Journal of Mathematical Research & Exposition
Aug., 2008, Vol. 28, No. 3, pp. 521–528
DOI:10.3770/j.issn:1000-341X.2008.03.009
Http://jmre.dlut.edu.cn

The New Criteria Robust Stability of Uncertain Neutral Systems with Discrete and Distributed Delays

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Abstract The robust stability of uncertain linear neutral systems with discrete and distributed delays is investigated. The uncertainties under consideration are norm bounded, and possibly time varying. By means of the equivalent equation of zero in the derivative of the Lyapunov-Krasovskii function, the proposed stability criteria are formulated in the form of a linear matrix inequality and it is easy to check the robust stability of the considered systems. Numerical examples demonstrate that the proposed criteria are effective.

Keywords stability; discrete delay; distributed delay; neutral system; linear matrix inequality (LMI).

Document code A
MR(2000) Subject Classification 34K20; 34K40
Chinese Library Classification O175.7

1. Introduction

The problem of stability of time-delay systems of neutral type has received considerable attention in the last two decades[1]. Current efforts on this topic can be divided into two categories, namely delay-independent stability criteria and delay-dependent stability criteria.

For linear time-delay systems of neutral type, some delay-independent stability conditions were obtained. They were formulated in terms of matrix measure and matrix norm[1] or the existence of a positive definite solution to an auxiliary algebraic Riccati matrix equation[3,4]. Although these conditions are easy to check, they required matrix measure to be negative or the parameters to be tuned.

A model transformation technique is often used to transform the neutral system with discrete delay to a neutral system with a distributed delay, and delay-dependent stability criteria are obtained by employing Lyapunov-Krasovskii functionals[5−8]. These results are usually less conservative than the delay-independent stability ones. Some of these results are still conservative since some model transformations will introduce additional dynamics as discussed in [9].

Recently, a new descriptor model transformation and a corresponding Lyapunov-Krasovskii functional have been introduced for stability analysis of systems with delays as outlined in [10].

Received date: 2006-07-28; **Accepted date**: 2007-01-19

The advantage of this transformation is to transform the original system to an equivalent descriptor form representation and will not introduce additional dynamics in the sense defined in [9]. Although the results in [10] are less conservative than some existing ones, they can be improved by employing the decomposition technique to get a larger bound for discrete delays. However, inequality used in the proving process caused more conservative and the stability bound of distributed delay was not given in [10].

In this paper, on the basis of the equivalent equation of the zero which is similiar to [11] in the derivative of a Lyapunov-Krasovskii functional, we investigate the robust stability of uncertain neutral systems with discrete and distributed delays. The robust stability problem of a considered system is transformed into the existence of some symmetric positive-definite matrices and the free weighting matrices. The stability criteria are formulated in the form of linear matrix inequalities (LMIs). Some numerical examples demonstrate that the results obtained in this paper are effective and are significant improvement over the existing criteria.

2. Problem statement

Consider the following linear neutral system with discrete and distributed delays:

$$\dot{x}(t) - C\dot{x}(t-h) = A(t)x(t) + B(t)x(t-r) + D(t)\int_{t-\tau}^{t} x(s)\mathrm{d}s, \tag{1}$$

$$x(t_0+\theta) = \varphi(\theta), \forall\theta \in [-\max\{h, r, \tau\}, 0], \tag{2}$$

where $x(t) \in R^n$ is the state, $h > 0$ is a constant neutral delay, $r > 0$ is a constant discrete delay, $\tau > 0$ is a constant distributed delay, $\varphi(.)$ is a continuous vector valued initial function, $C \in R^{n\times n}$ is a known constant matrix, and $A(t) \in R^{n\times n}, B(t) \in R^{n\times n}, D(t) \in R^{n\times n}$ are uncertain matrices, not known completely, except for that they are within a compact set Ω which we will refer to as the uncertainty set

$$(A(t), B(t), D(t)) \in \Omega \subset R^{n\times 3n} \quad \text{for all} \quad t \in [0, \infty). \tag{3}$$

3. Main result

Define the operator $D(x_t) = x(t) - Cx(t-h)$. Throughout this paper, we assume that

A1. All the eigenvalues of matrix C are inside the unit circle.

