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On Some Transformation and Summation Formulae for Bivariate Basic Hypergeometric Series

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Abstract The purpose of this paper is to establish several transformation formulae for bivariate basic hypergeometric series by means of series rearrangement technique. From these transformations, some interesting summation formulae are obtained.

Keywords basic hypergeometric series; Watson's q-Whipple transformation; series rearrangement.

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1. Introduction and definitions

The q-shifted factorial is defined by

$$(a;q)_0 = 1, (a;q)_n = \prod_{k=0}^{n-1}(1-aq^k)$$

with $n \in N_+$. For $|q|<1$, the infinite product expression is defined by

$$(a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k).$$

In this paper, we shall make use of the following compact notations:

$$(a_1,a_2,\ldots,a_m;q)_n = (a_1;q)_n(a_2;q)_n\ldots(a_m;q)_n,$$

$$(a_1,a_2,\ldots,a_m;q)_\infty = (a_1;q)_\infty(a_2;q)_\infty\ldots(a_m;q)_\infty.$$

The basic hypergeometric series is defined by

$${}_r\Phi_s\left[\begin{matrix} a_1,a_2,\ldots,a_r \\ b_1,b_2,\ldots,b_s \end{matrix};q,z\right] = \sum_{n=0}^{\infty}\frac{(a_1,a_2,\ldots,a_r;q)_n}{(q,b_1,b_2,\ldots,b_s;q)_n}\,[(-1)^n q^{\binom{n}{2}}]^{1+s-r}z^n.$$

For the convergence conditions of the basic hypergeometric series, we refer to [3, p 5].

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Bivariate basic hypergeometric function is defined by

$$\Phi^{A:B;C}_{D:E;F}\left[\begin{array}{c} a_A : b_B; c_C \\ d_D : e_E; f_F \end{array} ;q,x,y\right] = \sum_{m,n=0}^{\infty} \frac{(a_A;q)_{m+n}(b_B;q)_m(c_C;q)_n}{(d_D;q)_{m+n}(q,e_E;q)_m(q,f_F;q)_n} \times$$
$$\left[(-1)^{m+n}q^{\binom{m+n}{2}}\right]^{D-A}\left[(-1)^m q^{\binom{m}{2}}\right]^{1+E-B}\left[(-1)^n q^{\binom{n}{2}}\right]^{1+F-C} x^m y^n.$$

We refer to [3, 10.2.10] for more details.

Concerning bivariate basic hypergeometric series, by means of formal power series method and series rearrangement technique, some authors have established many reduction and transformation formulae.

For more details, the reader is referred to Chu and Srivastava[2], Jia and Wang[5], Srivastava and Karlsson[7].

The main purpose of this paper is to establish several transformation formulae for bivariate basic hypergeometric series by means of series rearrangement technique. From these transformations, some interesting summation formulae are derived.

2. Main results

Proposition 2.1 *Let $\{\Omega(n)\}_{n=0}^{\infty}$ be an arbitrary complex sequence. Then we have*

$$\sum_{m,n=0}^{\infty}\Omega(m+n)\frac{(a,b;q)_m(aq^2;q^2)_m(1/b^2q;q)_n}{(a;q^2)_m(aq/b;q)_m}\frac{(x/b^2q)^m}{(q;q)_m}\frac{x^n}{(q;q)_n}$$
$$=\sum_{n=0}^{\infty}\Omega(n)\frac{(a/b^2q;q)_n}{(q;q)_n}\frac{1-aq^{2n-1}/b^2}{1-a/b^2q}\frac{(1/bq;q)_n}{(aq/b;q)_n}\,x^n \tag{1}$$

provided that both series in (1) are absolutely convergent.

