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Double Periodic Cubic Spline Spaces over Non-Uniform Type-2 Triangulations

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Abstract: In this paper, the dimension of the double periodic cubic C^1 spline space over non-uniform type-2 triangulations is determined and a local support basis is given.

Key words: double periodic; cubic spline; dimension; basis; non-uniform type-2 triangulation.
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1. Introduction

Let $\Omega = [0, x_m] \times [0, y_n]$ be a rectangle. For $0 = x_0 < x_1 < \cdots < x_m$ and $0 = y_0 < y_1 < \cdots < y_n$, Ω is divided into mn small rectangles by mesh lines $x = x_i$, $i = 1, 2, \cdots, m-1, y = y_j$, $j = 1, 2, \cdots, n-1$. The triangulation which results when all northeast diagonals in all small rectangles are drawn in is called a non-uniform type-1 triangulation and denoted by $\overline{\triangle}_{mn}^{(1)}$. And the triangulation which results when all northwest diagonals in all small rectangles are also drawn in is called a non-uniform type-2 triangulation and denoted by $\overline{\triangle}_{mn}^{(2)}$. Let $h_i = x_i - x_{i-1}$, $t_j = y_j - y_{j-1}$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$. If $h_i \equiv h$, $i = 1, 2, \cdots, m$, and $t_j \equiv t$, $j = 1, 2, \cdots, n$, then $\overline{\triangle}_{mn}^{(1)}$ and $\overline{\triangle}_{mn}^{(2)}$ are denoted by $\triangle_{mn}^{(1)}$ and $\triangle_{mn}^{(2)}$, respectively.

For $0 \le r \le k-1$, where r and k are integers, let $S_k^r(\overline{\triangle}_{mn}^{(i)})$ denote the vector space of functions in C^r whose restrictions to each triangular element of $\overline{\triangle}_{mn}^{(i)}$ are polynomials of total degree at most k. $S_k^r(\overline{\triangle}_{mn}^{(i)})$ is called a bivariate spline space over $\overline{\triangle}_{mn}^{(i)}$ with degree k and smoothness order r.

Let $ls^{(0,0)}(x, y, H_1, H_2, H_1', H_2')$, $ls^{(1,0)}(x, y, H_1, H_2, H_1', H_2')$, $ls^{(0,1)}(x, y, H_1, H_2, H_1', H_2')$, $ls^{(1,2)}(x, y, H_1, H_2, H_2')$ and $ls^{(2,1)}(x, y, H_1, H_1', H_2')$ be five piecewise C^1 cubic polynomials with their B-net representations being defined on their local supports as displayed in Figure 1–4, respectively, where all other B-net ordinates which were not displayed are vanished. We have[5]

Lemma 1 *Let $G = [0, m] \times [0, n]$, $G_1 = [0, m] \times [0, n-1]$, $G_2 = [1, m] \times [0, n]$ and $\Gamma = \{(0,0), (1,0), (0,1)\}$. Then the set*

$$\{ls^{\alpha}(x - x_i, y - y_j, h_i, h_{i+1}, t_j, t_{j+1}) : (i, j) \in G \cap \mathbb{Z}^2, \alpha \in \Gamma\}$$

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$$\bigcup\{ls^{(1,2)}(x-x_i,y-y_j,h_i,h_{i+1},t_{j+1}):\ (i,j)\in G_1\cap \mathbb{Z}^2\}$$

$$\bigcup\{ls^{(2,1)}(x-x_i,y-y_j,h_i,t_j,t_{j+1}):\ (i,j)\in G_2\cap \mathbb{Z}^2\} \tag{1}$$

forms a basis of the space $S_3^1(\overline{\triangle}_{mn}^{(2)})$, *where* $\mathbb{Z}^2=\{(i,j): i$ *and* j *are arbitrary integers*$\}$. *Without loss of generality, we set* $h_0=t_0=h_{m+1}=t_{n+1}=1$.