A1 implies that the operator $D(x_t)$ is stable. For the stability of systems (1)–(2), we have the following result.

Theorem 1 *Under A1, the systems described by (1) and (2) are asymptotically stable if there exist symmetric positive definite matrices* P, R, Q_1, Q_2, Q_3 *and any appropriate dimensional* *matrices $N_j (j = 1, \ldots, 5)$ such that the following LMI holds:*

$$\phi = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} \\ \phi_{12}^{\mathrm{T}} & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} \\ \phi_{13}^{\mathrm{T}} & \phi_{23}^{\mathrm{T}} & \phi_{33} & \phi_{34} & \phi_{35} & \phi_{36} \\ \phi_{14}^{\mathrm{T}} & \phi_{24}^{\mathrm{T}} & \phi_{34}^{\mathrm{T}} & \phi_{44} & \phi_{45} & \phi_{46} \\ \phi_{15}^{\mathrm{T}} & \phi_{25}^{\mathrm{T}} & \phi_{35}^{\mathrm{T}} & \phi_{45}^{\mathrm{T}} & \phi_{55} & \phi_{56} \\ \phi_{16}^{\mathrm{T}} & \phi_{26}^{\mathrm{T}} & \phi_{36}^{\mathrm{T}} & \phi_{46}^{\mathrm{T}} & \phi_{56}^{\mathrm{T}} & \phi_{66} \end{pmatrix} < 0, \tag{4}$$

where

$$\phi_{11} = A^{\mathrm{T}}(t)P + PA(t) + N_1 A(t) + A^{\mathrm{T}}(t)N_1^{\mathrm{T}} + \tau Q_1 + Q_2 + Q_3,$$

$$\phi_{12} = PB(t) + A^{\mathrm{T}}(t)N_2^{\mathrm{T}} + N_1 B(t), \quad \phi_{13} = -A^{\mathrm{T}}(t)PC + A^{\mathrm{T}}(t)N_3^{\mathrm{T}},$$

$$\phi_{14} = A^{\mathrm{T}}(t)N_4^{\mathrm{T}} - N_1, \quad \phi_{15} = A^{\mathrm{T}}(t)N_5^{\mathrm{T}} + N_1 C, \quad \phi_{16} = \tau PD(t) + \tau N_1 D(t),$$

$$\phi_{22} = -Q_2 + N_2 B(t) + B^{\mathrm{T}}(t)N_2^{\mathrm{T}}, \quad \phi_{23} = B^{\mathrm{T}}(t)N_3^{\mathrm{T}} - B^{\mathrm{T}}(t)PC, \quad \phi_{24} = B^{\mathrm{T}}(t)N_4^{\mathrm{T}} - N_2,$$

$$\phi_{25} = N_2 C + B^{\mathrm{T}}(t)N_5^{\mathrm{T}}, \quad \phi_{26} = \tau N_2 D(t), \quad \phi_{33} = -Q_3, \quad \phi_{34} = -N_3, \quad \phi_{35} = N_3 C,$$

$$\phi_{36} = \tau N_3 D(t) - \tau C^{\mathrm{T}} PD(t), \quad \phi_{44} = R - N_4 - N_4^{\mathrm{T}}, \quad \phi_{45} = N_4 C - N_5^{\mathrm{T}}, \quad \phi_{46} = \tau N_4 D(t),$$

$$\phi_{55} = -R + N_5 C + C^{\mathrm{T}} N_5^{\mathrm{T}}, \quad \phi_{56} = \tau N_5 D(t), \quad \phi_{66} = -\tau Q_1.$$

