $[x_0, a_0] = [x_1, a_1] = [x_0, a_1] = [x_1, a_0] = 0$. Then $[x_0, a] = [x_1, a] = 0$. Hence $x_0, x_1 \in C_L(A)$, i.e., $C_L(A)$ is graded.

Let $x \in C_L(A)$ and $y \in L$. Since $a = a_0 + a_1 \in A$, we have $a_0, a_1 \in A$ by virtue of the gradation of A. Then we have $[[x, y], a_0] = [[x_0 + x_1, y_0 + y_1], a_0] = 0$ and $[[x, y], a_1] = [[x_0 + x_1, y_0 + y_1], a_1] = 0$ by virtue of the graded Jacobi identity and $[[x, y], a] = 0$. Hence $C_L(A)$ is an ideal of L.

Let $z \in C_L(A), z = z_0 + z_1, z_0 \in L_{\bar{0}}, z_1 \in L_{\bar{1}}$. We have $z_0 \in C_L(A)$ and $z_1 \in C_L(A)$ since $C_L(A)$ is graded. Then $[z^{[p]}, A] = [z_0^{[p]} + z_1^{[p]}, A] = [z_0^{[p]}, A] + [z_1^{[p]}, A] = (\mathrm{ad}z_0)^p(A) + (\mathrm{ad}z_1)^{2p}(A) = \{0\}$. So $C_L(A)^{[p]} \subseteq C_L(A)$, i.e., $C_L(A)$ is a restricted ideal of L.

Using similar methods, we can show that $C(L)$ and $C(A)$ are graded and restricted ideals of L. □

Lemma 2.2 *Let $(L, [p])$ be a restricted Lie superalgebra over* $\mathbf{F}$. *If A is a complete p-ideal of L, then there exists a p-ideal B of L such that $L = A \bigoplus B$.*

Proof Let A be a complete p-ideal of L, then $C_L(A)$ is restricted ideal of L by means of Theorem 1.1.4, i.e., $[x, A] \subseteq A$ for any $x \in L$. So $\mathrm{ad}x|_A \in \mathrm{Der}A = \mathrm{ad}A$, i.e., there is $y \in A$ such that $\mathrm{ad}x|_A = \mathrm{ad}y$. Then $[x-y, A] = 0$ and $x - y \in C_L(A)$, i.e., $L = A + C_L(A)$. Since A is a complete ideal of L, we have $A \cap C_L(A) \subseteq C(A) = \{0\}$. Let $B = C_L(A)$. So $L = A \bigoplus B$. □

Lemma 2.3 *Let $(L, [p])$ be a restricted Lie superalgebra over $\mathbf{F}$ such that $L = A \bigoplus B$, where A and B are ideals of L. Then the following identities hold:*

(1) $C(L_{\bar{0}}) = C(A_{\bar{0}}) \bigoplus C(B_{\bar{0}})$.

(2) $C(L) = C(A) \bigoplus C(B)$.

Proof (1) Let $L_{\bar{0}} = A_{\bar{0}} \bigoplus B_{\bar{0}}$. Then $[A_{\bar{0}}, L_{\bar{0}}] = [A_{\bar{0}}, A_{\bar{0}} \bigoplus B_{\bar{0}}] \subseteq [A_{\bar{0}}, A_{\bar{0}}] + [A, B] \subseteq A_{\bar{0}}$ since $[A, B] \subseteq A \cap B = \{0\}$, i.e., $A_{\bar{0}}$ is an ideal of $L_{\bar{0}}$. By virtue of Theorem 2.1, $C(A_{\bar{0}})$ is an ideal of $L_{\bar{0}}$. Similarly, $B_{\bar{0}}$ and $C(B_{\bar{0}})$ are ideals of $L_{\bar{0}}$. Since $[C(A_{\bar{0}}) \bigoplus C(B_{\bar{0}}), L_{\bar{0}}] = [C(A_{\bar{0}}) \bigoplus C(B_{\bar{0}}), A_{\bar{0}} \bigoplus B_{\bar{0}}] = [C(A_{\bar{0}}), A_{\bar{0}}] + [C(A_{\bar{0}}), B_{\bar{0}}] + [C(B_{\bar{0}}), B_{\bar{0}}] + [C(B_{\bar{0}}), A_{\bar{0}}] \subseteq [A_{\bar{0}}, B_{\bar{0}}] = \{0\}$ by $[A, B] = \{0\}$, we have $C(A_{\bar{0}}) \bigoplus C(B_{\bar{0}}) \subseteq C(L_{\bar{0}})$. Now let $x \in C(L_{\bar{0}})$ and $x = a + b$, $a \in A_{\bar{0}}$, $b \in B_{\bar{0}}$. We have $[a, A_{\bar{0}}] = [x, A_{\bar{0}}] - [b, A_{\bar{0}}] = \{0\}$ since $[A_{\bar{0}}, B_{\bar{0}}] = \{0\}$. So $a \in C(A_{\bar{0}})$. Similarly, $b \in C(B_{\bar{0}})$. Thus $C(A_{\bar{0}}) \bigoplus C(B_{\bar{0}}) = C(L_{\bar{0}})$.