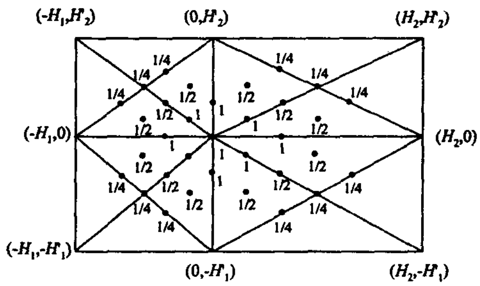

Figure 1: Local support and B-net representation of $ls^{(0,0)}(x,y,H_1,H_2,H_1',H_2')$.

A double periodic spline space $\widetilde{S}_k^r(\overline{\triangle}_{mn}^{(i)})$ is defined to be the subspace of $S_k^r(\overline{\triangle}_{mn}^{(i)})$,

$$\widetilde{S}_k^r(\overline{\triangle}_{mn}^{(i)})=\Big\{s\in S_k^r(\overline{\triangle}_{mn}^{(i)}):\ \frac{\partial^d s(x,y)}{\partial x^d}\mid_{x=0}\equiv \frac{\partial^d s(x,y)}{\partial x^d}\mid_{x=x_m},$$

$$\frac{\partial^d s(x,y)}{\partial y^d}\mid_{y=0}\equiv \frac{\partial^d s(x,y)}{\partial y^d}\mid_{y=y_n}, d=0,1,\cdots,r\Big\}. \tag{2}$$

For such spaces, an important task is to give their dimensions and bases. The first effort was made by Morsche[7] where the space $\widetilde{S}_3^1(\triangle_{mn}^{(1)})$ was discussed. Sha and Xuan[8] discussed the dimension of $\widetilde{S}_3^1(\triangle_{mn}^{(2)})$ and the interpolation and approximation by $\widetilde{S}_3^1(\triangle_{mn}^{(2)})$. Dimensions and local support bases of $\widetilde{S}_2^1(\triangle_{mn}^{(2)})$ and $\widetilde{S}_2^1(\overline{\triangle}_{mn}^{(2)})$ were given by Liu[1] and Liu and Shu[6], respectively. And two kinds of interpolation and their approximation by $\widetilde{S}_2^1(\triangle_{mn}^{(2)})$ were studied by Liu[2,4]. In addition, by using a so-called integral representation of bivariate splines, the dimension of $\widetilde{S}_k^1(\triangle_{mn}^{(1)})$ with degree $k\geq 4$ was also determined by Liu[3]. Recently, a class of interpolation by $\widetilde{S}_4^1(\triangle_{mn}^{(2)})$ and a class of transfinite interpolation and approximation by $\widetilde{S}_3^1(\triangle_{mn}^{(2)})$ are separately discussed by Xuan[9] and You[10].

In this paper, by employing the local support basis of $S_3^1(\overline{\triangle}_{mn}^{(2)})$ given in Lemma 1, the dimension of the double periodic C^1 cubic spline space $\widetilde{S}_3^1(\overline{\triangle}_{mn}^{(2)})$ is determined and a local support basis is constructed.

2. The double periodic cubic spline space $\widetilde{S}_3^1(\overline{\triangle}_{mn}^{(2)})$

According to Eq.(2) and Lemma 1, a sufficient and necessary condition for $s(x,y)\in \widetilde{S}_3^1(\overline{\triangle}_{mn}^{(2)})$ is that $s(x,y)$ can be expressed as

$$s(x,y)=\sum_{p=0}^{m}\sum_{q=0}^{n}A_{p,q}ls^{(0,0)}(x-x_p,y-y_q,h_p,h_{p+1},t_q,t_{q+1})+$$

$$\sum_{p=0}^{m}\sum_{q=0}^{n} B_{p,q} ls^{(1,0)}(x-x_p, y-y_q, h_p, h_{p+1}, t_q, t_{q+1})+$$

$$\sum_{p=0}^{m}\sum_{q=0}^{n} C_{p,q} ls^{(0,1)}(x-x_p, y-y_q, h_p, h_{p+1}, t_q, t_{q+1})+$$

$$\sum_{p=0}^{m}\sum_{q=0}^{n-1} D_{p,q} ls^{(1,2)}(x-x_p, y-y_q, h_p, h_{p+1}, t_{q+1})+$$

$$\sum_{p=1}^{m}\sum_{q=0}^{n} E_{p,q} ls^{(2,1)}(x-x_p, y-y_q, h_p, t_q, t_{q+1}) \tag{3}$$