Proof Choose a Lyapunov-Krasovskii functional candidate for systems (1) as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t),$$

where

$$V_1(t) = D^{\mathrm{T}}(x_t)PD(x_t), \quad V_2(t) = \int_{t-h}^{t} \dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s,$$

$$V_3(t) = \int_{-\tau}^{0}\int_{t+\theta}^{t} x^{\mathrm{T}}(s)Q_1 x(s)\mathrm{d}s\mathrm{d}\theta, \quad V_4(t) = \int_{t-r}^{t} x^{\mathrm{T}}(s)Q_2 x(s)\mathrm{d}s,$$

$$V_5(t) = \int_{t-h}^{t} x^{\mathrm{T}}(s)Q_3 x(s)\mathrm{d}s,$$

where P, R, Q_1, Q_2, Q_3 are symmetric positive definite matrices. According to (1), for any appropriate dimensional matrices $N_j (j = 1, \ldots, 5)$ one has

$$\begin{aligned} &2\{x^{\mathrm{T}}(t)N_1 + x^{\mathrm{T}}(t-r)N_2 + x^{\mathrm{T}}(t-h)N_3 + \dot{x}^{\mathrm{T}}(t)N_4 + \dot{x}^{\mathrm{T}}(t-h)N_5\}\times \\ &\{A(t)x(t) + B(t)x(t-r) + D(t)\int_{t-\tau}^{t} x(s)\mathrm{d}s - \dot{x}(t) + C\dot{x}(t-h)\} = 0. \end{aligned} \tag{5}$$

Using (5) and calculating the derivative along the solutions of systems of (1) yields

$$\begin{aligned} \dot{V}(t) =& 2\frac{1}{\tau}\int_{t-\tau}^{t} \big\{x^{\mathrm{T}}(t)A^{\mathrm{T}}(t)Px(t) + x^{\mathrm{T}}(t-r)B^{\mathrm{T}}(t)Px(t) - x^{\mathrm{T}}(t)A^{\mathrm{T}}(t)PCx(t-h) - \\ & x^{\mathrm{T}}(t-r)B^{\mathrm{T}}(t)PCx(t-h)\big\}\mathrm{d}s + \frac{2}{\tau}\int_{t-\tau}^{t} x^{\mathrm{T}}(s)\tau D^{\mathrm{T}}(t)Px(t)\mathrm{d}s + \\ & \frac{2}{\tau}\int_{t-\tau}^{t} x^{\mathrm{T}}(s)\tau D^{\mathrm{T}}(t)PCx(t-h)\mathrm{d}s + \frac{1}{\tau}\int_{t-\tau}^{t} \dot{x}^{\mathrm{T}}(t)R\dot{x}(t)\mathrm{d}s - \end{aligned}$$

$$\frac{1}{\tau}\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(t-h)R\dot{x}(t-h)\mathrm{d}s+\frac{1}{\tau}\int_{t-\tau}^{t}x^{\mathrm{T}}(t)\tau Q_1x(t)\mathrm{d}s-\frac{1}{\tau}\int_{t-\tau}^{t}x^{\mathrm{T}}(s)\tau Q_1x(s)\mathrm{d}s+$$
$$\frac{1}{\tau}\int_{t-\tau}^{t}x^{\mathrm{T}}(t)Q_2x(t)\mathrm{d}s-\frac{1}{\tau}\int_{t-\tau}^{t}x^{\mathrm{T}}(t-r)Q_2x(t-r)\mathrm{d}s+\frac{1}{\tau}\int_{t-\tau}^{t}x^{\mathrm{T}}(t)Q_3x(t)\mathrm{d}s-$$
$$\frac{1}{\tau}\int_{t-\tau}^{t}x^{\mathrm{T}}(t-h)Q_3x(t-h)\mathrm{d}s+2\left\{x^{\mathrm{T}}(t)\ N_1+x^{\mathrm{T}}(t-r)N_2+x^{\mathrm{T}}(t-h)N_3+\right.$$
$$\left.\dot{x}^{\mathrm{T}}(t)N_4+\dot{x}^{\mathrm{T}}(t-h)N_5\right\}\times\left\{A(t)x(t)+B(t)x(t-r)+D(t)\int_{t-\tau}^{t}x(s)\mathrm{d}s-\right.$$
$$\left.\dot{x}(t)+C\dot{x}(t-h)\right\}=:\frac{1}{\tau}\int_{t-\tau}^{t}\xi^{\mathrm{T}}\phi\xi\mathrm{d}s,$$

