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F-Covers for Right Type-A Semigroups

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Abstract A right adequate semigroup of type F is defined as a right adequate semigroup which is an F-rpp semigroup. A right adequate semigroup T of type F is called an F-cover for a right type-A semigroup S if S is the image of T under an $\mathcal{L}^*$-homomorphism. In this paper, we will prove that any right type-A monoid has F-covers and then establish the structure of F-covers for a given right type-A monoid. Our results extend and enrich the related results for inverse semigroups.

Keywords right type-A semigroup; F-rpp semigroup; left cancellative monoid; $\mathcal{L}^*$-homomorphism; $*$-homomorphism; F-cover.

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1. Introduction

An inverse semigroup S is called F-inverse if there exists a group congruence σ on S such that each σ-class has greatest element with respect to the natural partial order $\leq$ on S. McFadden and O'Carroll [19] pointed out that the concept of F-inverse semigroups is indeed a generalization of residuated inverse semigroups. Later on, Edwards [3] defined analogously F-regular semigroups and F-orthodox semigroups and showed that an F-regular semigroup is indeed an F-orthodox semigroup.

A semigroup is called rpp if for any $a \in S$, aS^1, regarded as an S-system, is projective. Dually, lpp semigroups may be defined. In [4], Fountain pointed out that a semigroup S is rpp if and only if every $\mathcal{L}^*$-class of S contains at least one idempotent. Following Fountain, a semigroup is called abundant if and only if it is both rpp and lpp. As in [4], an rpp semigroup S is said to be a right adequate semigroup if the set of idempotents of S forms a commutative subsemigroup, that is, a semilattice. It is easy to see that each $\mathcal{L}^*$-class of a right adequate semigroup contains exactly one idempotent. For convenience, we denote by a^* the idempotent related to $a \in S$ by $\mathcal{L}^*$. A right adequate semigroup S is called a right type-A semigroup if for any $a, e^2 = e \in S$,

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$ea = a(ea)^*$. Dually, we define left adequate semigroups and left type-A semigroups. We call a semigroup S an adequate semigroup if S is both a left adequate semigroup and a right adequate semigroup. Also, we call S a type-A semigroup if S is both a left type-A semigroup and a right type-A semigroup. Regular semigroups are abundant semigroups and inverse semigroups are type-A semigroups.

In order to generalize the F-regular semigroups, Guo [9] defined F-abundant semigroup. So-called an F-abundant semigroup is an abundant semigroup in which there exists a cancellative congruence σ such that each σ-class contains a greatest element with respect to the Lawson order $\leq$. In the same reference, Guo established the structure of a class of F-abundant semigroups, namely, strongly F-abundant semigroups by utilizing an SF-system. In [20], Ni, Chen and the second author obtained a structure of general F-abundant semigroups.

Parallelizing F-inverse semigroups, Li, Shum and the second author [10] defined F-rpp semigroups. We call an rpp semigroup S an F-rpp semigroup if there exists a left cancellative monoid congruence ρ on S such that each ρ-class contains a greatest element with respect to the Lawson partial order $\leq_\ell$ on S (for the Lawson orders, see [17]). By introducing SFR-systems, they established a structure for strongly F-rpp semigroups. Recently, Huang, Chen and the second author [16] obtained the structure of general F-rpp semigroups. In [1], Cui and Guo established the structure of right adequate semigroups of type F (such a semigroup is a right adequate semigroup which is F-rpp). Recently, the authors [2] gave a new structure of right adequate semigroups of type F.

An F-inverse (E-unitary inverse) semigroup T is called an F-cover (E-unitary cover) for an inverse semigroup S if S is an idempotent-separating homomorphic image of T. It is well known that any inverse semigroup has F-covers (E-unitary covers). Along this direction, it is found that any orthodox semigroup has E-unitary covers. Guo [9] established the structure of E-unitary covers for an orthodox semigroup. Fountain pointed out that any left type-A semigroup has proper covers (such proper covers are analogue of E-unitary covers in the range of left type-A semigroups). Later on, Guo and Xie [13] gave a construction method of proper covers for a left type-A semigroups. Guo and Tian [12], Huang, Guo and Shen [15] considered the problems on proper covers for left GC-lpp semigroups. This raises a natural problem: whether do left type-A semigroups have any F-covers? This is the aim of this paper.