and satisfies the following double periodic conditions

$$\begin{cases} s(x,y)\mid_{x=0,\ y_{j-1}\le y\le y_j} - s(x,y)\mid_{x=x_m,\ y_{j-1}\le y\le y_j}\equiv 0,\ j=1,\cdots,n, \\ s(x,y)\mid_{x_{i-1}\le x\le x_i,\ y=0} - s(x,y)\mid_{x_{i-1}\le x\le x_i,\ y=y_n}\equiv 0,\ i=1,\cdots,m, \\ \frac{\partial s(x,y)}{\partial x}\mid_{x=0,\ y_{j-1}\le y\le y_j} - \frac{\partial s(x,y)}{\partial x}\mid_{x=x_m,\ y_{j-1}\le y\le y_j}\equiv 0,\ j=1,\cdots,n, \\ \frac{\partial s(x,y)}{\partial y}\mid_{x_{i-1}\le x\le x_i,\ y=0} - \frac{\partial s(x,y)}{\partial y}\mid_{x_{i-1}\le x\le x_i,\ y=y_n}\equiv 0,\ i=1,\cdots,m. \end{cases} \tag{4}$$

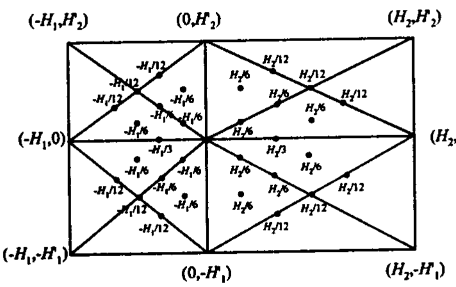

Figure 2

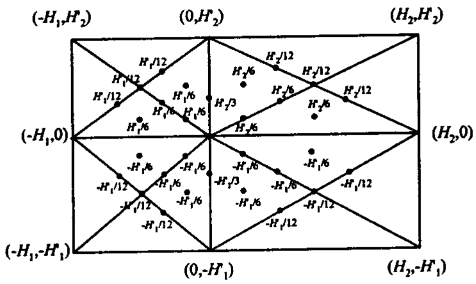

Figure 3

Figure 2. Local support and B-net representation of $ls^{(1,0)}(x,y,H_1,H_2,H_1',H_2')$.

Figure 3. Local support and B-net representation of $ls^{(0,1)}(x,y,H_1,H_2,H_1',H_2')$.

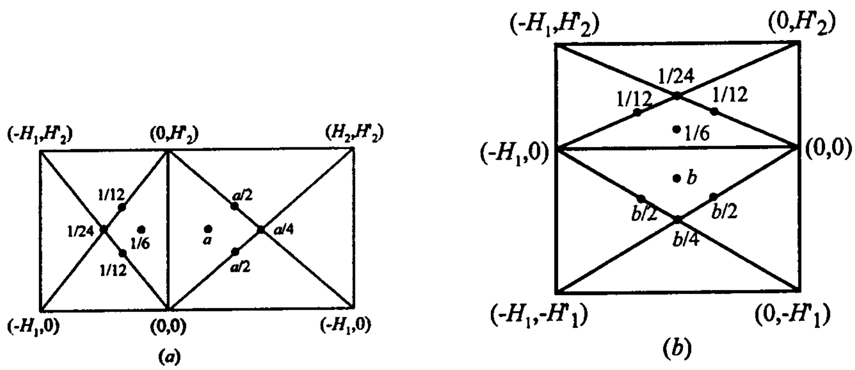

Figure 4. Local support and B-net representation: (a) $ls^{(1,2)}(x,y,H_1,H_2,H_1',H_2')$ with $a=-\frac{H_2}{6H_1}$; (b) $ls^{(2,1)}(x,y,H_1,H_2,H_1',H_2')$ with $b=-\frac{H_1'}{6H_2'}$