where $\xi=\{x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-r)\ x^{\mathrm{T}}(t-h)\ \dot{x}^{\mathrm{T}}(t)\ \dot{x}^{\mathrm{T}}(t-h)\ x^{\mathrm{T}}(s)\}^{\mathrm{T}}$ and ϕ are defined in (4). If $\phi<0$ for any $\xi\neq 0$. This implies that both the systems (1) and (2) with stable operator $D(x_t)$ are asymptotically stable by Theorem 9.8.1 in [1]. This completes the proof. □

By Proposition 1, it is easy to obtain the following corollary for the nominal system of systems (1)–(2), that is the system

$$\dot{x}(t)-C\dot{x}(t-h)=Ax(t)+Bx(t-r)+D\int_{t-\tau}^{t}x(s)\mathrm{d}s \tag{6}$$

with the initial condition (2).

Corollary 1 *Under A1, the systems described by (6) and (2) are asymptotically stable if there exist symmetric positive definite matrices P,R,Q_1,Q_2,Q_3 and any appropriate dimensional matrices $N_j(j=1,\ldots,5)$ such that the following LMI holds:*

$$\phi_0=\begin{pmatrix}\phi_{11}&\phi_{12}&\phi_{13}&\phi_{14}&\phi_{15}&\phi_{16}\\ \phi_{12}^{\mathrm{T}}&\phi_{22}&\phi_{23}&\phi_{24}&\phi_{25}&\phi_{26}\\ \phi_{13}^{\mathrm{T}}&\phi_{23}^{\mathrm{T}}&\phi_{33}&\phi_{34}&\phi_{35}&\phi_{36}\\ \phi_{14}^{\mathrm{T}}&\phi_{24}^{\mathrm{T}}&\phi_{34}^{\mathrm{T}}&\phi_{44}&\phi_{45}&\phi_{46}\\ \phi_{15}^{\mathrm{T}}&\phi_{25}^{\mathrm{T}}&\phi_{35}^{\mathrm{T}}&\phi_{45}^{\mathrm{T}}&\phi_{55}&\phi_{56}\\ \phi_{16}^{\mathrm{T}}&\phi_{26}^{\mathrm{T}}&\phi_{36}^{\mathrm{T}}&\phi_{46}^{\mathrm{T}}&\phi_{56}^{\mathrm{T}}&\phi_{66}\end{pmatrix}<0, \tag{7}$$

where

$$\phi_{11}=A^{\mathrm{T}}P+PA+N_1A+A^{\mathrm{T}}N_1^{\mathrm{T}}+\tau Q_1+Q_2+Q_3,\quad \phi_{12}=PB+A^{\mathrm{T}}N_2^{\mathrm{T}}+N_1B,$$
$$\phi_{13}=-A^{\mathrm{T}}PC+A^{\mathrm{T}}N_3^{\mathrm{T}},\quad \phi_{14}=A^{\mathrm{T}}N_4^{\mathrm{T}}-N_1,\quad \phi_{15}=A^{\mathrm{T}}N_5^{\mathrm{T}}+N_1C,$$
$$\phi_{16}=\tau PD+\tau N_1D,\quad \phi_{22}=-Q_2+N_2B+B^{\mathrm{T}}N_2^{\mathrm{T}},\quad \phi_{23}=B^{\mathrm{T}}N_3^{\mathrm{T}}-B^{\mathrm{T}}PC,$$
$$\phi_{24}=B^{\mathrm{T}}N_4^{\mathrm{T}}-N_2,\quad \phi_{25}=N_2C+B^{\mathrm{T}}N_5^{\mathrm{T}},\quad \phi_{26}=\tau N_2D,\quad \phi_{33}=-Q_3,$$
$$\phi_{34}=-N_3,\quad \phi_{35}=N_3C,\quad \phi_{36}=\tau N_3D-\tau C^{\mathrm{T}}PD,\quad \phi_{44}=R-N_4-N_4^{\mathrm{T}},$$
$$\phi_{45}=N_4C-N_5^{\mathrm{T}},\quad \phi_{46}=\tau N_4D,\quad \phi_{55}=-R+N_5C+C^{\mathrm{T}}N_5^{\mathrm{T}},$$
$$\phi_{56}=\tau N_5D,\quad \phi_{66}=-\tau Q_1.$$