Proof By means of the series rearrangement, we may rewrite the left-hand side of (1) as

$$\sum_{n=0}^{\infty}\Omega(n)\sum_{m=0}^{n}\frac{(a;q)_m(aq^2;q^2)_m(b;q)_m(1/b^2q;q)_{n-m}}{(q;q)_m(a;q^2)_m(aq/b;q)_m(q;q)_{n-m}}\,x^{n-m}(x/b^2q)^m$$
$$=\sum_{n=0}^{\infty}\Omega(n)\frac{(1/b^2q;q)_n}{(q;q)_n}\,x^n\sum_{m=0}^{n}\frac{(a,q\sqrt{a},-q\sqrt{a},b,q^{-n};q)_m}{(q,\sqrt{a},-\sqrt{a},aq/b,b^2q^{2-n};q)_m}\,q^m$$
$$=\sum_{n=0}^{\infty}\Omega(n)\frac{(1/b^2q;q)_n}{(q;q)_n}\,x^n\;{}_5\Phi_4\left[\begin{array}{ccccc} a, & q\sqrt{a}, & -q\sqrt{a}, & b, & q^{-n} \\ & \sqrt{a}, & -\sqrt{a}, & aq/b, & b^2q^{2-n}\end{array};q,q\right].$$

And then making use of the following summation formula

$${}_5\Phi_4\left[\begin{array}{ccccc} a, & q\sqrt{a}, & -q\sqrt{a}, & b, & q^{-n} \\ & \sqrt{a}, & -\sqrt{a}, & aq/b, & b^2q^{2-n}\end{array};q,q\right]=\frac{(a/b^2;q)_{n-1}(1/bq;q)_n(1-aq^{2n-1}/b^2)}{(aq/b;q)_n(1/b^2q;q)_n}$$

(see [8, 3.4.1.7]), we get

$$\sum_{n=0}^{\infty} \frac{\Omega(n)}{(q;q)_n}\ x^n\ \frac{(a/b^2;q)_{n-1}(1/bq;q)_n(1-aq^{2n-1}/b^2)}{(aq/b;q)_n}$$
$$= \sum_{n=0}^{\infty} \Omega(n)\frac{(a/b^2q;q)_n(1/bq;q)_n(1-aq^{2n-1}/b^2)}{(q;q)_n(aq/b;q)_n(1-a/b^2q)}\ x^n,$$

which completes the proof. □

Theorem 2.2 *For* $(|x|, |x/b^2q|) < 1$, *we have*

$${}_4\Phi_3\left[\begin{matrix} a/b^2q, & q\sqrt{a/b^2q}, & -q\sqrt{a/b^2q}, & 1/bq \\ & \sqrt{a/b^2q}, & -\sqrt{a/b^2q}, & aq/b \end{matrix} ;q,x\right]$$
$$= \frac{(x/b^2q;q)_\infty}{(x;q)_\infty}\ {}_4\Phi_3\left[\begin{matrix} a, & q\sqrt{a}, & -q\sqrt{a}, & b \\ & \sqrt{a}, & -\sqrt{a}, & aq/b \end{matrix} ;q,x/b^2q\right]. \tag{2}$$

Proof Taking $\Omega(n) = 1$ in Proposition 2.1 and making use of the q-binomial theorem

$${}_1\Phi_0\left[\begin{matrix} a \\ - \end{matrix} ;q,z\right] = \frac{(az;q)_\infty}{(z;q)_\infty},$$

we obtain

$$\sum_{n=0}^{\infty} \frac{(a/b^2q;q)_n}{(q;q)_n}\frac{1-aq^{2n-1}/b^2}{1-a/b^2q}\frac{(1/bq;q)_n}{(aq/b;q)_n}\ x^n$$
$$= \sum_{m=0}^{\infty} \frac{(a,b;q)_m(aq^2;q^2)_m}{(q;q)_m(aq/b;q)_m(a;q^2)_m}\ \left(x/b^2q\right)^m \sum_{n=0}^{\infty}\frac{(1/b^2q;q)_n}{(q;q)_n}\ x^n,$$

i.e.,

$${}_4\Phi_3\left[\begin{matrix} a/b^2q, & q\sqrt{a/b^2q}, & -q\sqrt{a/b^2q}, & 1/bq \\ & \sqrt{a/b^2q}, & -\sqrt{a/b^2q}, & aq/b \end{matrix} ;q,x\right]$$
$$= \frac{(x/b^2q;q)_\infty}{(x;q)_\infty}\ {}_4\Phi_3\left[\begin{matrix} a, & q\sqrt{a}, & -q\sqrt{a}, & b \\ & \sqrt{a}, & -\sqrt{a}, & aq/b \end{matrix} ;q,x/b^2q\right]. \quad □$$