The above conditions are equivalent to the following eight sets of linear equations:

$$\begin{cases} A_{i,0}-A_{i,n}=0, & i=0,1,\cdots,m, \\ A_{0,j}-A_{m,j}=0, & j=0,n, \end{cases} \tag{5}$$

$$A_{0,j}-A_{m,j}=0,\quad j=1,2,\cdots,n-1, \tag{6}$$

$$\begin{cases} 2B_{i-1,0}-B_{i,0}-2B_{i-1,n}+B_{i,n}=0, & i=1,2,\cdots,m, \\ B_{i-1,0}-2B_{i,0}-B_{i-1,n}+2B_{i,n}=0, & i=1,2,\cdots,m, \\ B_{0,j}+B_{m,j}=0, & j=0,n \end{cases} \tag{7}$$

$$B_{0,j}+B_{m,j}=0,\quad j=1,2,\cdots,n-1, \tag{8}$$

$$C_{i,0}+C_{i,n}=0,\quad i=1,2,\cdots,m-1, \tag{9}$$

$$\begin{cases} 2C_{0,j-1}-C_{0,j}-2C_{m,j-1}+C_{m,j}=0, & j=1,2,\cdots,n, \\ C_{0,j-1}-2C_{0,j}-C_{m,j-1}+2C_{m,j}=0, & j=1,2,\cdots,n, \\ C_{i,0}+C_{i,n}=0, & i=0,m \end{cases} \tag{10}$$

$$W_jD_{0,j}+Z_jD_{m,j}=0,\quad j=0,1,\cdots,n-1, \tag{11}$$

$$X_iE_{i,0}+Y_iE_{i,n}=0,\quad i=1,2,\cdots,m, \tag{12}$$

where

$$X_i=\frac{\sqrt{h_i^2+y_1^2}}{y_1^2},\quad Y_i=\frac{\sqrt{h_i^2+1}}{t_n},\quad W_j=\frac{\sqrt{t_j^2+1}}{x_1t_j},\quad Z_j=\frac{\sqrt{h_m^2+t_j^2}}{t_jh_m^2}. \tag{13}$$

Which can also be expressed in matrix form:

$$\mathbf{AF}=\mathbf{0}, \tag{14}$$

where

$$\mathbf{A}=\mathrm{diag}\left(\mathbf{A}_1,\mathbf{A}_2,\cdots,\mathbf{A}_8\right), \tag{15}$$

with

$$\mathbf{A}_1=\begin{pmatrix} 1 & & & & & -1 & & & & \\ & 1 & & & & & -1 & & & \\ & & \ddots & & & & & \ddots & & \\ & & & 1 & & & & & -1 & \\ & & & & 1 & & & & & -1 \\ 1 & 0 & \dots & 0 & -1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 1 & 0 & \dots & 0 & -1 \end{pmatrix}_{(m+3)\times 2(m+1)},$$

$$\mathbf{A}_2=\begin{pmatrix} 1 & & & -1 & & \\ & \ddots & & & \ddots & \\ & & 1 & & & -1 \end{pmatrix}_{(n-1)\times 2(n-1)},$$

$$\mathbf{A}_3=\begin{pmatrix} \mathbf{A}_{31} \\ \mathbf{A}_{32} \end{pmatrix},$$

$$\mathbf{A}_{31}=\begin{pmatrix} 2 & -1 & & & -2 & 1 & & \\ & \ddots & \ddots & & & \ddots & \ddots & \\ & & 2 & -1 & & & -2 & 1 \end{pmatrix}_{m\times 2(m+1)},$$

$$\mathbf{A}_{32}=\begin{pmatrix} 1 & -2 & & & & & -1 & 2 & & & \\ & 1 & -2 & & & & & -1 & 2 & & \\ & & \ddots & \ddots & & & & & \ddots & \ddots & \\ & & & 1 & -2 & & & & & -1 & 2 \\ 1 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & 1 \end{pmatrix}_{(m+2)\times 2(m+1)},$$

$$\mathbf{A}_4=\begin{pmatrix} 1 & & & 1 & & \\ & \ddots & & & \ddots & \\ & & 1 & & & 1 \end{pmatrix}_{(n-1)\times 2(n-1)},$$

$$\mathbf{A}_5=\begin{pmatrix} 1 & & & 1 & & \\ & \ddots & & & \ddots & \\ & & 1 & & & 1 \end{pmatrix}_{(m-1)\times 2(m-1)},$$

$$\mathbf{A}_6=\begin{pmatrix} \mathbf{A}_{61} \\ \mathbf{A}_{62} \end{pmatrix},$$

