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Adjacent Strong Edge Chromatic Number of Series-Parallel Graphs

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Abstract: In this paper, we will study the adjacent strong edge coloring of series-parallel graphs, and prove that series-parallel graphs of $\Delta(G) = 3$ and 4 satisfy the conjecture of adjacent strong edge coloring using the double inductions and the method of exchanging colors from the aspect of configuration property. For series-parallel graphs of $\Delta(G) \geq 5$, $\Delta(G) \leq \chi'_{as}(G) \leq \Delta(G) + 1$. Moreover, $\chi'_{as}(G) = \Delta(G) + 1$ if and only if it has two adjacent vertices of maximum degree, where $\Delta(G)$ and $\chi'_{as}(G)$ denote the maximum degree and the adjacent strong edge chromatic number of graph G respectively.

Key words: series-parallel graph; adjacent strong edge coloring; adjacent strong edge chromatic number.
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1. Introduction

The strong edge coloring of graphs comes from computer science and has a very strong background. For any graph, it is very difficult to determine its strong edge chromatic number. For graph $G(V, E)$, its k–edge coloring f is a k-partition $E = E_1 \cup E_2 \cup \cdots \cup E_k$ of edge set E of G, such that any two edges of $E_i (i = 1, 2, \cdots, k)$ are non-adjacent. For a k–edge coloring f of G, if for any $u, v \in V(G)$, $f[u] \neq f[v]$, where $f[u] = \{f(uw) | uw \in E(G)\}$, then we call f a strong edge coloring of $G(V, E)$, denoted by k-SEC and $\chi'_s(G) = \min\{k | \text{there exists a } k - SEC \text{ of } G\}$ strong edge chromatic number of graph G. For a k-edge coloring f of graph G, if for any $uv \in E(G)$, $f[u] \neq f[v]$, where $f[u] = \{f(uw) | uw \in E(G)\}$, then we call f an adjacent strong edge coloring of $G(V, E)$, denoted by k-ASEC and $\chi'_{as}(G) = \min\{k | \text{there exists a } k - ASEC \text{ of } G\}$ adjacent strong edge chromatic number of graph G. LIU Lin-zhong, ZHANG Zhong-fu, WANG Jian-fang determined the adjacent strong edge chromatic number of paths, cycles, complete graphs, completly muti-partite graphs, wheels, outerplanar graphs and Halin graphs in [1] and put forward the conjecture of adjacent strong edge coloring according to their results: for any 2-connected graph G of $|V(G)| \geq 3$, $G \neq C_5$, $\Delta(G) \leq \chi'_{as}(G) \leq \Delta(G) + 2$. MA De-shan, LIU Lin-zhong, ZHANG Zhong-fu studied the adjacent strong edge coloring of 1-tree graphs in [2].

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A graph is called series-parallel graph (in short SP graph) if it contains no subgraph homeomorphic to K_4. Duffin[3] showed that a connected SP graph can be obtained from a K_2 by repeatly applying the following operations: duplication or replacing an edge by a path of length 2. Dirac[4] proved that the chromatic number of any SP graph is at most three. Seymour[5] proved that the edge chromatic number of any SP multi-graph G is $\max\{\Delta(G), \eta'(G)\}$, where $\eta'(G) = \max\{| E(G[V'])||k : V' \subseteq V(G), |V'| = 2k+1, k \geq 1\}$. WU Jian-liang etc. studied some parameters of SP graph[6].

Unless stated otherwise, all the graphs dealt in this paper are finite, undirected, simple and loopless. Let $G = (V, E)$ be a graph, where $V = V(G)$ is its vertices set and $E = E(G)$ is its edges set. For a graph G, we denote the maximum degree, the minimum degree, the degree of vertex v and the set of vertices adjacent to v by $\Delta(G)$, $\delta(G)$, $d_G(v)$ and $N_G(v)$ respectively, in short Δ, δ, $d(v)$ and $N(v)$ if no confusion. The terms and notations undefined in this paper can be found in [7].

In order to prove these main results, we need the following lemmas:

Lemma 1[6] *Let G be an SP graph of $\delta(G) \geq 2$. Then one of the following conditions holds:*

(i) G has an edge uv such that $d(u) + d(v) = 4$.

(ii) G has an edge uv such that $d(u) + d(v) = 5$.

(iii) G has a 4-vertex w adjacent to two non-adjacent 2-vertices u and v such that $N(w)\backslash\{u, v\} = (N(u) \cup N(v))\backslash\{w\}$.

(iv) G has a 4-vertex w adjacent to two non-adjacent 2-vertices u and v such that $N(u)\backslash\{w\} = N(v)\backslash\{w\} \subset N(w)$.

(v) G has three pairwise non-adjacent 2-vertices u, v and w, such that $N(u) = N(v)$ and $N(u) \cap N(w) \neq \emptyset$.

Lemma 2 *Let G be a 2-connected SP graph of $\Delta = 3$. If G does not contain (i) and (v) of Lemma 1, then it must do edge $e = uv$, such that $d(u) + d(v) = 5$. Let $N(u) = \{v, x\}$, $N(v) = \{u, y_1, y_2\}$. Then $x \in \{y_1, y_2\}$.*

Proof The notations used here are the same as the ones of Lemma 1. Since G does not contain (i) and (v) of Lemma 1, so it must do a group vertices satisfying (ii) of Lemma 1. Now we suppose that this group vertices does not satisfy the conclusion of Lemma 2. Let $G^* = G - \{u\} + vx$. It is obvious that G^* is still a 2-connected SP graph. According to the constructional process of G^*, we know that G^* does not contain a group vertices satisfying (i), (ii) and (v) of Lemma 1. A contradiction! This completes the proof.

Lemma 3 *Let G be a 2-connected SP graph of $\Delta = 4, 5$. If G does not contain (i), (iii) and (iv) of Lemma 1, then one of the following conditions holds:*

(a) There exists an edge $e = uv$, such that $d(u) + d(v) = 5$. Let $N(u) = \{v, x\}$, $N(v) = \{u, y_1, y_2\}$. Then $x \in \{y_1, y_2\}$.

(b) There exist three pairwise non-adjacent 2-vertices u, v, w, such that $N(u) = N(v) = \{x, y\}$ and $N(u) \cap N(w) \neq \emptyset$, $xy \in E(G)$.