In this paper, we shall prove that any right type-A semigroup has F-covers and then provide a method constructing F-covers for any right type-A semigroups.

2. Preliminaries

Throughout this paper we use the notions and terminologies of Fountain [4] and Howie [14]. Now, we provide some known results repeatedly used without mentions in the sequel.

Lemma 2.1 ([4]) *Let S be a semigroup and $a, b \in S$. Then the following statements are equivalent:*

(1) $a\mathcal{L}^*b$.

(2) For all $x, y \in S^1$, $ax = ay$ if and only if $bx = by$.

Evidently, $\mathcal{L}^*$ is a right congruence while $\mathcal{R}^*$ is a left congruence. In general, we have $\mathcal{L} \subseteq \mathcal{L}^*$ and $\mathcal{R} \subseteq \mathcal{R}^*$. But if a, b are regular elements, $a\mathcal{L}^*b$ $[a\mathcal{R}^*b]$ if and only if $a\mathcal{L}b$ $[a\mathcal{R}b]$. For the sake of convenience, we use a^* to denote a typical idempotent $\mathcal{L}^*$-related to a, and $a^\dagger$ to denote that $\mathcal{R}^*$-related to a.

Proposition 2.2 ([4]) *If S is a right adequate semigroup with semilattice of idempotents E, then*

(1) For all $a, b \in S$, $(ab)^* = (a^*b)^*$.

(2) For all $a, b \in S$, $(ab)^*\omega b^*$.

Let S be an rpp semigroup. As in [17], we define a relation S by

$$x \leq_\ell y \text{ if and only if } L^*(x) \subseteq L^*(y) \text{ and there exists } f \in E(S) \bigcap L^*_x \text{ such that } x = yf,$$

where $L^*(x)$ is the left *-ideal generated by x (see [5]) and L^*_x denotes the $\mathcal{L}^*$-class of S containing x. Then $\leq_\ell$ is a partial order on S. Dually, we may define the partial order $\leq_r$ on an lpp semigroup. If S is an abundant semigroup, we define the partial order $\leq$ on S as $\leq_\ell \cap \leq_r$. Equivalently, for $a, b \in S$, $a \leq b$ if and only if there exist $e, f \in E(S)$ such that $a = eb = bf$ (see [17]). We shall denote $G_a = \{s \in S : s \leq_\ell a\}$ for $a \in S$.

Lemma 2.3 ([17]) *Let S be an rpp semigroup. Then for $x, y \in S$ and $e \in E(S)$, the following statements hold:*

(1) $\leq_\ell$ is a partial order on S, in particular, $\leq_\ell$ coincides with the usual idempotent order ω on $E(S)$, that is, $e\omega f$ if and only if $e = ef = fe$.

(2) If $x \leq_\ell e$, then $x^2 = x$ in S.

(3) If $x \leq_\ell y$ and y is a regular element in S, then x is also a regular element in S.

(4) Let $y^* \in L^*_y \bigcap E(S)$ and $\omega(y^*) = \{f : f\omega y^*\}$. Then $x \leq_\ell y$ if and only if for all [some] y^*, there exists $f \in \omega(y^*)$ such that $x = yf$.

(5) If $x \leq_\ell y$ and $x\mathcal{L}^*y$, then $x = y$.

A congruence ρ on a semigroup S is called a left cancellative monoid congruence on S if S/ρ is a left cancellative monoid.

By an F-rpp semigroup, we mean an rpp semigroup in which there exists a left cancellative monoid congruence σ on S such that each σ-class of S contains a greatest element with respect to the Lawson order $\leq_\ell$. In this case, the σ is indeed the smallest left cancellative monoid congruence on S (see [10]). In what follows, we use σ to denote the smallest left cancellative monoid congruence on S if it exists.

Assume that S is an F-rpp semigroup. We denote by M the set of greatest elements in all σ-classes of S and m_a the greatest element in σ_a. In general, M does not form a subsemigroup of S. Now define a multiplication $\circ$ as follows:

$$m \circ n = \text{the greatest element of the } \sigma\text{-class of } S \text{ containing } mn.$$

It is not difficult to check that $(M, \circ)$ is a semigroup isomorphic to S/σ.

Lemma 2.4 ([15]) *If S is an F-abundant semigroup, then*

(1) $G_1 = E$, where 1 is the identity of S.

(2) $G_m G_n \subseteq G_{m\circ n}$ for any $m, n \in M(S)$.