Now we consider the norm bounded uncertainty described by

$$A(t) = A + \Delta A(t), B(t) = B + \Delta B(t), D(t) = D + \Delta D(t), \tag{8}$$

where

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) & \Delta D(t) \end{bmatrix} = LF(t)\begin{bmatrix} E_a & E_b & E_d \end{bmatrix}, \tag{9}$$

where $F(t) \in R^{p\times q}$ is an unknown real and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$\sigma_{\max}(F(t)) \leq 1 \tag{10}$$

and L, E_a, E_b and E_d are known real constant matrices which characterize how the uncertainty enters the nominal matrices A, B, D.

Now we state the following result.

Theorem 2 *Under A1, the systems described by (1) and (2), with uncertainty described by (8)-(10), are asymptotically stable if there exist symmetric positive definite matrices $\tilde{P}, \tilde{R}, \tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3$ and any appropriate dimensional matrices $\tilde{N}_j (j = 1, \ldots, 5)$ such that the following LMI holds:*

$$\tilde{\phi} = \begin{pmatrix} \tilde{\phi}_{11} & \tilde{\phi}_{12} & \tilde{\phi}_{13} & \tilde{\phi}_{14} & \tilde{\phi}_{15} & \tilde{\phi}_{16} & \tilde{\phi}_{17} & E_a \\ \tilde{\phi}_{12}^{\mathrm{T}} & \tilde{\phi}_{22} & \tilde{\phi}_{23} & \tilde{\phi}_{24} & \tilde{\phi}_{25} & \tilde{\phi}_{26} & \tilde{N}_2 L & E_b \\ \tilde{\phi}_{13}^{\mathrm{T}} & \tilde{\phi}_{23}^{\mathrm{T}} & \tilde{\phi}_{33} & \tilde{\phi}_{34} & \tilde{\phi}_{35} & \tilde{\phi}_{36} & \tilde{\phi}_{37} & 0 \\ \tilde{\phi}_{14}^{\mathrm{T}} & \tilde{\phi}_{24}^{\mathrm{T}} & \tilde{\phi}_{34}^{\mathrm{T}} & \tilde{\phi}_{44} & \tilde{\phi}_{45} & \tilde{\phi}_{46} & \tilde{N}_4 L & 0 \\ \tilde{\phi}_{15}^{\mathrm{T}} & \tilde{\phi}_{25}^{\mathrm{T}} & \tilde{\phi}_{35}^{\mathrm{T}} & \tilde{\phi}_{45}^{\mathrm{T}} & \tilde{\phi}_{55} & \tilde{\phi}_{56} & \tilde{N}_5 L & 0 \\ \tilde{\phi}_{16}^{\mathrm{T}} & \tilde{\phi}_{26}^{\mathrm{T}} & \tilde{\phi}_{36}^{\mathrm{T}} & \tilde{\phi}_{46}^{\mathrm{T}} & \tilde{\phi}_{56}^{\mathrm{T}} & \tilde{\phi}_{66} & 0 & \tau E_d \\ \tilde{\phi}_{17}^{\mathrm{T}} & L^{\mathrm{T}}\tilde{N}_2^{\mathrm{T}} & \tilde{\phi}_{37}^{\mathrm{T}} & L^{\mathrm{T}}\tilde{N}_4^{\mathrm{T}} & L^{\mathrm{T}}\tilde{N}_5^{\mathrm{T}} & 0 & -I & 0 \\ E_a^{\mathrm{T}} & E_b^{\mathrm{T}} & 0 & 0 & 0 & \tau E_d^{\mathrm{T}} & 0 & -I \end{pmatrix} < 0, \tag{11}$$