The proof of Lemma 3 is similar to the one of Lemma 2.

Lemma 4[1] *For cycle C_n,*

$$\chi'_{as}(C_n) = \begin{cases} 3, & n \equiv 0(\text{mod}3), \\ 4, & n \not\equiv 0(\text{mod}3), \quad n \neq 5, \\ 5, & n = 5. \end{cases}$$

Lemma 5[2] *Let G be a 1-tree graph of maximum degree $\Delta \geq 4$ and $v \in V(G)$, $T = G - \{v\}$ be a tree. Then*

$$\chi'_{as}(G) = \begin{cases} \Delta, & E(G[V_\Delta]) = \emptyset, \\ \Delta+1, & E(G[V_\Delta]) \neq \emptyset. \end{cases}$$

Lemma 6 *Let $1, 2, 3, \cdots, n$ be n digits. Now we combine $n-2$ digits of these n digits arbitrarily, then the number combined may classify into at most two classes: each class contains at least one same digit.*

Proof We see the number combined by $n-2$ digits as $m \times n$ matrix, where $1 \leq m \leq n$. The ith line of the matrix consists of digit i and $n-3$ digits which are behind digit i orderly, and other bits are 0. Its form is as follows:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & \cdots & \cdots & \cdots & \cdots & \cdots & n-2 & 0 & 0 \\ 0 & 2 & 3 & 4 & \cdots & \cdots & \cdots & \cdots & \cdots & n-2 & n-1 & 0 \\ 0 & 0 & 3 & 4 & 5 & \cdots & \cdots & \cdots & \cdots & \cdots & n-1 & n \\ 1 & 0 & 0 & 4 & 5 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & n \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \vdots & \vdots \\ 1 & 2 & 3 & 4 & \cdots & \cdots & n-5 & 0 & 0 & n-2 & n-1 & n \\ 1 & 2 & 3 & 4 & 5 & \cdots & \cdots & n-4 & 0 & 0 & n-1 & n \\ 1 & 2 & 3 & 4 & 5 & \cdots & \cdots & \cdots & n-3 & 0 & 0 & n \end{bmatrix}.$$

Obviously, this matrix contains digit $n-2$ in the lines $1, 2, \cdots, n-2$ and digit 1 in the lines $n-1, n$. Thus we may classify the number combined arbitrarily into two classes: class 1 contains digit 1 and class 2 contains digit $n-2$.

Now we give a lemma which is the key section in the proof of theorems. In order to simplify the proof of theorems, we proof it as a special lemma.

Lemma 7 *Let $v \in V(G)$, $G^* = G + \{vx, vy\}$, $x, y \notin V(G)$. If G has an adjacent strong edge coloring $\sigma : E(G) \to C$, where when $E(G[V_{\Delta(G)}]) = \emptyset$, $|C| = \Delta(G)$, when $E(G[V_{\Delta(G)}]) \neq \emptyset$, $|C| = \Delta(G)+1$, then G^* has an adjacent strong edge coloring $\sigma^* : E(G^*) \to C^*$, where when $E(G^*[V_{\Delta(G^*)}]) = \emptyset$, $|C^*| = \Delta(G^*)$, when $E(G^*[V_{\Delta(G^*)}]) \neq \emptyset$, $|C^*| = \Delta(G^*)+1$.*

Proof We keep the coloring σ of G unchanged and extend σ to the coloring σ^* of G^*.

Case 1 $d(v) = \Delta(G)$ or $\Delta(G)-1$. Then v is the only vertex of maximum degree in G^*. Let $\sigma^*(vx) \in C^* \backslash \sigma[v]$, $\sigma^*(vy) \in C^* \backslash \sigma[v] \backslash \sigma^*(vx)$.

Case 2 $1 \leq d(v) \leq \Delta(G)-2$. Let $N_G(v) = \{v_1, v_2, \cdots, v_m\}, m = d_G(v)$, and let the degree of $v_{i_1}, v_{i_2}, \cdots, v_{i_t}$ which are the adjacent vertices of v be equal to the degree of v. Then $\sigma[v]$ must be the subset of $\sigma[v_{i_1}], \sigma[v_{i_2}], \cdots, \sigma[v_{i_t}]$ (Otherwise, if for the vertex v_{i_s}, $\sigma[v] \not\subset \sigma[v_{i_s}]$. Namely, there exists at least one color $\alpha \notin \sigma[v_{i_s}]$ in $\sigma[v]$. Thus however we color the edge vx, vy, if only it satisfies the proper edge coloring, then we must have $\sigma^*[v] \neq \sigma^*[v_{i_s}]$). By Lemma 6, we may classify $v_{i_1}, v_{i_2}, \cdots, v_{i_t}$ into at most two classes: each class lacks the same color. Without loss of generality, we suppose the absent colors are α_1, α_2 respectively. Thus we let $\sigma^*(vx) = \alpha_1, \sigma^*(vy) = \alpha_2$.

2. Main results and proofs

The following graphs G are all 2-connected.

Theorem 1 *Let G be an SP graph of order p and $\Delta(G) = \Delta = p-1$. Then $n_\Delta \leq 2$.*

Proof Suppose, to the contrary, that $n_\Delta \geq 3$. Without loss of generality, we assume that $d(u) = d(v) = d(w) = \Delta = p-1$, thus u, v, w form a triangle. $\forall x \in V(G)$ and $x \neq u, v, w$, then $ux, vx, wx \in E(G)$. Thus $G[\{u, v, w, x\}] = K_4$, a contradiction with the definition of SP graph. Therefore $n_\Delta \leq 2$.

Theorem 2 *Suppose G is an SP graph of order p, and $\Delta(G) = \Delta$. Then $|E(G)| \geq 2\Delta - 1$.*

Proof Let v be a vertex of maximum degree in G. Since G is 2-connected, so $G - \{v\}$ is connected. Therefore $|E(G^*)| \geq |V(G^*)| - 1$. Again since G is a simple graph, $\Delta \leq p-1$, so $|V(G^*)| = |V(G)| - 1 = p - 1 \geq \Delta$. Thus $|E(G)| = |E(G^*)| + \Delta \geq |V(G^*)| - 1 + \Delta \geq 2\Delta - 1$.