(2) By virtue of Theorem 2.1, $C(A)$ and $C(B)$ are ideals of L. Then $[A, B] = C(A) \bigcap C(B) = C(A) \bigcap B = A \bigcap C(B) = \{0\}$ since A and B are graded ideals of L and $A \bigcap B = \{0\}$. So $[C(A) \bigoplus C(B), L] = [C(A) \bigoplus C(B), A \bigoplus B] = [C(A), A] + [C(A), B] + [C(B), B] + [C(B), A] \subseteq [A, B] = \{0\}$, i.e., $C(A) \bigoplus C(B) \subseteq C(L)$. Now let $x \in C(L)$ and $x = a + b, a \in A, b \in B$. We have $[a, A] = [x, A] - [b, A] = \{0\}$ since $[A, B] = \{0\}$. So $a \in C(A)$. Similarly, $b \in C(B)$. Thus $C(A) \bigoplus C(B) = C(L)$. □

Lemma 2.4 *Let $(L, [p])$ be a restricted Lie superalgebra over $\mathbf{F}$ such that $L = A \bigoplus B$. If $C(L) = \{0\}$, then $\mathrm{ad}L = \mathrm{ad}A \bigoplus \mathrm{ad}B$ and $\mathrm{Der}L = \mathrm{Der}A \bigoplus \mathrm{Der}B$.*

Proof For any $D \in \mathrm{Der}A$, extend it to a linear transformation on L by setting $D(a + b) = D(a)$ for $a \in A$ and $b \in B$. Obviously, $D \in \mathrm{Der}L$ and $\mathrm{Der}A \subset \mathrm{Der}L$. Similarly, $\mathrm{Der}B \subset \mathrm{Der}L$. Let $a \in A_\alpha, b \in B$ and $D \in (\mathrm{Der}L)_\beta$. Then $[D(a), b] = D[a, b] - (-1)^{\alpha\beta}[a, D(b)] = -(-1)^{\alpha\beta}[a, D(b)] \in A \bigcap B$. Since $A \bigcap B = \{0\}$, $[Da, b] = [a, Db] = 0$. Let $D(a) = a_1 + b_1$, where $a_1 \in A$ and $b_1 \in B$. Then $[D(a), b] = [a_1, b_1] + [b_1, b_1] = 0$ for any $b \in B$ and $b_1 \in C(B)$. By virtue of Lemma 2.3, we have $C(L) = C(A) \bigoplus C(B) = \{0\}, b_1 = 0$. So $D(a) = a_1 \in A$. Therefore $D(A) \subseteq A$. Similarly, $D(B) \subseteq B$.