Theorem 2.3 *For* $(|a/b^3cd|, |aq/bcd|) < 1$, *we have*

$$\Phi^{2:4;1}_{2:3;0}\left[\begin{matrix} c,d : a, q\sqrt{a}, -q\sqrt{a}, b; 1/b^2q \\ a/b^2c, a/b^2d : \sqrt{a}, -\sqrt{a}, aq/b; - \end{matrix} ;q, a/b^3cd, aq/bcd\right]$$
$$= \frac{(a/b^2, aq/bc, aq/bd, a/b^2cd;q)_\infty}{(aq/b, a/b^2c, a/b^2d, aq/bcd;q)_\infty}. \tag{3}$$

Proof Taking $\Omega(n) = \frac{(c,d;q)_n}{(a/b^2c,a/b^2d;q)_n}$, $x = aq/bcd$ in Proposition 2.1 and using the nonterminating ${}_6\Phi_5$ summation formula

$${}_6\Phi_5\left[\begin{matrix} a, & q\sqrt{a}, & -q\sqrt{a}, & b, & c, & d \\ & \sqrt{a}, & -\sqrt{a}, & aq/b, & aq/c, & aq/d \end{matrix} ;q, aq/bcd\right]$$
$$= \frac{(aq, aq/bc, aq/bd, aq/cd;q)_\infty}{(aq/b, aq/c, aq/d, aq/bcd;q)_\infty} \tag{4}$$

yields the conclusion (see [3, 2.7.1]). □

When $c = q^{-N}$ in (3), we obtain the following terminating form.

Theorem 2.4 *We have*

$$\Phi^{2:4;1}_{2:3;0}\left[\begin{matrix} q^{-N}, d : a, q\sqrt{a}, -q\sqrt{a}, b; 1/b^2q \\ aq^N/b^2, a/b^2d : \sqrt{a}, -\sqrt{a}, aq/b; - \end{matrix} ; q, aq^N/b^3d, aq^{1+N}/bd\right]$$
$$= \frac{(a/b^2, aq/bd; q)_N}{(aq/b, a/b^2d; q)_N}. \tag{5}$$

Theorem 2.5 *We have*

$$\Phi^{4:4;1}_{4:3;0}\left[\begin{matrix} c, d, e, q^{-N} : a, b, q\sqrt{a}, -q\sqrt{a}; 1/b^2q \\ a/b^2c, a/b^2d, a/b^2e, aq^N/b^2 : \sqrt{a}, -\sqrt{a}, aq/b; - \end{matrix} ; q, a^2q^N/b^5cde, a^2q^{1+N}/b^3cde\right]$$
$$= \frac{(a/b^2, a/b^2de; q)_N}{(a/b^2d, a/b^2e; q)_N}\, {}_4\Phi_3\left[\begin{matrix} q^{-N}, & d, & e, & aq/bc \\ & aq/b, & a/b^2c, & b^2deq^{1-N}/a \end{matrix} ; q, q\right].$$

Proof The proof is completed by taking $\Omega(n) = \frac{(c,d,e,q^{-N};q)_n}{(a/b^2c,a/b^2d,a/b^2e,aq^N/b^2;q)_n}$, $x = \frac{a^2q^{1+N}}{b^3cde}$ in Proposition 2.1 and using Watson's transformation formula for a terminating very-well-poised ${}_8\Phi_7$ series

$${}_8\Phi_7\left[\begin{matrix} a, & q\sqrt{a}, & -q\sqrt{a}, & b, & c, & d, & e, & q^{-n} \\ & \sqrt{a}, & -\sqrt{a}, & aq/b, & aq/c, & aq/d, & aq/e, & aq^{1+n} \end{matrix} ; q, a^2q^{2+n}/bcde\right]$$

$$= \frac{(aq, aq/de; q)_n}{(aq/d, aq/e; q)_n}\, {}_4\Phi_3\left[\begin{matrix} q^{-n}, & d, & e, & aq/bc \\ & aq/b, & aq/c, & deq^{-n}/a \end{matrix} ; q, q\right].$$ □