Theorem 3 *Let G be an SP graph of order p, $\Delta(G) = \Delta(\geq 4)$, $|E(G)| = 2\Delta - 1$. Then*

$$\chi'_{as}(G) = \begin{cases} \Delta, & E(G[V_\Delta]) = \emptyset, \\ \Delta + 1, & E(G[V_\Delta]) \neq \emptyset. \end{cases}$$

Proof Let v be a vertex of maximum degree in G. Since G is 2-connected, so $G - \{v\}$ is connected. Therefore $|E(G - v)| = |E(G)| - \Delta = \Delta - 1 \geq |V(G - v)| - 1 = p - 2$. Namely, $\Delta \geq p - 1$. Again since $\Delta \leq p - 1$, so $\Delta = p - 1$. By Theorem 1, there are two possibilities:

(1) $n_\Delta = 1$. Since G is 2–connected and $E(G) = 2\Delta - 1$, so $G^* = G - v$ is a connected graph of order $p - 1$ and $|E(G^*)| = \Delta - 1 = p - 2$. By the properties of trees, we know that G^* is a tree. Again by the construct of G^*, G is a 1-tree graph. By Lemma 5, $\chi'_{as}(G) = \Delta$.

(2) $n_\Delta = 2$. we suppose, without loss of generality, that $d(u) = d(v) = \Delta$, $N(u) \cap N(v) = \{v_1, v_2, \ldots, v_{p-2}\}$. Then $uv \in E(G)$. Again by the definition of SP graph, we obtain that $v_iv_j \notin E(G), i, j = 1, 2, \ldots, p-2$ and $i \neq j$. Now we give a $(\Delta+1)$–adjacent strong edge coloring π of G as follows:

$$\pi(vv_i) = i, i = 1, 2, \ldots, p-2,$$
$$\pi(uv_i) = i + 1, i = 1, 2, \ldots, p-2,$$
$$\pi(uv) = p.$$

Therefore, $\chi'_{as}(G) \leq \Delta + 1$.

Obviously, For any graph G, $\chi'_{as}(G) \geq \begin{cases} \Delta, & E(G[V_\Delta]) = \emptyset, \\ \Delta + 1, & E(G[V_\Delta]) \neq \emptyset. \end{cases}$

Therefore, when $n_\Delta = 2$, $\chi'_{as}(G) = \Delta + 1$.

Theorem 4 *Let G be an SP graph of $\Delta = 3$. Then $\chi'_{as}(G) \leq 5$.*

Proof We will prove the conclusion by induction on the order $p(G)$ of G. Let $C = \{1, 2, 3, 4, 5\}$. When $p(G) = 4$, then G is F_4. It is obvious that the conclusion is true. Suppose that the conclusion is true for the order $p(G) < p (p \geq 5)$. For SP graph of order p, by Lemma 2, there are three possibilities:

Case 1 G has an edge uv such that $d(u) + d(v) = 4$. Let $N(u) = \{v, x\}$, $N(v) = \{u, y\}$. Then $x \neq y$(Otherwise $x \equiv y$ is a cut-vertex).

Let $G^* = G - \{u\} + vx$. By the induction, we know that G^* has a 5–adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a 5–adjacent strong edge coloring σ of G. Let $\sigma(ux) = \sigma^*(vx)$. For the coloring of edge uv, there are three possibilities:

Case 1.1 Both x and y are 2-vertices. Then let $\sigma(uv) = C \backslash \sigma^*[x] \backslash \sigma^*[y]$.

Case 1.2 Either x or y is a 2-vertex. We suppose, without loss of generality, that x is a 2-vertex. Then let $\sigma(uv) \in C \backslash \sigma^*[x] \backslash \sigma^*(vy)$.

Case 1.3 Neither x nor y is a 2-vertex. Then let $\sigma(uv) \in C \backslash \sigma^*(ux) \backslash \sigma^*(vy)$.

The coloring of other edges is the same to σ^*.

Case 2 G has an edge uv such that $d(u) + d(v) = 5$.

By Lemma 2, we know that $N(u) \cap N(v) \neq \emptyset$. Let $N(u) = \{v, x\}$, $N(v) = \{u, x, y_1\}$. Since G is a 2-connected SP graph, so x must be a 3-vertex. Let $N(x) = \{u, v, x_1\}$.

Let $G^* = G - \{u\}$. Obviously, G^* is a 2-connected SP graph of order less than p. If $\Delta(G^*) = 2$, then by Lemma 4, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. If $\Delta(G^*) = 3$, by the induction, G^* has a 5–adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a 5-adjacent strong edge coloring σ of G.

Case 2.1 Neither x_1 nor y_1 is a 3-vertex. Then let $\sigma(ux) \in C \backslash \sigma^*[x]$,
$\sigma(uv) = C \backslash \sigma^*(xx_1) \backslash \sigma^*(xv) \backslash \sigma^*(vy_1) \backslash \sigma(ux)$.

Case 2.2 Both x_1 and y_1 are 3-vertices. If $\sigma^*[x] \subset \sigma^*[x_1]$, then let $\sigma(ux) \in C \backslash \sigma^*[x_1]$. If $\sigma^*[x] \not\subset \sigma^*[x_1]$, then let $\sigma(ux) \in C \backslash \sigma^*[x]$. If $\sigma^*[v] \subset \sigma^*[y_1]$ and $\sigma(ux) \in \sigma^*[y_1]$, then let $\sigma(uv) = C \backslash \sigma^*[y_1] \backslash \sigma^*(xx_1)$. If $\sigma^*[v] \subset \sigma^*[y_1]$, but $\sigma(ux) \notin \sigma^*[y_1]$, then let $\sigma(uv) = C \backslash \sigma^*[y_1] \backslash \sigma(ux)$. If $\sigma^*[v] \not\subset \sigma^*[y_1]$, then let $\sigma(uv) = C \backslash \sigma^*[v] \backslash \sigma(ux) \backslash \sigma^*(xx_1)$.

Case 2.3 Either x_1 or y_1 is a 3-vertex. Let x_1 be a 3-vertex. Similarly to Case 2.2, we first color edge ux, then let $\sigma(uv) = C \backslash \sigma^*(xx_1) \backslash \sigma^*(xv) \backslash \sigma^*(vy_1) \backslash \sigma(ux)$.