$$\mathbf{A}_{61}=\begin{pmatrix} 2 & -1 & & & -2 & 1 & & \\ & \ddots & \ddots & & & \ddots & \ddots & \\ & & 2 & -1 & & & -2 & 1 \end{pmatrix}_{n\times 2(n+1)},$$

$$\mathbf{A}_{62}=\begin{pmatrix} 1 & -2 & & & & & -1 & 2 & & & \\ & 1 & -2 & & & & & -1 & 2 & & \\ & & \ddots & \ddots & & & & & \ddots & \ddots & \\ & & & 1 & -2 & & & & & -1 & 2 \\ 1 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & 1 \end{pmatrix}_{(n+2)\times 2(n+1)},$$

$$\mathbf{A}_7=\begin{pmatrix} W_0 & & & Z_0 & & \\ & \ddots & & & \ddots & \\ & & W_{n-1} & & & Z_{n-1} \end{pmatrix}_{n\times 2n},$$

$$\mathbf{A}_8=\begin{pmatrix} X_1 & & & Y_1 & & \\ & \ddots & & & \ddots & \\ & & X_m & & & Y_m \end{pmatrix}_{m\times 2m},$$

and

$$\mathbf{F}=\begin{pmatrix} \mathbf{F}_1 & \mathbf{F}_2 & \mathbf{F}_3 & \mathbf{F}_4 & \mathbf{F}_5 & \mathbf{F}_6 & \mathbf{F}_7 & \mathbf{F}_8 \end{pmatrix}^T$$

with

$$\begin{aligned}
\mathbf{F}_1 &= (\ A_{0,0}\quad A_{1,0}\quad \cdots\quad A_{m,0}\quad A_{0,n}\quad A_{1,n}\quad \cdots\quad A_{m,n}\ )^T,\\
\mathbf{F}_2 &= (\ A_{0,1}\quad A_{0,2}\quad \cdots\quad A_{0,n-1}\quad A_{m,1}\quad A_{m,2}\quad \cdots\quad A_{m,n-1}\ )^T,\\
\mathbf{F}_3 &= (\ B_{0,0}\quad B_{1,0}\quad \cdots\quad B_{m,0}\quad B_{0,n}\quad B_{1,n}\quad \cdots\quad B_{m,n}\ )^T,\\
\mathbf{F}_4 &= (\ B_{0,1}\quad B_{0,2}\quad \cdots\quad B_{0,n-1}\quad B_{m,1}\quad B_{m,2}\quad \cdots\quad B_{m,n-1}\ )^T,\\
\mathbf{F}_5 &= (\ C_{0,0}\quad C_{1,0}\quad \cdots\quad C_{m,0}\quad C_{0,n}\quad C_{1,n}\quad \cdots C_{m,n}\ )^T,\\
\mathbf{F}_6 &= (\ C_{0,1}\quad C_{0,2}\quad \cdots\quad C_{0,n-1}\quad C_{m,1}\quad C_{m,2}\quad \cdots\quad C_{m,n-1}\ )^T,\\
\mathbf{F}_7 &= (\ D_{1,0}\quad D_{2,0}\quad \cdots\quad D_{m,0}\quad D_{1,n}\quad D_{2,n}\quad \cdots\quad D_{m,n}\ )^T,\\
\mathbf{F}_8 &= (\ E_{0,0}\quad E_{0,1}\quad \cdots\quad E_{0,n-1}\quad E_{m,0}\quad E_{m,1}\quad \cdots\quad E_{m,n-1}\ )^T.
\end{aligned}$$

By applying elementary row transformations to $\mathbf{A}$, the equation (14) can be transformed into

$$\mathbf{BF} = \mathbf{0}, \tag{16}$$

where

$$\mathbf{B} = \operatorname{diag}(\mathbf{B}_1, \mathbf{A}_2, \mathbf{B}_3, \mathbf{A}_4, \mathbf{A}_5, \mathbf{B}_6, \mathbf{A}_7, \mathbf{A}_8), \tag{17}$$

with

$$\mathbf{B}_1 = \begin{pmatrix}
1 & & & & & -1 & & & \\
 & \ddots & & & & & \ddots & & \\
 & & 1 & & & & & -1 & \\
 & & & 1 & & & & & -1 \\
 & & & & 1 & & & & -1
\end{pmatrix}_{(m+2)\times 2(m+1)},$$

