第25卷第4期 数学研究与评论 Vol.25, No.4
2005年11月 JOURNAL OF MATHEMATICAL RESEARCH AND EXPOSITION Nov., 2005

Article ID: 1000-341X(2005)04-0582-01 **Document code**: A

A Pair of General Series-Transformation Formulas

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MSC(2000): 65B10, 39A70, 41A80
CLC number: O173.1

Very recently, we have found that the method used in our recent paper[1] (appeared in 2005) could be extended to obtain two general series-transformation formulas for formal power series defined over the complex number field. As usual, Δ, Δ^k, D, and D^k denote, respectively, the difference and differential operators with $\Delta f(t) = f(t+1) - f(t)$, $Df(t) = (d/dt)f(t)$ and $\Delta^0 = D^0 = 1$ (the identity operator). What we have obtained are the following two general transformation formulas (formal expansion formulas)

$$\sum_{k=0}^{\infty} f(k)\varphi^{(k)}(0)\frac{t^k}{k!} = \sum_{k=0}^{\infty} \Delta^k f(0)\varphi^{(k)}(t)\frac{t^k}{k!} \tag{1}$$

$$\sum_{k=0}^{\infty} f(k)\varphi^{(k)}(0)\frac{t^k}{k!} = \sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(0)A_k(t,\varphi) \tag{2}$$

where $A_k(t,\varphi)$ are generalized Eulerian functions defined via $\varphi(t)$, namely

$$A_k(t,\varphi) = \sum_{j=0}^{k} S(k,j)\varphi^{(j)}(t)t^j \quad (A_0 = \varphi(t)), \tag{3}$$

$S(k,j)$ denoting the Stirling numbers of the second kind.

It is not difficult to find that both (1) and (2) imply a great variety of classical formulas and new identities, including summation formulas for various types of power series. All details will appear elsewhere.

References:

[1] HE T X, HSU L C, SHIUE P J S. et al. *A symbolic operator approach to several summation formulas for power series* [J]. J. Comput. Appl. Math., 2005, **177**: 17–33.

Received date: 2005-08-09