The coloring of other edges is the same to σ^*.

Case 3 G has three pairwise non-adjacent 2-vertices u,v and w, such that $N(u)=N(v)$ and $N(u)\cap N(w)\neq\emptyset$. Since G is a 2-connected SP graph, so $xy\notin E(G)$ and y is a 3-vertex.

Let $G^*=G-\{u\}+xy$. By the induction, G^* has a 5-adjacent strong edge coloring $\sigma^*: E(G)\to C$. Now we extend σ^* to a 5–adjacent strong edge coloring σ of G. Let $\sigma(uy)=\sigma^*(xy)$, $\sigma(ux)=C\backslash\sigma^*[x]$. The coloring of other edges is the same to σ^*.

From the above described, we obtain the following: for any SP graph G of $\Delta=3$, $\chi'_{as}(G)\leq 5$. □

Theorem 5 *Let G be an SP graph of $\Delta=4$. Then $\chi'_{as}(G)\leq 5$.*

Proof We prove the conclusion by induction on the order $p(G)$ of G. Let $C=\{1,2,3,4,5\}$. When $p(G)=5$, by enumerating all the coloring of G, it is obvious that the conclusion is true. Suppose the conclusion is true for the order $p(G)<p(p\geq 6)$. For any SP graph of order p, by Lemma 3, there are five possibilities:

Case 1 G has an edge uv such that $d(u)+d(v)=4$. Let $N(u)=\{v,x\}$, $N(v)=\{u,y\}$. Then $x\neq y$(Otherwise $x\equiv y$ is a cut-vertex). The proof is the same to Case 1 of Theorem 4.

Case 2 G has an edge uv such that $d(u)+d(v)=5$. By Lemma 3, $N(u)\cap N(v)\neq\emptyset$. Let $N(u)=\{v,x\}$, $N(v)=\{u,x,y_1\}$. Let $G^*=G-\{u\}$. Obviously, G^* is a 2-connected SP graph of order less than p. If $\Delta(G^*)=3$, then by Theorem 4, G^* has a 5-adjacent strong edge coloring $\sigma^*: E(G^*)\to C$. If $\Delta(G^*)=4$, by the induction, G^* has a 5–adjacent strong edge coloring $\sigma^*: E(G^*)\to C$. Now we extend σ^* to a 5-adjacent strong edge coloring σ of G.

Case 2.1 Let x be a 3-vertex and $N(x)=\{u,v,x_1\}$. If $\sigma(vx)\in\sigma^*[x_1]$, then let $\sigma(ux)\in C\backslash\sigma^*[x_1]$, otherwise let $\sigma(ux)\in C\backslash\sigma^*[x]$. If y_1 is a 3-vertex and $\sigma^*(vx)\in\sigma^*[y_1]$, $\sigma(ux)\in\sigma^*[y_1]$, let $\sigma(uv)=C\backslash\sigma^*[y_1]\backslash\sigma(xx_1)$. If y_1 is a 3-vertex, $\sigma^*(vx)\in\sigma^*[y_1]$, but $\sigma(ux)\notin\sigma^*[y_1]$, then let $\sigma(uv)=C\backslash\sigma^*[y_1]\backslash\sigma(ux)$, otherwise let $\sigma(uv)=C\backslash\sigma^*(xx_1)\backslash\sigma^*(vx)\backslash\sigma^*(vy_1)\backslash\sigma(ux)$.

Case 2.2 Let x be a 4-vertex and $N(x)=\{u,v,x_1,x_2\}$. If neither x_1 nor x_2 is a 4-vertex, then let $\sigma(ux)=C\backslash\sigma^*[x]$. If either x_1 or x_2 is a 4-vertex, we suppose, without loss of generality, that x_1 is a 4-vertex. If $\{\sigma^*(vx),\sigma^*(xx_2)\}\subset\sigma^*[x_1]$, then let $\sigma(ux)=C\backslash\sigma^*[x_1]$. If $\{\sigma^*(vx),\sigma^*(xx_2)\}\not\subset\sigma^*[x_1]$, then let $\sigma(ux)\in C\backslash\sigma^*[x]$. If both x_1 and x_2 are 4-vertices, let $C'_{\sigma^*}(x)=C\backslash\sigma^*(xx_1)\backslash\sigma^*(xx_2)$, $C'_{\sigma^*}(x_1)=C\backslash\sigma^*[x_1]$, $C'_{\sigma^*}(x_2)=C\backslash\sigma^*[x_2]$. Thus $C'_{\sigma^*}(x_1),C'_{\sigma^*}(x_2)\in C'_{\sigma^*}(x)$(otherwise however we color the edge ux, if only it is the proper edge coloring, then it also satisfies the definition of adjacent strong edge coloring). Let $\sigma(ux)\in C'_{\sigma^*}(x)\backslash\{C'_{\sigma^*}(x_1),C'_{\sigma^*}(x_2)\}$. Relet $\sigma(vx)=C\backslash\sigma^*(xx_1)\backslash\sigma^*(xx_2)\backslash\sigma^*(vy_1)\backslash\sigma(ux)$. For the coloring of edge uv, it is similar to Case 2.1.

The coloring of other edges is the same to σ^*.

Case 3 G has a 4-vertex w adjacent to two non-adjacent 2-vertices u and v such that $N(w)\backslash\{u,v\}=(N(u)\cup N(v))\backslash\{w\}=\{x,y\}$.

Case 3.1 If $xy \in E(G)$, then x, y must be 4-vertices. Let $N(x) = \{u, w, y, x_1\}$, $N(y) = \{u, w, y, y_1\}$. Let $G^* = G - \{u\}$. By the induction, G^* has a 5–adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a 5-adjacent strong edge coloring σ of G.