$$\mathbf{B}_3 = \begin{pmatrix}
1 & & & & & -1 & & & & \\
 & 1 & & & & & -1 & & & \\
 & & \ddots & & & & & \ddots & & \\
 & & & 1 & & & & & -1 & \\
0 & 0 & \ldots & 0 & 1 & 0 & 0 & & 0 & -1 \\
1 & 0 & \ldots & 0 & 1 & 0 & 0 & \ldots & 0 & 0
\end{pmatrix}_{(m+2)\times 2(m+1)},$$

$$\mathbf{B}_6 = \begin{pmatrix}
1 & & & & & -1 & & & & \\
 & 1 & & & & & -1 & & & \\
 & & \ddots & & & & & \ddots & & \\
 & & & 1 & & & & & -1 & \\
0 & 0 & \ldots & 0 & 1 & 0 & 0 & & 0 & -1 \\
1 & 0 & \ldots & 0 & 1 & 0 & 0 & \ldots & 0 & 0
\end{pmatrix}_{(n+2)\times 2(n+1)}.$$

It is easy to see that all the row vectors in the coefficient matrix $\mathbf{B}$ are linearly independent, so the rank of the matrix **B** is 4m+4n+3. It follows from equation (16) that

$$\left\{\begin{array}{ll} A_{i,n}=A_{i,0}, & i=0,1,\cdots,m,\\ A_{m,j}=A_{0,j}, & j=0,1,\cdots,n-1,\\ B_{i,n}=B_{i,0}, & i=0,1,\cdots,m,\\ B_{m,j}=-B_{0,j}, & j=0,1,\cdots,n-1,\\ C_{i,n}=-C_{i,0}, & i=0,1,\cdots,m-1,\\ C_{m,j}=C_{0,j}, & j=0,1,\cdots,n,\\ D_{m,j}=-\dfrac{W_j}{Z_j}D_{0,j}, & j=0,1,\cdots,n-1,\\ E_{i,n}=-\dfrac{X_i}{Y_i}E_{i,0}, & i=1,2,\cdots,m. \end{array}\right. \tag{18}$$

For convenience of statement, we denote

$\widetilde{ls}^{\alpha}(x-x_i,y-y_j,h_i,h_{i+1},t_j,t_{j+1})$

$=ls^{\alpha}(x-x_i,y-y_j,h_i,h_{i+1},t_j,t_{j+1}),\alpha\in\Gamma,i=1,\cdots,m-1,\ j=1,\cdots,n-1,$

$\widetilde{ls}^{(1,2)}(x-x_i,y-y_j,h_i,h_{i+1},t_{j+1})$

$=ls^{(1,2)}(x-x_i,y-y_j,h_i,h_{i+1},t_{j+1}),i=1,\cdots,m-1,\ j=0,\cdots,n-1,$

$\widetilde{ls}^{(2,1)}(x-x_i,y-y_j,h_i,t_j,t_{j+1})$

$=ls^{(2,1)}(x-x_i,y-y_j,h_i,t_j,t_{j+1}),i=1,\cdots,m,\ j=1,\cdots,n-1,$

$\widetilde{ls}^{(0,0)}(x-x_i,y,h_i,h_{i+1},1,t_1)$

$=ls^{(0,0)}(x-x_i,y,h_i,h_{i+1},1,t_1)+ls^{(0,0)}(x-x_i,y-y_n,h_i,h_{i+1},t_n,1),i=1,\cdots,m-1,$

$\widetilde{ls}^{(1,0)}(x-x_i,y,h_i,h_{i+1},1,t_1)$

$=ls^{(1,0)}(x-x_i,y,h_i,h_{i+1},1,t_1)+ls^{(1,0)}(x-x_i,y-y_n,h_i,h_{i+1},t_n,1),i=1,\cdots,m-1,$

$\widetilde{ls}^{(0,1)}(x-x_i,y,h_i,h_{i+1},1,t_1)$

$=ls^{(0,1)}(x-x_i,y,h_i,h_{i+1},1,t_1)-ls^{(0,1)}(x-x_i,y-y_n,h_i,h_{i+1},t_n,1),i=1,\cdots,m-1,$