Let $D \in \mathrm{Der}L$ and $a + b \in A + B$, where $a \in A$ and $b \in B$. Define E and F by $E(a+b) = D(a)$ and $F(a + b) = D(b)$. Then $E \in \mathrm{Der}A$ and $F \in \mathrm{Der}B$. Hence $D = E + F \in \mathrm{Der}A + \mathrm{Der}B$. Since $\mathrm{Der}A \bigcap \mathrm{Der}B = \{0\}$, $\mathrm{Der}L = \mathrm{Der}A \bigoplus \mathrm{Der}B$ as a vector space. Let $D \in (\mathrm{Der}A)_\alpha, E \in (\mathrm{Der}L)_\beta$ and $b \in B$. Then $[E, D]b = EDb - (-1)^{\alpha\beta}DEb = 0$. Hence $\mathrm{Der}A$ is an ideal of $\mathrm{Der}L$. Similarly, $\mathrm{Der}B$ is an ideal of $\mathrm{Der}L$. □

Lemma 2.5 *Let $(L, [p])$ be a restricted Lie superalgebra over $\mathbf{F}$. Then $L^{[p]} \subseteq L_{\bar{0}}$ or $L^{[p]} \subseteq C(L)$.*

Proof Let $x = x_0 + x_1$, $x_0 \in L_{\bar{0}}$, $x_1 \in L_{\bar{1}}$. Then $\mathrm{ad}x^{[p]} = \mathrm{ad}(x_0 + x_1)^{[p]} = (\mathrm{ad}x_0)^p + (\mathrm{ad}x_1)^{2p}$ by Definition 1.1. For any $y \in L_\alpha$, $\alpha \in Z_2$, we have $\mathrm{ad}x^{[p]}(y) = [(x_0 + x_1)^{[p]}, y] = [x_{\bar{0}}^{[p]}, y] + [x_{\bar{1}}^{[p]}, y]$ $= (\mathrm{ad}x_0)^p(y) + (\mathrm{ad}x_1)^{2p}(y) \in L_\alpha$. We investigate into their degrees, then $x^{[p]} \in L_{\bar{0}}$ or $x^{[p]} \in C(L)$. So $L^{[p]} \subseteq L_{\bar{0}}$ or $L^{[p]} \subseteq C(L)$. □

Theorem 2.6[7] *Let $(L, [p])$ be a restricted Lie algebra over $\mathbf{F}$, and L be the direct sum of ideals $I_1, I_2, \cdots, I_n$, each of which has center zero. Then $I_1, I_2, \cdots, I_n$ are p-ideals of L.*

Theorem 2.7 *Let $(L, [p])$ be a restricted Lie superalgebra over $\mathbf{F}$ such that $C(L_{\bar{0}}) = C(L) = \{0\}$. If L is the direct sum of ideals $I_1, I_2, \cdots, I_n$, then $I_1, I_2, \cdots, I_n$ are p-ideals of L.*

Proof Let A and B be ideals of L such that $L = A \bigoplus B$. We claim that A and B are p-ideals of L. By virtue of Lemma 2.3, we have $C(L_{\bar{0}}) = C(A_{\bar{0}}) \bigoplus C(B_{\bar{0}})$ and $C(L) = C(A) \bigoplus C(B)$. So $C(A_{\bar{0}}) = C(B_{\bar{0}}) = C(A) = C(B) = \{0\}$ since $C(L_{\bar{0}}) = C(L) = \{0\}$. Since $[A_{\bar{0}}, L_{\bar{0}}] = [A_{\bar{0}}, A_{\bar{0}} \bigoplus B_{\bar{0}}] \subseteq [A_{\bar{0}}, A_{\bar{0}}] + [A, B] \subseteq A_{\bar{0}}$ by virtue of $[A, B] \subseteq A \cap B = \{0\}$, $A_{\bar{0}}$ is an ideal of $L_{\bar{0}}$. Similarly, $B_{\bar{0}}$ is an ideal of $L_{\bar{0}}$. It is clear that $L_{\bar{0}} = A_{\bar{0}} \bigoplus B_{\bar{0}}$. According to Theorem 2.6, $A_{\bar{0}}$ and $B_{\bar{0}}$ are p-ideals of $L_{\bar{0}}$.