Case 3.1.1 Both x_1 and y_1 are 4-vertices. If $\sigma^*[x] \subset \sigma^*[x_1]$, then let $\sigma(ux) = C \backslash \sigma^*[x_1]$. If $\sigma^*[x] \not\subset \sigma^*[x_1]$, then let $\sigma(ux) \in C \backslash \sigma^*[x]$. If $\{\sigma^*[y] \backslash \sigma^*(vy)\} \subset \sigma^*[y_1]$, then let $\sigma(vy) = C \backslash \sigma^*[y_1]$. If $\{\sigma^*[y] \backslash \sigma^*(vy)\} \not\subset \sigma^*[y_1]$, then let $\sigma(vy) \in C \backslash \sigma^*(xy) \backslash \sigma^*(wy) \backslash \sigma^*(yy_1)$. First we recolor edge wy: if $\{\sigma(vy), \sigma^*(yy_1)\} \subset \{\sigma(ux), \sigma^*(xx_1), \sigma^*(wx)\}$, then let $\sigma(wy) = C \backslash \sigma^*(xx_1) \backslash \sigma^*(xy) \backslash \sigma^*(wx) \backslash \sigma(ux)$. If $\{\sigma(vy), \sigma^*(yy_1)\} \not\subset \{\sigma(ux), \sigma^*(xx_1), \sigma^*(wx)\}$, then let $\sigma(wy) = C \backslash \sigma^*(yy_1) \backslash \sigma^*(xy) \backslash \sigma(vy) \backslash \sigma^*(wx)$. Then we color edge uw: if $\sigma(wy) \in \{\sigma(ux), \sigma^*(xx_1)\}$, then let $\sigma(uw) = C \backslash \sigma^*(xx_1) \backslash \sigma^*(xy) \backslash \sigma(ux) \backslash \sigma^*(wx)$. If $\sigma(wy) \notin \{\sigma(ux), \sigma^*(xx_1)\}$, then let $\sigma(uw) \in C \backslash \sigma(ux) \backslash \sigma^*(wx) \backslash \sigma(wy)$. Then recolor edge vw: if $\{\sigma(uw), \sigma^*(wx)\} \subset \{\sigma(vy), \sigma^*(yy_1), \sigma^*(xy)\}$, then let $\sigma(vw) = C \backslash \sigma^*(yy_1) \backslash \sigma^*(xy) \backslash \sigma(wy) \backslash \sigma(vy)$. If $\{\sigma(uw), \sigma^*(wx)\} \not\subset \{\sigma(vy), \sigma^*(yy_1), \sigma^*(xy)\}$, then let $\sigma(vw) = C \backslash \sigma(uw) \backslash \sigma^*(wx) \backslash \sigma(wy) \backslash \sigma(vy)$. The coloring of other edges is the same to σ^*.

Case 3.1.2 Either x_1 or y_1 is not a 4-vertex. The proof is easy and we omit it.

Case 3.2 $xy \notin E(G)$. Let $G^* = G - \{u, v\} + xy$. If $\Delta(G^*) = 3$, by Theorem 4, G^* has a 5–adjacent strong edge coloring $\sigma^* : E(G) \to C$. If $\Delta(G^*) = 4$, then by induction, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G) \to C$. Now we extend σ^* to a 5-adjacent strong edge coloring σ of G. There are two possibilities: let $\sigma(ux) = \sigma(vy) = \sigma^*(xy)$. If $\sigma^*[w] \subset \sigma^*[x]$, then let $\sigma(uw) \in C \backslash \sigma^*[x]$. If $\sigma^*[w] \not\subset \sigma^*[x]$, then let $\sigma(uw) \in C \backslash \sigma^*[w] \backslash \sigma(ux)$. If $\{\sigma^*[w], \sigma(uw)\} \subset \sigma^*[y]$, then let $\sigma(vw) = C \backslash \sigma^*[y]$. If $\{\sigma^*[w], \sigma(uw)\} \not\subset \sigma^*[y]$, then let $\sigma(vw) = C \backslash \sigma^*(wx) \backslash \sigma^*(wy) \backslash \sigma(uw) \backslash \sigma(vy)$. The coloring of other edges is the same to σ^*.

Case 4 G has a 4-vertex w adjacent to two non-adjacent 2-vertices u and v such that $N(u) \backslash \{w\} = N(v) \backslash \{w\} \subset N(w)$. Let $N(u) = N(v) = \{x, w\}$, $N(w) = \{x, u, v, w_1\}$. Then x must be a 4-vertex. Let $N(x) = \{u, v, w, x_1\}$.

Let $G^* = G - \{u\}$. If $\Delta(G^*) = 3$, by Theorem 4, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G) \to C$. If $\Delta(G^*) = 4$, by the induction, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G) \to C$. Now we extend σ^* to a 5-adjacent strong edge coloring σ of G. If $\sigma^*[x] \subset \sigma^*[x_1]$, then let $\sigma(ux) \in C \backslash \sigma^*[x_1]$. If $\sigma^*[x] \not\subset \sigma^*[x_1]$, then let $\sigma(ux) \in C \backslash \sigma^*[x]$. If $\sigma^*[w] \subset \sigma^*[w_1]$, then let $\alpha = C \backslash \sigma^*[w_1]$. If $\alpha = \sigma(ux)$, then we first exchange the two colors of edge vx and ux, then let $\sigma(uw) = \alpha$. If $\sigma^*[w] \not\subset \sigma^*[w_1]$, then let $\sigma(uw) = C \backslash \sigma^*(vw) \backslash \sigma^*(ww_1) \backslash \sigma^*(wx) \backslash \sigma(ux)$. The coloring of other edges is the same to σ^*.

Case 5 G has three pairwise non-adjacent 2-vertices u, v and w, such that $N(u) = N(v)$ and $N(u) \cap N(w) \neq \emptyset$. By Lemma 3, we know that $xy \in E(G)$. Again since G is a 2-connected SP graph, so y is a 4-vertex. Let $N(y) = \{u, v, x, y_1\}$. Let $G^* = G - \{u\}$. If $\Delta(G^*) = 3$, by Theorem 4, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G) \to C$. If $\Delta(G^*) = 4$, then by the induction, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G) \to C$. Now we extend σ^* to a 5-adjacent strong edge coloring σ of G. If $\sigma^*[y] \subset \sigma^*[y_1]$, then let $\sigma(uy) = C \backslash \sigma^*[y_1]$. If $\sigma^*[y] \not\subset \sigma^*[y_1]$, then let $\sigma(uy) \in C\backslash\sigma^*[y]$. If $\sigma^*[x] \subset \{\sigma^*[y], \sigma(uy)\}$, then let $\sigma(ux) = C\backslash\sigma^*[y]\backslash\sigma(uy)$. If $\sigma^*[x] \not\subset \{\sigma^*[y], \sigma(uy)\}$, then let $\sigma(ux) \in C\backslash\sigma^*[x]\backslash\sigma(uy)$. The coloring of other edges is the same to σ^*.