$\widetilde{ls}^{(0,0)}(x,y-y_j,1,h_1,t_j,t_{j+1})$

$=ls^{(0,0)}(x,y-y_j,1,h_1,t_j,t_{j+1})+ls^{(0,0)}(x-x_m,y-y_j,h_m,1,t_j,t_{j+1}),j=1,\cdots,n-1,$

$\widetilde{ls}^{(1,0)}(x,y-y_j,1,h_1,t_j,t_{j+1})$

$=ls^{(1,0)}(x,y-y_j,1,h_1,t_j,t_{j+1})-ls^{(1,0)}(x-x_m,y-y_j,h_m,1,t_j,t_{j+1}),j=1,\cdots,n-1,$

$\widetilde{ls}^{(0,1)}(x,y-y_j,1,h_1,t_j,t_{j+1})$

$=ls^{(0,1)}(x,y-y_j,1,h_1,t_j,t_{j+1})+ls^{(0,1)}(x-x_m,y-y_j,h_m,1,t_j,t_{j+1}),j=1,\cdots,n-1,$

$\widetilde{ls}^{(1,2)}(x,y-y_i,1,h_1,t_{j+1})$

$=ls^{(1,2)}(x,y-y_j,1,h_1,t_{j+1})-\dfrac{W_j}{Z_j}ls^{(1,2)}(x-x_m,y-y_j,h_m,1,t_{j+1}),j=0,\cdots,n-1,$

$\widetilde{ls}^{(2,1)}(x-x_i,y,h_i,1,t_1)$

$=ls^{(2,1)}(x-x_i,y,h_i,1,t_1)-\dfrac{X_i}{Y_i}ls^{(2,1)}(x-x_i,y-y_n,h_i,t_n,1),i=1,\cdots,m,$

$\widetilde{ls}^{(0,0)}(x,y,1,h_1,1,t_1)$

$=ls^{(0,0)}(x,y,1,h_1,1,t_1)+ls^{(0,0)}(x-x_m,y,h_m,1,1,t_1)+$

$\quad ls^{(0,0)}(x,y-y_n,1,h_1,t_n,1)+ls^{(0,0)}(x-x_m,y-y_n,h_m,1,t_n,1),$

$\widetilde{ls}^{(1,0)}(x,y,1,h_1,1,t_1)$

$=ls^{(1,0)}(x,y,1,h_1,1,t_1)-ls^{(1,0)}(x-x_m,y,h_m,1,1,t_1)+$

$$ls^{(1,0)}(x, y-y_n, 1, h_1, t_n, 1) - ls^{(1,0)}(x-x_m, y-y_n, h_m, 1, t_n, 1),$$

$$\widetilde{ls}^{(0,1)}(x, y, 1, h_1, 1, t_1)$$
$$= ls^{(0,1)}(x, y, 1, h_1, 1, t_1) + ls^{(0,1)}(x-x_m, y, h_m, 1, 1, t_1) -$$
$$ls^{(0,1)}(x, y-y_n, 1, h_1, t_n, 1) - ls^{(0,1)}(x-x_m, y-y_n, h_m, 1, t_n, 1).$$