Lemma 2.5 ([1]) *If S is a right adequate semigroup of type F, then*

(1) S is a right type A semigroup (that is, a right adequate semigroup in which for any $a, e^2 = e \in S$, $ea = a(ea)^*$).

(2) The smallest left cancellative monoid congruence σ on S is equal to $\{(a,b) \in S \times S : (\exists e^2 = e \in S)\ ae = be\}$.

(3) $\sigma \cap \mathcal{L}^* = \mathrm{id}$ (the identity relation on S).

Let ϕ be a homomorphism of the semigroup S into another T. ϕ is called an $\mathcal{L}^*$-homomorphism if for any $a, b \in S$, $\phi(a) = \phi(b)$ implies that $a\mathcal{L}^*b$. Obviously, any $\mathcal{L}^*$-homomorphism of S into T is idempotent-separating when S is a right adequate semigroup.

Assume S is a right adequate semigroup. Then every $\mathcal{L}^*$-class of S contains a unique idempotent. We denote the unique idempotent in L_a^* by a^*. So, we may regard a right adequate semigroup as an algebra with a binary operation of multiplication and a unary operation $*$. We will refer to such algebras as $*$-semigroups. From universal algebras, we have the notions of $*$-subsemigroup, $*$-homomorphism and $*$-congruence. It is not difficult to see that a homomorphism is a $*$-homomorphism if and only if it preserves the $\mathcal{L}^*$-classes.

3. F-covers

To begin with, we formulate the definition of F-covers.

Definition 3.1 *Let S be a right type-A semigroup and T a right adequate semigroup of type F. Then T is called an F-cover for S if there exists a surjective $*$-homomorphism φ of T onto S which also is an $\mathcal{L}^*$-homomorphism of T into S.*

The aim of this section is to prove the following theorem.

Theorem 3.2 *Any right type-A monoid has F-covers.*

To prove Theorem 3.2, we need to recall the construction of free right type-A semigroups.

Let X be a non-empty set. Let F_X be the free monoid on X and partially order on F_X by putting $u \leq v$ if and only if u is a final segment of v. For any subset A of F_X, we write

$$\max A = \{a \in A : \ a \text{ is maximal in } A \text{ under } \leq\}.$$

Now let

$$E_X = \{A : \ A \subset F_X, A \text{ is finite and non-empty, } A = \max A\}.$$

Thus E_X is the set of all finite suffix codes over X. For $A, B \in E_X$, let $AB = \max(A \cup B)$. Then E_X is a semilattice. In fact, if we consider F_X as partial order by the dual of note, the following statements are equivalent for members A, B of E_X where we use $\leq$ for the order relation in E_X as well as that in F_X:

$A \leq B$; $AB = A$; $\max(A \cup B) = A$; for each $b \in B$, there is $a \in A$ such that $b \leq a$; each element in B is a final segment of some element in A.

For $w \in F_X$, $A \subset F_X$, we put $A \cdot w = \{aw : a \in A\}$. Clearly, $A \cdot w \in E_X$ and we have an action of F_X on E_X. Furthermore, if $w \in F_X, A, B \in E_X$, then it is routine to verify that

$$(AB) \cdot w = (A \cdot w)(B \cdot w)$$

and consequently the action is order-preserving. We form the set

$$A_X = \{(w, A) \in F_X^1 \times E_X : w \leq a \text{ for some } a \in A\}.$$

On A_X, define a multiplication by

$$(w, A) \circ (v, B) = (wv, A \cdot v \wedge B).$$

Then $(A_X, \circ)$ is a free right type-A semigroup [6, P.138-141].

Lemma 3.3 ([6, Proposition 2.6]) *Let (v, A), (w, B) be elements of A_X. Then*

(1) $(v, A)\sigma(w, B)$ if and only if $v = w$.

(2) $(v, A)\mathcal{L}^*(w, B)$ if and only if $A = B$.

Lemma 3.4 ([6, Theorem 2.8]) *Let S be a right type-A semigroup. Then S is the image of some A_X under an $\mathcal{L}^*$-homomorphism which also is a $*$-homomorphism.*

For a non-empty subset $A \subseteq F_X$, we denote by $[A]^{\downarrow}$ the order ideal of $(F_X, \leq)$ generated by A.