where

$$\tilde{\phi}_{11} = A^{\mathrm{T}}\tilde{P} + \tilde{P}A + \tilde{N}_1 A + A^{\mathrm{T}}\tilde{N}_1^{\mathrm{T}} + \tau\tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3, \quad \tilde{\phi}_{12} = \tilde{P}B + A^{\mathrm{T}}\tilde{N}_2^{\mathrm{T}} + \tilde{N}_1 B,$$

$$\tilde{\phi}_{13} = -A^{\mathrm{T}}\tilde{P}C + A^{\mathrm{T}}\tilde{N}_3^{\mathrm{T}}, \quad \tilde{\phi}_{14} = A^{\mathrm{T}}\tilde{N}_4^{\mathrm{T}} - \tilde{N}_1, \quad \tilde{\phi}_{15} = A^{\mathrm{T}}\tilde{N}_5 + \tilde{N}_1 C,$$

$$\tilde{\phi}_{16} = \tau\tilde{P}D + \tau\tilde{N}_1 D, \quad \tilde{\phi}_{17} = \tilde{N}_1 L + \tilde{P}L, \quad \tilde{\phi}_{22} = -\tilde{Q}_2 + \tilde{N}_2 B + B^{\mathrm{T}}\tilde{N}_2^{\mathrm{T}},$$

$$\tilde{\phi}_{23} = B^{\mathrm{T}}\tilde{N}_3^{\mathrm{T}} - B^{\mathrm{T}}\tilde{P}C, \quad \tilde{\phi}_{24} = B^{\mathrm{T}}\tilde{N}_4^{\mathrm{T}} - \tilde{N}_2, \quad \tilde{\phi}_{25} = \tilde{N}_2 C + B^{\mathrm{T}}\tilde{N}_5^{\mathrm{T}},$$

$$\tilde{\phi}_{26} = \tau\tilde{N}_2 D, \quad \tilde{\phi}_{33} = -\tilde{Q}_3, \quad \tilde{\phi}_{34} = -\tilde{N}_3, \quad \tilde{\phi}_{35} = \tilde{N}_3 C,$$

$$\tilde{\phi}_{36} = \tau\tilde{N}_3 D - \tau C^{\mathrm{T}}\tilde{P}D, \quad \tilde{\phi}_{37} = \tilde{N}_3 L - C^{\mathrm{T}}\tilde{P}L, \quad \tilde{\phi}_{44} = \tilde{R} - \tilde{N}_4 - \tilde{N}_4^{\mathrm{T}},$$

$$\tilde{\phi}_{45} = \tilde{N}_4 C - \tilde{N}_5^{\mathrm{T}}, \quad \tilde{\phi}_{46} = \tau\tilde{N}_4 D, \quad \tilde{\phi}_{55} = -\tilde{R} + \tilde{N}_5 C + C^{\mathrm{T}}\tilde{N}_5^{\mathrm{T}},$$

$$\tilde{\phi}_{56} = \tau\tilde{N}_5 D, \quad \tilde{\phi}_{66} = -\tau\tilde{Q}_1, \quad \tilde{\phi}_{17} = \tilde{N}_1 L + \tilde{P}L, \quad \tilde{\phi}_{37} = \tilde{N}_3 L - C^{\mathrm{T}}\tilde{P}L.$$

Proof $\tilde{\phi} < 0$ can be written as

$$\phi_0 + \begin{pmatrix} PL + N_1L \\ N_2L \\ N_3L - C^{\mathrm{T}}PL \\ N_4L \\ N_5L \\ 0 \end{pmatrix} F(t) \begin{pmatrix} E_a & E_b & 0 & 0 & 0 & \tau E_d \end{pmatrix} +$$

$$\begin{pmatrix} E_a^{\mathrm{T}} \\ E_b^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ \tau E_d^{\mathrm{T}} \end{pmatrix} F^{\mathrm{T}}(t) \begin{pmatrix} L^{\mathrm{T}}P + L^{\mathrm{T}}N_1^{\mathrm{T}} & L^{\mathrm{T}}N_2^{\mathrm{T}} & L^{\mathrm{T}}N_3^{\mathrm{T}} - L^{\mathrm{T}}PC & L^{\mathrm{T}}N_4^{\mathrm{T}} & L^{\mathrm{T}}N_5^{\mathrm{T}} & 0 \end{pmatrix} < 0.$$