Proposition 2.6 *Let $\{\Omega(n)\}_{n=0}^{\infty}$ be an arbitrary complex sequence. Then we have*

$$\sum_{m,n=0}^{\infty} \Omega(m+n)\frac{(aq^{1/2}/c; q)_{2m+n}}{(aq; q)_{2m+n}}\frac{(a, c; q)_m (q^{1/2}/c; q)_n}{(aq/c; q)_m}\frac{(c^{-1}xq^{3/2})^m}{(q; q)_m}\frac{x^n}{(q; q)_n}$$
$$= \frac{1+\sqrt{a}}{2\sqrt{a}}\sum_{n=0}^{\infty}\Omega(n)\frac{(aq^{1/2}/c, \sqrt{q}, \sqrt{aq}/c, q\sqrt{a}/c; q)_n}{(q, aq/c, q\sqrt{a}, \sqrt{aq}; q)_n}x^n -$$
$$\frac{1-\sqrt{a}}{2\sqrt{a}}\sum_{n=0}^{\infty}\Omega(n)\frac{(aq^{1/2}/c, \sqrt{q}, -\sqrt{aq}/c, -q\sqrt{a}/c; q)_n}{(q, aq/c, -q\sqrt{a}, -\sqrt{aq}; q)_n}x^n, \tag{6}$$

provided that all the series involved are absolutely convergent.

Proof By the series rearrangement, we reformulate the left-hand side in (6) as follows

$$\sum_{n=0}^{\infty}\Omega(n)\frac{(aq^{1/2}/c; q)_n}{(aq; q)_n}x^n\sum_{m=0}^{n}\frac{(a, c, aq^{n+1/2}/c; q)_m}{(q, aq/c, aq^{n+1}; q)_m}\frac{(q^{1/2}/c; q)_{n-m}}{(q; q)_{n-m}}(c^{-1}q^{3/2})^m$$
$$= \sum_{n=0}^{\infty}\Omega(n)\frac{(aq^{1/2}/c, q^{1/2}/c; q)_n}{(q, aq; q)_n}x^n \sum_{m=0}^{n}\frac{(a, c, aq^{n+\frac{1}{2}}/c, q^{-n}; q)_m}{(q, aq/c, aq^{n+1}, cq^{\frac{1}{2}-n}; q)_m}q^{2m}.$$

Then, by means of the following summation formula

$$
{}_4\Phi_3\left[\begin{matrix} a, & c, & aq^{N+1/2}/c, & q^{-N} \\ & aq/c, & cq^{1/2-N}, & aq^{N+1} \end{matrix}; q, q^2\right]
$$
$$
= \frac{1+\sqrt{a}}{2\sqrt{a}}\frac{(aq,\sqrt{q},\sqrt{aq}/c,q\sqrt{a}/c;q)_N}{(aq/c,\sqrt{q}/c,\sqrt{aq},q\sqrt{a};q)_N} - \frac{1-\sqrt{a}}{2\sqrt{a}}\frac{(aq,\sqrt{q},-\sqrt{aq}/c,-q\sqrt{a}/c;q)_N}{(aq/c,\sqrt{q}/c,-\sqrt{aq},-q\sqrt{a};q)_N},
$$

(see [6]), we have

$$
\frac{1+\sqrt{a}}{2\sqrt{a}}\sum_{n=0}^{\infty}\Omega(n)\frac{(aq^{1/2}/c,\sqrt{q},\sqrt{aq}/c,q\sqrt{a}/c;q)_n}{(q,aq/c,q\sqrt{a},\sqrt{aq};q)_n}x^n -
$$
$$
\frac{1-\sqrt{a}}{2\sqrt{a}}\sum_{n=0}^{\infty}\Omega(n)\frac{(aq^{1/2}/c,\sqrt{q},-\sqrt{aq}/c,-q\sqrt{a}/c;q)_n}{(q,aq/c,-q\sqrt{a},-\sqrt{aq};q)_n}x^n,
$$

as desired. □

Theorem 2.7 We have

$$
\sum_{m,n=0}^{\infty}\frac{(q^{5/4}\sqrt{a/c},-q^{5/4}\sqrt{a/c};q)_{m+n}}{(q^{1/4}\sqrt{a/c},-q^{1/4}\sqrt{a/c};q)_{m+n}}\frac{(aq^{1/2}/c;q)_{2m+n}}{(aq;q)_{2m+n}}\frac{(a,c;q)_m(q^{1/2}/c;q)_n}{(aq/c;q)_m}\frac{q^m}{(q;q)_m}\frac{(cq^{-1/2})^n}{(q;q)_n}
$$
$$
= \frac{1+\sqrt{a}}{2\sqrt{a}}\frac{(aq^{3/2}/c,\sqrt{a},c;q)_\infty}{(aq/c,q\sqrt{a},c/\sqrt{q};q)_\infty} - \frac{1-\sqrt{a}}{2\sqrt{a}}\frac{(aq^{3/2}/c,-\sqrt{a},c;q)_\infty}{(aq/c,-q\sqrt{a},c/\sqrt{q};q)_\infty}.
$$