Lemma 3.5 *If $A \in E_X$, then $A = \max([A]^{\downarrow})$.*

Proof Clearly $A \subseteq [A]^{\downarrow}$. We show first that every element of A is maximal in the set $[A]^{\downarrow}$. Suppose $a \in A$ is not maximal in $[A]^{\downarrow}$. Then there exists $a' \in [A]^{\downarrow}$ such that $a < a'$. By definition $a' \leq a''$ for some $a'' \in A$. But this implies $a, a'' \in A$ distinct comparable elements of A, which is a contradiction. Thus $A \subseteq \max([A]^{\downarrow})$. Conversely, let $b \in \max([A]^{\downarrow})$. By definition $b \leq c$ for some $c \in A$. But b is maximal, and so $b = c \in A$. Thus $A = \max([A]^{\downarrow})$.

Proof of Theorem 3.2 By Lemma 3.4, we need only to prove that A_X is F-rpp. To this end, we let (w, A) be any element of A_X, then by Lemma 3.3, $\sigma_{(w,A)} = \{(w, B) \in A_X : w \in B\}$. Now let $x \in \max([w]^{\downarrow})$. Then $x \leq w$ and so by definition, $B \leq [w]^{\downarrow}$. Clearly $(w, \max([w]^{\downarrow}) \in \sigma_{(w,A)}$ and is the greatest element of $\sigma_{(w,A)}$. Consequently, A_X is F-rpp and we complete the proof. $\Box$

4. The structure of F-covers

In this section we will establish the structure of F-covers for a right type-A semigroup. $\mathcal{P}(S)$ is a set containing all subset of S. In what follows, we denote $A \cdot B := \{ab : a \in A, b \in B\}$ for any $A, B \in \mathcal{P}(S)$. It is easy to see that $(\mathcal{P}(S), \cdot)$ is a commutative semigroup.

Definition 4.1 *Let S be a monoid with identity 1, T a semigroup. A mapping $\phi : S \to \mathcal{P}(T)$ is called a pre-homomorphism if the following conditions are satisfied:*

(1) $\phi(1) = E(T)$;

(2) *for all* $x, y \in S, \phi(x)\phi(y) \subseteq \phi(xy)$.

Definition 4.2 *Let* M *be a left cancellative monoid,* T *a right type-*A *monoid. If* ϕ *is a mapping from* M *into* $\mathcal{P}(T)$*. Then* $(M, T; \phi)$ *is called an* $\mathcal{FRA}^*$*-system if*

(FRA1) ϕ *is pre-homomorphism;*

(FRA2) *for all* $m \in M$*, there exists* $m_a \in T$ *such that* $\phi(m) = G_{m_a}$.

Given an $\mathcal{FRA}$-system $(M, T; \phi)$, put

$$FRA(M, T; \phi) = FRA = \{(m, x) \in M \times T : x \in \phi(m)\}.$$

By routine computing, FRA is a subsemigroup of $M \times T$. Moreover, we may prove

Lemma 4.3 *The above semigroup* FRA *is a monoid.*

Proof By $(FRA2)$, there exists $e \in E(T)$ such that $\phi(1) = G_e$ so that e is the identity of $E(T)$. Hence e is the identity of T. The following result is immediate.

Lemma 4.4 $E(FRA) = \{(1, f) : f \in E(T)\}$ *and is isomorphic to* $E(T)$*. Moreover,* $E(FRA)$ *has* $(1, e)$ *as its identity.*

Proposition 4.5 *Let* $(M, T; \phi)$ *be an* $\mathcal{FRA}$*-system.*

(1) *For all* $(m, x), (n, y) \in FRA$*,* $(m, x)\mathcal{L}^*(n, y)$ *if and only if* $x\mathcal{L}^*y$.

(2) *For all* $(m, x), (n, y) \in FRA$*,* $(m, x) \leq_\ell (n, y)$ *if and only if* $m = n, x \leq_\ell y$.

(3) *For all* $(m, x), (n, y) \in FRA$*,* $(m, x)\sigma(n, y)$ *if and only if* $m = n, x\sigma y$.

(4) FRA *is a right adequate monoid of type* F.

Proof (1) First of all, we verify that $(m, x)\mathcal{L}^*(1, x^*)$. Now let $(\delta, t), (\gamma, u) \in FRA$ with $(m, x)(\delta, t) = (m, x)(\gamma, u)$. Then $(m\delta, xt) = (m\gamma, xu)$, and so $m\delta = m\gamma, xt = xu$. Thereby $\delta = \gamma$ and $x^*t = x^*u$. Thus $(1, x^*)(\delta, t) = (1, x^*)(\gamma, u)$. Together with the fact $(m, x)(1, x^*) = (m, x)$, we have that $(m, x)\mathcal{L}^*(1, x^*)$.