From the above described, we obtain the following: for any SP graph G of $\Delta = 4$, $\chi'_{as}(G) \leq 5$. □

Theorem 6 *Let G be an SP graph of $\Delta = 5$. Then*

$$\chi'_{as}(G) = \begin{cases} 5, & E(G[V_\Delta]) = \emptyset, \\ 6, & E(G[V_\Delta]) \neq \emptyset. \end{cases}$$

Proof We prove the conclusion by induction on the order $p(G)$ of G. We suppose that $E(G[V_\Delta]) = \emptyset$, $C = \{1,2,3,4,5\}$. When $p(G) = 6$, G is as Graph 1, by enumerating all the coloring of G, it is obvious that the conclusion is true. Suppose the conclusion is true for the order $p(G) < p (p \geq 7)$. For any SP graph G of order p, by Lemma 3, there are five possibilities.

Case 1 G has an edge uv such that $d(u) + d(v) = 4$. Let $N(u) = \{v, x\}$, $N(v) = \{u, y\}$. Then $x \neq y$ (otherwise $x \equiv y$ is a cut-vertex). The proof is the same to Case 1 of Theorem 4.

Case 2 G has an edge uv such that $d(u) + d(v) = 5$. By Lemma 3, $N(u) \cap N(v) \neq \emptyset$. Let $N(u) = \{v, x\}$, $N(v) = \{u, x, y_1\}$. Let $G^* = G - \{u\}$. Obviously, G^* is a 2-connected SP graph of order less than p. If $\Delta(G^*) = 4$, then by Theorem 5, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. If $\Delta(G^*) = 5$, by the induction, G^* has a 5–adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a 5-adjacent strong edge coloring σ of G:

Case 2.1 x is a 5-vertex. Let $\sigma(ux) = C\backslash\sigma^*[x]$. If y_1 is a 3-vertex and $\sigma^*(vx) \in \sigma^*[y_1]$, then let $\sigma(uv) \in C\backslash\sigma^*[y_1]\backslash\sigma(ux)$. Otherwise let $\sigma(uv) \in C\backslash\sigma^*[v]\backslash\sigma(ux)$.

Case 2.2 x is not a 5-vertex. The proof is the same to the one of Case 2 of Theorem 5.

Case 3 G has a 4-vertex w adjacent to two non-adjacent 2-vertices u and v such that $N(w)\backslash\{u, v\} = (N(u) \cup N(v))\backslash\{w\} = \{x, y\}$.

Case 3.1 $xy \in E(G)$. Let $G^* = G - \{u\}$. Obviously, G^* is a 2-connected SP graph of order less than p. If $\Delta(G^*) = 4$, then by Theorem 5, G^* has a 5–adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. If $\Delta(G^*) = 5$, by the induction, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a 5-adjacent strong edge coloring σ of G.

If neither x nor y is a 5-vertex, the proof is the similar to the one of Case 3.1 of Theorem 5.

If either x or y is a 5-vertex, without loss of generality, we suppose that x is a 5-vertex. Let $\sigma(ux) = C\backslash\sigma^*[x]$. If y is a 4-vertex and $\sigma^*[w] \not\subset \sigma^*[y]$, then let $\sigma(uw) = C\backslash\sigma^*[w]\backslash\sigma(ux)$. If y is a 4-vertex, $\sigma^*[w] \subset \sigma^*[y]$ and $\sigma(ux) \in \sigma^*[y]$, then let $\sigma(uw) = C\backslash\sigma^*[y]$. If y is a 4-vertex, $\sigma^*[w] \subset \sigma^*[y]$ and $\sigma(ux) \notin \sigma^*[y]$, then we first exchange the two colors of edge wx and ux. Since the color of edge uw is restricted by four colors colored edges wx, wv, wy, ux, so we may color edge uw according to the proper adjacent strong edge coloring. The coloring of other edges is the same to σ^*.

Case 3.2 $xy \notin E(G)$. Let $G^* = G - \{u\}$. Obviously, G^* is a 2-connected SP graph of order less than p. If $\Delta(G^*) = 4$, then by Theorem 5, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. If $\Delta(G^*) = 5$, by the induction, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a 5–adjacent strong edge coloring σ of G. If both x and y are 5-vertices. Let $\sigma(ux) = C\backslash\sigma^*[x]$, $\sigma(uw) = C\backslash\sigma^*[w]\backslash\sigma(ux)$. The coloring of other edges is the same to σ^*. If at least one of x, y is not a 5-vertex, the proof is the similar to Case 3.2 of Theorem 5.

Case 4 G has a 4-vertex w adjacent to two non-adjacent 2-vertices u and v such that $N(u)\backslash\{w\} = N(v)\backslash\{w\} \subset N(w)$.

Let $N(u) = N((v) = \{x, w\}$, $N(w) = \{x, u, v, w_1\}$. Let $G^* = G - \{u\}$. If $\Delta(G^*) = 4$, by Theorem 5, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. If $\Delta(G^*) = 5$, by the induction, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a 5-adjacent strong edge coloring σ of G. If x is a 4-vertex, the proof is the same to Case 4 of Theorem 5. If x is a 5-vertex, let $N(x) = \{u, v, w, x_1, x_2\}$, then neither x_1 nor x_2 is a 5-vertex. Let $\sigma(ux) = C\backslash\sigma^*[x]$. If w_1 is not a 4-vertex or w_1 is a 4-vertex and $\sigma^*[w] \not\subset \sigma^*[w_1]$, then let $\sigma(uw) = C\backslash\sigma^*[w]\backslash\sigma(ux)$. If w_1 is a 4-vertex and $\sigma^*[w] \subset \sigma^*[w_1]$, then let $\alpha = C\backslash\sigma^*[w_1]$. If $\alpha \neq \sigma(ux)$, then let $\sigma(uw) = \alpha$. If $\alpha = \sigma(ux)$, then we first exchange two colors of edge vx and ux, then let $\sigma(uw) = \alpha$. The coloring of other edges is the same to σ^*.