Proof Putting $\Omega(n)=\frac{(q^{5/4}\sqrt{a/c},-q^{5/4}\sqrt{a/c};q)_n}{(q^{1/4}\sqrt{a/c},-q^{1/4}\sqrt{a/c};q)_n}$, $x=q^{-1/2}c$ in (6) and making use of the non-terminating ${}_6\Phi_5$ summation formula (4) gives the desired result. □

Theorem 2.8 We have

$$
\sum_{m,n=0}^{\infty}\frac{(q^{5/4}\sqrt{a/c},-q^{5/4}\sqrt{a/c},d,q^{-N};q)_{m+n}}{(q^{1/4}\sqrt{a/c},-q^{1/4}\sqrt{a/c},aq^{3/2}/cd,aq^{3/2+N}/c;q)_{m+n}}\times
$$
$$
\frac{(aq^{1/2}/c;q)_{2m+n}}{(aq;q)_{2m+n}}\frac{(a,c;q)_m(q^{1/2}/c;q)_n}{(aq/c;q)_m}\frac{(ac^{-1}d^{-1}q^{N+5/2})^m}{(q;q)_m}\frac{(aq^{N+1}/d)^n}{(q;q)_n}
$$
$$
= \frac{1+\sqrt{a}}{2\sqrt{a}}\frac{(aq^{3/2}/c,\sqrt{aq};q)_N}{(aq/c,q\sqrt{a};q)_N}\ {}_4\Phi_3\left[\begin{matrix} q^{-N}, & \sqrt{q}, & \sqrt{aq}/c, & \sqrt{aq}/d \\ & aq^{3/2}/cd, & \sqrt{aq}, & q^{1/2-N}/\sqrt{a}\end{matrix}; q, q\right] -
$$
$$
\frac{1-\sqrt{a}}{2\sqrt{a}}\frac{(aq^{3/2}/c,-\sqrt{aq};q)_N}{(aq/c,-q\sqrt{a};q)_N}\ {}_4\Phi_3\left[\begin{matrix} q^{-N}, & \sqrt{q}, & -\sqrt{aq}/c, & -\sqrt{aq}/d \\ & aq^{3/2}/cd, & -\sqrt{aq}, & -q^{1/2-N}/\sqrt{a}\end{matrix}; q, q\right].
$$

Proof Taking $\Omega(n)=\frac{(q^{5/4}\sqrt{a/c},-q^{5/4}\sqrt{a/c},d,q^{-N};q)_n}{(q^{1/4}\sqrt{a/c},-q^{1/4}\sqrt{a/c},aq^{3/2}/cd,aq^{3/2+N}/c;q)_n}$, $x=aq^{N+1}/d$ in (6) and using Watson's q-Whipple transformation formula gives the desired result. □

Proposition 2.9 *Let $\Omega(n)_{n=0}^{\infty}$ be an arbitrary complex sequence. Then we have*

$$
\sum_{m,n=0}^{\infty}\Omega(m+n)\frac{(a,b;q)_m}{(abq^{\frac{1}{2}};q)_m}\frac{(a,b;q)_n}{(abq^{\frac{1}{2}};q)_n}\frac{(xq^{\frac{1}{2}})^m}{(q;q)_m}\frac{x^n}{(q;q)_n}
$$
$$
= \sum_{n=0}^{\infty}\Omega(n)\frac{(a,b,\sqrt{ab},-\sqrt{ab};q^{1/2})_n}{(q^{1/2},ab,q^{\frac{1}{4}}\sqrt{ab},-q^{\frac{1}{4}}\sqrt{ab};q^{1/2})_n}x^n. \quad (7)
$$