Now, by the above proof, we have

$$\begin{aligned}(m, x)\mathcal{L}^*(n, y) &\Leftrightarrow (1, x^*)\mathcal{L}^*(1, y^*) \Leftrightarrow x^*y^* = x^*, y^*x^* = y^* \\ &\Leftrightarrow x^*\mathcal{L}y^* \Leftrightarrow x\mathcal{L}^*y.\end{aligned}$$

(2) It follows from the computation:

$$\begin{aligned}(m, x) \leq_\ell (n, y) &\Leftrightarrow (\exists(1, u) \in \omega(1, y^*))(m, x) = (n, y)(1, u) \\ &\Leftrightarrow m = n, x \leq_\ell y.\end{aligned}$$

(3) By Lemmas 2.5 (2) and 4.4, the proof is a routine checking.

(4) By (3), $\sigma_{(m,x)} = \{(m, y) \in FRA : y \in \phi(m), y\sigma x\}$ for any $(m, x) \in FRA$. Note that $\phi(m) = G_{m_a}$ for some $m_a \in T$. We observe that (m, m_a) is the greatest element in $\sigma_{(m,x)}$, and consequently FRA is a right adequate semigroup of type F.

Consider the mapping $\varphi : (m, x) \mapsto x$ from FRA to T. It is a routine calculation to show that φ is an $\mathcal{L}^*$-homomorphism which is a $*$-homomorphism. Now, the following lemma is immediate.

Lemma 4.6 *The projection $\varphi : (m, x) \mapsto x$ from FRA to T is an idempotent-separating homomorphism.*

We now arrive at the structure theorem of F-covers for right type-A monoids

Theorem 4.7 *Let $(M, T; \phi)$ be an $\mathcal{FRA}$-system. If $T = \cup_{m\in M}\phi(m)$, then $FRA(M, T; \phi)$ is an F-cover for right type-A monoid T.*

Conversely, any F-cover for the right type-A monoid T can be constructed in this manner.

Proof Notice that $T = \cup_{m\in M}\phi(m)$. We observe that the projection of $FRA(M, T; \phi)$ to T is surjective. Hence by Proposition 4.5, we only need to prove the converse part. For this, we assume that φ is a surjective $*$-homomorphism and $\mathcal{L}^*$-homomorphism of a right type-A monoid S of type F into the right adequate monoid T. We denote by M the set of greatest elements in all σ-classes of S. Under the multiplication $\circ$ defined by

$m \circ n =$ the greatest element of the σ-class of S containing mn.

It is not difficult to check that $(M, \circ)$ is a semigroup isomorphic to S/σ.

Define

$$\theta :\ M \to \mathcal{P}(\mathcal{T});\ m \mapsto (G_m)\varphi.$$

Since $S = \cup_{m\in M}G_m$ and φ is surjective, we have $T = \cup_{m\in M}(G_m)\varphi$. Clearly, $E(S)\varphi \subseteq E(T)$. Conversely, for any $x \in E(T)$, there exists $a \in S$ such that $a\varphi = x$ since φ is surjective. Note that φ is a $*$-homomorphism, it is easy to see that $a\varphi\, \mathcal{L}^* a^*\varphi$, hence $a^*\varphi = a\varphi$ since T is a right adequate semigroup. It follows that $E(T) \subseteq E(S)\varphi$. Thus $E(T) = E(S)\varphi$, hence $\theta(1) = E(T)$. On the other hand, since T is a right type-A semigroup, it is easy to check that $a \leq_\ell x, b \leq_\ell y$ imply that $ab \leq_\ell xy$ for any $a, b, x, y \in T$. This can derive that $(G_m)\varphi \cdot (G_n)\varphi \subseteq (G_{m\circ n})\varphi$ for any $m, n \in M$. We have now proved that θ is a pre-homomorphism from M into $\mathcal{P}(T)$.