Case 5 G has three pairwise non-adjacent 2-vertices u, v and w, such that $N(u) = N(v)$ and $N(u) \cap N(w) \neq \emptyset$.

Case 5.1 $xy \notin E(G)$. Let $G^* = G - \{u\} + xy$. By the induction, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a 5-adjacent strong edge coloring σ of G. Let $\sigma(uy) = \sigma^*(xy)$, $\sigma(ux) \in C\backslash\sigma^*[x]$.

Case 5.2 $xy \in E(G)$. Let $G^* = G - \{u\}$. If $\Delta(G^*) = 4$, by Theorem 5, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. If $\Delta(G^*) = 5$, by the induction, G^* has a 5-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a 5-adjacent strong edge coloring σ of G. If y is a 4-vertex, the proof is the same to Case 5 of Theorem 5. If y is a 5-vertex, let $\sigma(uy) = C\backslash\sigma^*[y]$, $\sigma(ux) = C\backslash\sigma^*[x]\backslash\sigma(uy)$. The coloring of other edges is the same to σ^*.

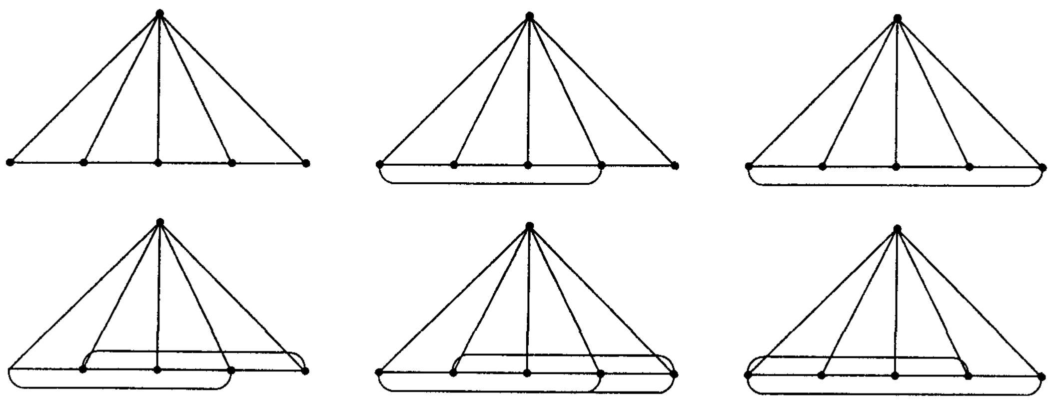
For the case of $E(G[V_\Delta]) \neq \emptyset$, the proof is similar to the one of Theorem 7.

Theorem 7 *Let G be an SP graph of $\Delta \geq 5$. Then*

$$\chi'_{as}(G) = \begin{cases} \Delta(G), & E(G[V_\Delta]) = \emptyset, \\ \Delta(G) + 1, & E(G[V_\Delta]) \neq \emptyset. \end{cases}$$

Proof Because the proof is very complicated, here we only consider the proofs of the worst case. We will prove the conclusion by double inductions on the edge number $q(G)$ and the maximum degree $\Delta(G)$ of G. We suppose that $E(G[V_\Delta]) \neq \emptyset$, $C = \{1, 2, 3, \cdots, \Delta, \Delta + 1\}$. For the case of

$E(G[V_\triangle]) = \emptyset$, the proof is the similar to the one of Theorem 6. When $\Delta(G) = 5$, by Theorem 6, the conclusion is true. When $q(G) = 2\Delta(G) - 1$, by Theorem 3, the conclusion is true. We assume that $\Delta(G) < \Delta(\Delta \geq 6)$ (hypothesis 1) or $q(G) < q(q \geq 2\Delta(G))$ (hypothesis 2), the conclusion is true. Now we proof that when $\Delta(G) = \Delta$ and $q(G) = q$, the conclusion is true. For any SP graph G of order p, by Lemma 1, there are five possibilities:



Graph 1

Case 1 G has an edge uv such that $d(u) + d(v) = 4$. Let $N(u) = \{v, x\}$, $N(v) = \{u, y\}$. Then $x \neq y$ (otherwise $x \equiv y$ is a cut-vertex). The proof is the similar to the one of Case 1 of Theorem 4.

Case 2 G has an edge uv such that $d(u) + d(v) = 5$. Let $N(u) = \{v, x\}$, $N(v) = \{u, x, y_1, y_2\}$.

Case 2.1 $N(x) \cap N(v) \neq \emptyset$. Let $x = y_2$. Let $G^* = G - \{u\}$. Obviously, G^* is a 2-connected SP graph of order less than p. If $\Delta(G^*) < \Delta$, then by the hypothesis 1, G^* has a $(\Delta + 1)$–adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. If $\Delta(G^*) = \Delta$, by the hypothesis 2, G^* has a $(\Delta + 1)$–adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a $(\Delta + 1)$–adjacent strong edge coloring σ of G. In the worst case, x is a vertex of maximum degree, $N(x) = \{u, v, x_1, x_2, \cdots, x_{\Delta-2}\}$, and x_i is the vertices of maximum degree, $i = 1, 2, \cdots, \Delta - 2$, y_1 is a 3-vertex. By Lemma 7, we may proceed the proper adjacent strong edge coloring in edge ux, vx. Then let $\sigma(uv) \in C \backslash \sigma^*[y_1] \backslash \sigma(ux) \backslash \sigma(vx)$.

Case 2.2 $N(x) \cap N(v) = \emptyset$. Let $G^* = G - \{u\} + vx$. By hypothesis 2, G^* has a $(\Delta + 1)$–adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a $(\Delta + 1)$-adjacent strong edge coloring σ of G. In the worst case, both y_1 and y_2 are 3-vertices and x is a 2-vertex. Let $\sigma(ux) = \sigma^*(vx)$. Now we color edge uv. If $\{\sigma(vy_1), \sigma(vy_2)\}$ is a subset of $\sigma^*[y_1]$(or $\sigma^*[y_2]$) (assume $\sigma^*[y_2] \backslash \{\sigma(vy_1), \sigma(vy_2)\} = C'$), then let $\sigma(uv) \in C \backslash \sigma^*[y_1] \backslash C' \backslash \sigma(ux)$. If $\{\sigma(vy_1), \sigma(vy_2)\}$ is not a subset of $\sigma^*[y_1]$ and $\sigma^*[y_2]$, then let $\sigma(uv) \in C \backslash \sigma^*[v]$.