By means of Lemma 2.5, we have $L^{[p]} \subseteq L_{\bar{0}}$ since $C(L) = \{0\}$. Then $A_{\bar{1}}^{[p]} \subseteq L_{\bar{0}}$ and $B_{\bar{1}}^{[p]} \subseteq L_{\bar{0}}$. Let $x_1, x_2, \cdots, x_n$ be a basis of $A_{\bar{0}}$ and $y_1, y_2, \cdots, y_m$ be a basis of $B_{\bar{0}}$. So there are $\alpha_1, \alpha_2, \cdots, \alpha_n$ and $\beta_1, \beta_2, \cdots, \beta_m$ such that $a^{[p]} = \sum_{i=1}^n \alpha_i x_i + \sum_{j=1}^m \beta_j y_j$ for any $a \in A_{\bar{1}}$. Therefore we have $[a^{[p]}, L] = [\sum_{i=1}^n \alpha_i x_i + \sum_{j=1}^m \beta_j y_j, A \bigoplus B] = [\sum_{i=1}^n \alpha_i x_i, A] + [\sum_{j=1}^m \beta_j y_j, B]$. Since $[a^{[p]}, L] \subseteq A$, we have $[\sum_{j=1}^m \beta_j y_j, B] = \{0\}$. Clearly, $[\sum_{j=1}^m \beta_j y_j, A] = \{0\}$. Then $[\sum_{j=1}^m \beta_j y_j, L] = [\sum_{j=1}^m \beta_j y_j, A \bigoplus B] = \{0\}$, that is, $\sum_{j=1}^m \beta_j y_j \in C(L) = \{0\}$. Hence $\sum_{j=1}^m \beta_j y_j = 0$ and $a^{[p]} = \sum_{i=1}^n \alpha_i x_i + \sum_{j=1}^m \beta_j y_j = \sum_{i=1}^n \alpha_i x_i \in A_{\bar{0}}$ for any $a \in A_{\bar{1}}$. Thus $A_{\bar{1}}^{[p]} \subseteq A_{\bar{0}}$ and $A^{[p]} \subseteq A_{\bar{0}} \subseteq A$, i.e., A is a p-ideal of L. Similarly, we can show that B is a p-ideal of L.

By induction on n, we can show that if L is the direct sum of ideals $I_1, I_2, \cdots, I_n$ and $C(L_{\bar{0}}) = C(L) = \{0\}$, then $I_1, I_2, \cdots, I_n$ are p-ideals of L. □

Definition 2.2 *Let $(L, [p])$ be a restricted Lie superalgebra over $\mathbf{F}$. L is called a strongly completely restricted Lie superalgebra if $C(L) = \{0\}$, $C(L_{\bar{0}}) = \{0\}$ and $\mathrm{Der}L = \mathrm{ad}L$.*

Theorem 2.8 *Let L be a Lie superalgebra over $\mathbf{F}$ such that $L = A \bigoplus B$, where A and B are ideals of L. Then the following statements hold:*

(1) L is a strongly completely restricted Lie superalgebra if and only if A and B are strongly complete p-ideals of L.

(2) If L and A are strongly completely restricted, then B is strongly completely restricted.

Proof (1) If A and B are strongly complete p-ideals of L, then $C(A) = C(B) = C(A_{\bar{0}}) = C(B_{\bar{0}}) = \{0\}$. So we obtain $C(L) = C(A) + C(B) = \{0\}$ and $C(L_{\bar{0}}) = C(A_{\bar{0}}) + C(B_{\bar{0}}) = \{0\}$ by Lemma 2.3.

By virtue of Lemma 2.5, we have $A_{\bar{1}}^{[p]} \subseteq C(A)$ or $A_{\bar{1}}^{[p]} \subseteq A_{\bar{0}}$, and $B_{\bar{1}}^{[p]} \subseteq C(B)$ or $B_{\bar{1}}^{[p]} \subseteq B_{\bar{0}}$. Since $C(A) = C(B) = \{0\}$, we have $A_{\bar{1}}^{[p]} \subseteq A_{\bar{0}}$ and $B_{\bar{1}}^{[p]} \subseteq B_{\bar{0}}$. Then $A^{[p]} \subseteq A_{\bar{0}}$ and $B^{[p]} \subseteq B_{\bar{0}}$.