We can now form the semigroup $FRA(M, T; \theta)$. Furthermore, define

$$\psi :\ S \to FRA(M, T; \theta);\ s \mapsto \psi(s) = (m_s, s\varphi).$$

If $s\psi = t\psi$, then $m_s = m_t$ and $s\varphi = t\varphi$. Hence $(s, t) \in \sigma \cap \mathcal{L}^*$. Thereby $s = t$ since S is F-rpp. Thus ψ is injective.

For any $(m, t) \in FRA(M, T; \theta)$, we have $s \in G_m$ such that $t = s\varphi$. Note that $s \in G_m$, we have $(s, m) \in \sigma$, thus $(m, t) = s\psi$. Consequently, ψ is surjective.

Finally, suppose that $a, b \in S$,

$$\psi(a)\psi(b) = (m_a, a\varphi)(m_b, b\varphi) = (m_{ab}, (ab)\varphi) = \psi(ab).$$

Thus ψ is a homomorphism. We have now proved that ψ is an isomorphism of S onto $FRA(M, T; \theta)$. The proof is completed. $\square$

References

[1] CUI Ranran, GUO Xiaojiang. *A structure theorem for right adequate semigroups of type F* [J]. Int. Math. Forum, 2008, **3**(41-44): 2197–2206.

[2] CUI Ranran, GUO Xiaojiang. *Right Adequate Monoids og type F* [J]. J. Jiangxi Normal Unv. (Sci. Press), 2009, **33**: 5–8. (in Chinese)

[3] EDWARDS C C. *F-regular and F-orthodox semigroups* [J]. Semigroup Forum, 1980, **19**(4): 331–345.

[4] FOUNTAIN J. *Adequate semigroups* [J]. Proc. Edinburgh Math. Soc. (2), 1979, **22**(2): 113–125..

[5] FOUNTAIN J. *Abundant semigroups* [J]. Proc. London Math. Soc. (3), 1982, **44**(1): 103–129.

[6] FOUNTAIN J. *Free right type-A semigroups* [J]. Glasgow Math. J., 1991, **33**(2): 135–148.

[7] GUO Xiaojiang. *On the structure of E-unitary coverse for an orthodox semigroup* [J]. Adv. Math. (China), 1997, **26**: 181–182.

[8] GUO Xiaojiang. *Abundant left C-lpp proper semigroups* [J]. Southeast Asian Bull. Math., 2000, **24**(1): 41–50.

[9] GUO Xiaojiang. *F-abundant semigroups* [J]. Glasg. Math. J., 2001, **43**(2): 153–163.

[10] GUO Xiaojiang, LI Xiaoping, SHUM K P. *F-rpp semigroups* [J]. Int. Math. Forum, 2006, **1**(29-32): 1571–1585.

[11] GUO Xiaojiang, LUO Yanfeng. *The natural partial orders on abundant semigroups* [J]. Adv. Math. (China), 2005, **34**(3): 297–308.

[12] GUO Xiaojiang, TIAN Zhenji. *Right cancellative monoids, left regular bands, and left regular type A monoids* [J]. J. Math. Res. Exposition, 1999, **19**(3): 563–568. (in Chinese)

[13] GUO Xiaojiang, XIE Xiangyun. *A note on left type* A *semigroups* [J]. Semigroup Forum, 1999, **58**(2): 313–316.

[14] HOWIE J M. *An Introduction to Semigroup Theory* [M]. Academic Press, London, 1976.

[15] HUANG Hui, GUO Xiaojiang, X. Shen. *Proper covers for IC quasi-adequate semigroups* [J]. J. Jiangxi Normal Univ. (Nat. Sci.), 2007, **3**: 242–245.

[16] HUANG Hui, GUO Xiaojiang, CHEN Hui. *The structure of F-rpp semigroups* [J]. Int. Math. Forum, 2007, **2**(13-16): 771–779.

[17] LAWSON M V. *The natural partial order on an abundant semigroup* [J]. Proc. Edinburgh Math. Soc. (2), 1987, **30**(2): 169–186.

[18] MCALISTER D B. *Groups, semilattices and inverse semigroups (I), (II)* [J]. Trans. Amer. Math. Soc., 1974, **192**: 227–244

[19] MCFADDEN R, O'CARROLL L. *F-inverse semigroups* [J]. Proc. London Math. Soc. (3), 1971, **22**: 652–666.

[20] NI Xiangfei, CHEN Wei, GUO Xiaojiang. *F-abundant semigroups* [J]. J. Jiangxi Normal Univ. (Natural Sci.), 2005, **29**: 407–410.