Then, by putting (18) into (3), we have

$$\begin{aligned}
s(x,y) = {} & \sum_{p=1}^{m-1}\sum_{q=1}^{n-1}[A_{p,q}\widetilde{ls}^{(0,0)}(x-x_p, y-y_q, h_p, h_{p+1}, t_q, t_{q+1}) + \\
& B_{p,q}\widetilde{ls}^{(1,0)}(x-x_p, y-y_q, h_p, h_{p+1}, t_q, t_{q+1}) + \\
& C_{p,q}\widetilde{ls}^{(0,1)}(x-x_p, y-y_q, h_p, h_{p+1}, t_q, t_{q+1})] + \\
& \sum_{p=1}^{m-1}\sum_{q=0}^{n-1} D_{p,q}\widetilde{ls}^{(1,2)}(x-x_p, y-y_q, h_p, h_{p+1}, t_{q+1}) + \\
& \sum_{p=1}^{m}\sum_{q=1}^{n-1} E_{p,q}\widetilde{ls}^{(2,1)}(x-x_p, y-y_q, h_p, t_q, t_{q+1}) + \\
& \sum_{p=1}^{m-1}[A_{p,0}\widetilde{ls}^{(0,0)}(x-x_p, y, h_p, h_{p+1}, 1, t_1) + \\
& B_{p,0}\widetilde{ls}^{(1,0)}(x-x_p, y, h_p, h_{p+1}, 1, t_1) + \\
& C_{p,0}\widetilde{ls}^{(0,1)}(x-x_p, y, h_p, h_{p+1}, 1, t_1)] + \\
& \sum_{q=1}^{n-1}[A_{0,q}\widetilde{ls}^{(0,0)}(x, y-y_q, 1, h_1, t_q, t_{q+1}) + \\
& B_{0,q}\widetilde{ls}^{(1,0)}(x, y-y_q, 1, h_1, t_q, t_{q+1}) + \\
& C_{0,q}\widetilde{ls}^{(0,1)}(x, y-y_q, 1, h_1, t_q, t_{q+1})] + \\
& \sum_{q=0}^{n-1} D_{0,q}\widetilde{ls}^{(1,2)}(x, y-y_q, 1, h_1, t_{q+1}) + \\
& \sum_{p=1}^{m} E_{p,0}\widetilde{ls}^{(2,1)}(x-x_p, y, h_p, 1, t_1) + \\
& A_{0,0}\widetilde{ls}^{(0,0)}(x, y, 1, h_1, 1, t_1) + \\
& B_{0,0}\widetilde{ls}^{(1,0)}(x, y, 1, h_1, 1, t_1) + \\
& C_{0,0}\widetilde{ls}^{(0,1)}(x, y, 1, h_1, 1, t_1).
\end{aligned}$$

It is not difficult to prove that all the splines in the above expansion are linear independent, so we have obtained the following

Theorem 1

$$\dim \widetilde{S}_3^1(\overline{\triangle}_{mn}^{(2)}) = 5mn, \tag{19}$$

and the set

$$
\begin{aligned}
&\{\widetilde{ls}^{\alpha}(x-x_i,y-y_j,h_i,h_{i+1},t_j,t_{j+1}):\ \alpha\in\Gamma, i=1,2,\cdot sm-1, j=1,2,\cdots n-1\}\\
&\bigcup\{\widetilde{ls}^{(1,2)}(x-x_i,y-y_j,h_i,h_{i+1},t_{j+1}):\ i=1,\cdots,m, j=1,\cdots,n-1\}\\
&\bigcup\{\widetilde{ls}^{(2,1)}(x-x_i,y-y_j,h_i,t_j,t_{j+1}):\ i=1,\cdots,m-1, j=0,\cdots,n-1\}\\
&\bigcup\{\widetilde{ls}^{\alpha}(x-x_i,y,h_i,h_{i+1},1,t_1):\ \alpha\in\Gamma, i=1,\cdots,m-1\}\\
&\bigcup\{\widetilde{ls}^{\alpha}(x,y-y_j,1,h_1,t_j,t_{j+1}):\ \alpha\in\Gamma, j=1,\cdots,n-1\}\\
&\bigcup\{\widetilde{ls}^{(1,2)}(x,y-y_j,1,h_1,t_{j+1}):\ j=0,\cdots,n-1\}\\
&\bigcup\{\widetilde{ls}^{(2,1)}(x-x_i,y,h_i,1,t_1):\ i=1,\cdots,m\}\\
&\bigcup\{\widetilde{ls}^{(0,0)}(x,y,1,h_1,1,t_1)\}\bigcup\{\widetilde{ls}^{(1,0)}(x,y,1,h_1,1,t_1)\}\bigcup\{\widetilde{ls}^{(0,1)}(x,y,1,h_1,1,t_1)\}
\end{aligned}
$$

forms a local basis of $\tilde{S}^1_3(\overline{\triangle}^{(2)}_{mn})$.

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非均匀 II 型三角剖分上的双周期三次样条函数空间

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摘要: 本文确立了非均匀 II 型三角剖分上双周期三次样条函数空间的维数, 并给出了一组具有局部支集的基底.

关键词: 双周期; 三次样条; 维数; 基底; 非均匀 II 型三角剖分.