By Lemma 2.4 in [12], a sufficient condition for the above is

$$\lambda\phi_0 + \lambda^2 \begin{pmatrix} PL + N_1L \\ N_2L \\ N_3L - C^{\mathrm{T}}PL \\ N_4L \\ N_5L \\ 0 \end{pmatrix} \begin{pmatrix} L^{\mathrm{T}}P + L^{\mathrm{T}}N_1^{\mathrm{T}} & L^{\mathrm{T}}N_2^{\mathrm{T}} & L^{\mathrm{T}}N_3^{\mathrm{T}} - L^{\mathrm{T}}PC & L^{\mathrm{T}}N_4^{\mathrm{T}} & L^{\mathrm{T}}N_5^{\mathrm{T}} & 0 \end{pmatrix} +$$

$$\begin{pmatrix} E_a^{\mathrm{T}} \\ E_b^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ \tau E_d^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} E_a & E_b & 0 & 0 & 0 & \tau E_d \end{pmatrix} < 0$$

for some $\lambda > 0$. Introducing the following new variables $\tilde{P} = \lambda P, \tilde{R} = \lambda R, \tilde{Q}_i = \lambda Q_i$ $(i = 1, \ldots, 3)$, $\tilde{N}_i = \lambda N_i$ $(i = 1, \ldots, 5)$ and using Schur's complement [13] yields (11). □

4. Examples

In this section, two numerical examples are presented to illustrate our result and gain the distributed delay.

Example 1 Consider the following system

$$\dot{x}(t) - C\dot{x}(t-h) = A(t)x(t) + B(t)x(t-r) + D(t)\int_{t-\tau}^{t} x(s)\mathrm{d}s \tag{12}$$

with

$$A = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & -0.2 \\ 0 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.3 \end{pmatrix}, \quad D = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}.$$

Now we use the criterion in this paper to study the problem. The maximum value for the system to be asymptotically stable is $\tau_{\max} = 1.7818$.

Example 2 Consider the following system

$$\dot{x}(t) - C\dot{x}(t-h) = (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t-r) + (D + \Delta D(t)) \int_{t-\tau}^{t} x(s)\mathrm{d}s \quad (13)$$

with

$$A = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0.5 \\ 0.5 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}, \quad D = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$$

and $\Delta A(t), \Delta B(t), \Delta D(t)$ are unknown matrices satisfying $\|\Delta A(t)\| \leq \alpha, \|\Delta B(t)\| \leq \alpha$ and $\|\Delta D(t)\| \leq \alpha, \alpha \geq 0, \forall t$. The above system is the form of (8)–(10) with $L = \alpha I$ and $E_a = E_b = E_d = I$. For $\Delta A(t) = 0, \Delta B(t) = 0, \Delta D(t) = 0$ and $a = 0$ the system has been described in [14], where the discrete delay was given, but the allowable maximum bound of τ for the asymptotical stability of system (13) was not indicated. Now we use our criterion in this paper on the system (13) as $a = 0.30$, and we can get that the maximum for the system to be asymptotically stable is $\tau_{\max} = 2.5363$.

α	0.00	0.05	0.10	0.15	0.20	0.25
τ	2.5363	1.9126	1.4252	1.0461	0.7434	0.4942

Table 1: The maximal allowable delays τ of example2 for different values of α.

The effect of the uncertainty bound on the maximum time delay for stability $\tau_{\max}$ is also studied. Table 1 illustrates the numerical results for different α. We can see that as $\alpha \to 0$, the asymptotical stability limit for delay approaches the uncertainty-free case. As α increases, $\tau_{\max}$ decreases.

5. Conclusion

The robust stability problem for uncertain neutral systems with discrete and distributed delays has been investigated. Stability criteria have been obtained. Numerical examples have shown that the results obtained in this paper are very effective.

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