Proof By the summation formula

$$
{}_4\Phi_3\left[\begin{matrix} q^{-N}, & a, & b, & \frac{1}{ab}q^{\frac{1}{2}-N} \\ & \frac{1}{a}q^{1-N}, & \frac{1}{b}q^{1-N}, & abq^{\frac{1}{2}} \end{matrix}; q, q^2\right] = \frac{(ab;q)_N(a,b,-\sqrt{q};q^{1/2})_N}{(ab;q^{1/2})_N(a,b;q)_N},
$$

(see [1]), we may reformulate the left-hand side of (7):

$$
\begin{aligned}
&\sum_{n=0}^{\infty}\Omega(n)x^n\sum_{m=0}^{n}\frac{(a,b;q)_m}{(q,abq^{\frac{1}{2}};q)_m}\frac{(a,b;q)_{n-m}}{(q,abq^{\frac{1}{2}};q)_{n-m}}q^{\frac{1}{2}m}\\
&=\sum_{n=0}^{\infty}\Omega(n)x^n\frac{(a,b;q)_n}{(q,abq^{\frac{1}{2}};q)_n}\sum_{m=0}^{n}\frac{(a,b;q)_m}{(q,abq^{\frac{1}{2}};q)_m}\frac{(q^{-n},\frac{1}{ab}q^{\frac{1}{2}-n};q)_m}{(\frac{1}{a}q^{1-n},\frac{1}{b}q^{1-n};q)_m}q^{2m}\\
&=\sum_{n=0}^{\infty}\Omega(n)\frac{(a,b,\sqrt{ab},-\sqrt{ab};q^{1/2})_n}{(q^{1/2},ab,q^{\frac{1}{4}}\sqrt{ab},-q^{\frac{1}{4}}\sqrt{ab};q^{1/2})_n}x^n.
\end{aligned}
$$

This completes the proof. □

Theorem 2.10 We have

$$
\begin{aligned}
&\Phi^{7:4;4}_{7:3;3}\left[\begin{matrix} q^{-N/2}, & \sqrt{aq}, & -\sqrt{aq}, & c, & ab, & q^{1/4}\sqrt{ab}, & -q^{1/4}\sqrt{ab}: \\ \sqrt{a}, & -\sqrt{a}, & aq^{1/2}/b, & \sqrt{aq/b}, & -\sqrt{aq/b}, & aq^{1/2}/c, & aq^{(1+N)/2}: \end{matrix}\right.\\
&\left.\begin{matrix}\sqrt{a},-\sqrt{a},\sqrt{b},-\sqrt{b};\sqrt{a},-\sqrt{a},\sqrt{b},-\sqrt{b}\\ -\sqrt{q},q^{1/4}\sqrt{ab},-q^{1/4}\sqrt{ab};-\sqrt{q},q^{1/4}\sqrt{ab},-q^{1/4}\sqrt{ab}\end{matrix}; q^{1/2},-\frac{aq^{\frac{N}{2}+\frac{3}{2}}}{b^2c},-\frac{aq^{\frac{N+2}{2}}}{b^2c}\right]\\
&=\frac{(aq^{1/2},-q^{1/2}/b;q^{1/2})_N}{(\sqrt{aq/b},-\sqrt{aq/b};q^{1/2})_N}\;{}_4\Phi_3\left[\begin{matrix} aq^{\frac{1}{2}}/bc, & \sqrt{ab}, & -\sqrt{ab}, & q^{-N/2} \\ & aq^{\frac{1}{2}}/b, & aq^{\frac{1}{2}}/c, & -bq^{-N/2}\end{matrix}; q^{1/2},q^{1/2}\right].
\end{aligned}
$$

Proof It suffices to take $\Omega(n)=\frac{(\sqrt{aq},-\sqrt{aq},c,q^{-N/2};q^{1/2})_n(ab,q^{1/4}\sqrt{ab},-q^{1/4}\sqrt{ab};q^{1/2})_n}{(\sqrt{a},-\sqrt{a},aq^{1/2}/b,\sqrt{aq/b},-\sqrt{aq/b},aq^{1/2}/c,aq^{(1+N)/2};q^{1/2})_n}$ and $x=-\frac{aq^{(2+N)/2}}{b^2c}$ in (7) and then to make use of Watson's q-Whipple transformation formula. □

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