The coloring of other edges is the same to σ^*.

Case 3 G has a 4-vertex w adjacent to two non-adjacent 2-vertices u and v such that $N(w) \backslash \{u, v\} =$

$(N(u) \cup N(v)) \backslash \{w\} = \{x, y\}$.

Case 3.1 $xy \in E(G)$. Let $G^* = G - \{u, v\}$. Obviously, G^* is a 2-connected SP graph of order less than p. If $\Delta(G^*) < \Delta$, then by the hypothesis 1, G^* has a $(\Delta+1)$-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. If $\Delta(G^*) = \Delta$, by the hypothesis 2, G^* has a $(\Delta+1)$-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a $(\Delta+1)$-adjacent strong edge coloring σ of G. If both x and y are 4-vertices, the proof is similar to the one of Case 3.1 of Theorem 5. If both x and y are the vertices of maximum degree, and all the adjacent vertices of x, y are the vertices of maximum degree. Let $N(x) = \{u, w, y, x_1, x_2, \cdots, x_{\Delta-3}\}$, $N(y) = \{u, w, x, y_1, y_2, \cdots, y_{\Delta-3}\}$. By Lemma 7, we may proceed the proper adjacent strong edge coloring in edge ux, wx. Similarly, we may also proceed the proper adjacent strong edge coloring in edges vy, wy. Then let $\sigma(wu) \in C \backslash \sigma(ux) \backslash \sigma(wx) \backslash \sigma(wy)$, $\sigma(vw) \in C \backslash \sigma(wu) \backslash \sigma(wx) \backslash \sigma(wy) \backslash \sigma(vy)$. The coloring of other edges is the same as σ^*.

Case 3.2 $xy \notin E(G)$. The proof is similar to Case 3.2 of Theorem 5.

Case 4 G has a 4-vertex w adjacent to two non-adjacent 2-vertices u and v such that $N(u) \backslash \{w\} = N(v) \backslash \{w\} \subset N(w)$. Let $N(u) = N((v) = \{x, w\}$, $N(w) = \{x, u, v, w_1\}$. Let $G^* = G - \{u, v\}$. Obviously, G^* is a 2-connected SP graph of order less than p. If $\Delta(G^*) < \Delta$, then by the hypothesis 1, G^* has a $(\Delta+1)$-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. If $\Delta(G^*) = \Delta$, by the hypothesis 2, G^* has a $(\Delta+1)$-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a $(\Delta+1)$-adjacent strong edge coloring σ of G. In the worst case, x is a vertex of maximum degree. Let $N(x) = \{u, v, w, x_1, x_2, \cdots, x_{\Delta-3}\}$, and $x_i, i = 1, 2, \cdots, \Delta-3$ be the vertices of maximum degree, w_1 be a 4-vertex. By Lemma 7, we may proceed the proper adjacent strong edge coloring in edge ux, vx. Let $\sigma(wu) \in C \backslash \sigma^*[w_1] \backslash \sigma(ux) \backslash \sigma^*(wx)$, $\sigma(vw) \in C \backslash \sigma^*(ww_1) \backslash \sigma(uw) \backslash \sigma(vx) \backslash \sigma^*(wx)$. The coloring of other edges is the same as σ^*.

Case 5 G has three pairwise non-adjacent 2-vertices u, v and w, such that $N(u) = N(v)$ and $N(u) \cap N(w) \neq \emptyset$.

Case 5.1 $xy \notin E(G)$. The proof is similar to the one of Case 5.1 of Theorem 6.

Case 5.2 $xy \in E(G)$. Let $G^* = G - \{u, v\}$. Obviously, G^* is a 2-connected SP graph of order less than p. If $\Delta(G^*) < \Delta$, then by the hypothesis 1, G^* has a $(\Delta+1)$-adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. If $\Delta(G^*) = \Delta$, by the hypothesis 2, G^* has a $(\Delta+1)$–adjacent strong edge coloring $\sigma^* : E(G^*) \to C$. Now we extend σ^* to a $(\Delta+1)$-adjacent strong edge coloring σ of G. In the worst case, y is a vertex of maximum degree. Let $N(y) = \{u, v, x, y_1, y_2, \cdots, y_{\Delta-3}\}$, and $y_i, i = 1, 2, \cdots, \Delta-3$ be the vertices of maximum degree. By Lemma 7, we may proceed the proper adjacent strong edge coloring in edges uy, vy. Let $\sigma(ux) \in C \backslash \sigma^*(xy) \backslash \sigma(uy) \backslash \sigma^*(wx)$, $\sigma(vx) \in C \backslash \sigma(vy) \backslash \sigma(wx) \backslash \sigma^*(xy) \backslash \sigma(ux)$. The coloring of other edges is the same as σ^*.

From the above described, we obtain the following: for any SP graph G of $\Delta \geq 5$,

$$\chi'_{as}(G) = \begin{cases} \Delta(G), & E(G[V_\Delta]) = \emptyset, \\ \Delta(G) + 1, & E(G[V_\Delta]) \neq \emptyset. \end{cases}$$

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系列平行图的邻强边色数

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摘要: 本文研究了系列平行图的邻强边染色. 从图的结构性质出发, 利用双重归纳和换色的方法证明了对于 $\Delta(G)=3,4$ 的系列平行图满足邻强边染色猜想; 对于 $\Delta(G)\geq 5$ 的系列平行图 G, 有 $\Delta(G)\leq \chi'_{as}(G)\leq \Delta(G)+1$, 且 $\chi'_{as}(G)=\Delta(G)+1$ 当且仅当存在两个最大度点相邻, 其中 $\Delta(G)$ 和 $\chi'_{as}(G)$ 分别表示图 G 的最大度和邻强边色数.

关键词: 系列平行图; 邻强边染色; 邻强边色数.