By Definition 1.1, we have $L^{[p]} = (A \bigoplus B)^{[p]} = [(A_{\bar{0}} + B_{\bar{0}}) \bigoplus (A_{\bar{1}} + B_{\bar{1}})]^{[p]} = (A_{\bar{0}} + B_{\bar{0}})^{[p]} + (A_{\bar{1}} + B_{\bar{1}})^{[p]} \subseteq A \bigoplus B = L$. If $C(L) = \{0\}$, then $\mathrm{ad}L = \mathrm{ad}A \bigoplus \mathrm{ad}B$ and $\mathrm{Der}L = \mathrm{Der}A \bigoplus \mathrm{Der}B$ by means of Theorem 2.4. We have $\mathrm{Der}A = \mathrm{ad}A$ and $\mathrm{Der}B = \mathrm{ad}B$ since A and B are complete ideals of L, so $\mathrm{Der}L = \mathrm{ad}L$. Hence L is a strongly completely restricted Lie superalgebra.

If L is a strongly completely restricted Lie superalgebra, then $C(L) = \{0\}$ and $C(L_{\bar{0}}) = \{0\}$. By Lemma 2.3, we have $C(L) = C(A) \bigoplus C(B)$ and $C(L_{\bar{0}}) = C(A_{\bar{0}}) \bigoplus C(B_{\bar{0}})$, so $C(A) = C(B) = \{0\}$ and $C(A_{\bar{0}}) = C(B_{\bar{0}}) = \{0\}$. Since A and B are ideals of L and $C(L) = C(L_{\bar{0}}) = \{0\}$, A and B are p-ideals of L by Theorem 2.6. By means of Theorem 2.4, we have $\mathrm{ad}L = \mathrm{ad}A \bigoplus \mathrm{ad}B$ and $\mathrm{Der}L = \mathrm{Der}A \bigoplus \mathrm{Der}B$ since $C(L) = \{0\}$. Since L is completely restricted Lie superalgebra, we obtain $\mathrm{Der}L = \mathrm{ad}L$. Then $\mathrm{ad}A \bigoplus \mathrm{ad}B = \mathrm{Der}A \bigoplus \mathrm{Der}B$. So $\mathrm{Der}A = \mathrm{ad}A$ and $\mathrm{Der}B = \mathrm{ad}B$. Hence A and B are strongly completely restricted ideals of L.

(2) It is clear by (1). □

Lemma 2.9[4] *Let L be a Lie superalgebra over $\mathbf{F}$. If the Killing form on $L = L_{\bar{0}} \bigoplus L_{\bar{1}}$ is nondegenerate, then $\mathrm{Der}L = \mathrm{ad}L$, its restriction to $L_{\bar{0}}$ is nondegenerate, and L is semisimple.*

Lemma 2.10[7] *Let L be a Lie algebra over $\mathbf{F}$. If the Killing form on L is nondegenerate, then $\mathrm{Der}L = \mathrm{ad}L$ and L is semisimple.*

Theorem 2.11 *Let $(L, [p])$ be a restricted Lie superalgebra over $\mathbf{F}$. If the Killing form on $L = L_{\bar{0}} \bigoplus L_{\bar{1}}$ is nondegenerate, then L is strongly complete and $L_{\bar{0}}$ is complete.*

Proof If the Killing form on $L = L_{\bar{0}} \bigoplus L_{\bar{1}}$ is nondegenerate, then $\mathrm{Der}L = \mathrm{ad}L$, its restriction to $L_{\bar{0}}$ is nondegenerate, and L is semisimple by Lemma 2.9. So $C(L) = \{0\}$ and L is complete. By Lemma 2.10, $L_{\bar{0}}$ is semisimple, then $C(L_{\bar{0}}) = \{0\}$. Hence L is strongly complete. By virtue of Theorem 2.4, we have $\mathrm{Der}L_{\bar{0}} = \mathrm{ad}L_{\bar{0}}$ and $C(L_{\bar{0}}) = \{0\}$. Thus $L_{\bar{0}}$ is complete. □

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关于完备限制李超代数

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摘要: 我们分别给出了完备限制李超代数和强完备限制李超代数的一个充分必要条件，同时得到了一些完备限制李超代数的重要性质.

关键词: 完备限制李超代数；强完备限制李超代数；非退化 Killing 型.