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The Inexistence of Symmetrical, Orthonormal and Compactly Supported Three-Band Wavelet System with the Length $(6, m, n)$

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Abstract This paper proves the inexistence of symmetric, orthonormal and commpactly supported three-band wavelet system with the length $(6, m, n)$ in terms of the method of polyphase matrix representation.

Keywords wavelets; polyphase matrix; filters.

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1. Introduction

The construction of wavelet filters with better properties is becoming more and more important, because of its widespread and successful applications in signal and image processing. It is well known that there are some defects about 2-band wavelet. In her celebrated paper [1], Daubechies proved that up to an integer shift and possible sign change, except for the Haar scaling function (i.e., the characteristic function of the interval $[0, 1]$), there were not any dyadic orthonormal scaling functions which are symmetric about some point on R. In recent years, in order to construct wavelets with better properties, many researchers generalized the construction of 2-band wavelet based on the Daubechies' theory, such as M-band ($M \geq 3$) wavelet, multiwavelet theory, complex wavelet and framework theory.

For M-band wavelet, Heller [2], Steffen et al [3] and Wellend [4] constructed compactly supported orthogonal scaling functions ϕ which were not symmetrical, but did not consider the construction of the corresponding wavelet function ψ. Belogay [5], Bi et al [6], Chui and Lian [7] constructed compactly supported orthogonal symmetric or antisymmetric scaling functions ϕ for dilations $M \geq 3$. Furthermore, they also proposed the method of constructing the corresponding symmetric or antisymmetric wavelet function ψ. However, it is not an evident process to construct wavelet function ψ from the conresponding scaling function ϕ when $M = 3$. In the

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literature [7], Chui and Lian proposed a perfect construction of wavelet system and also gave some wavelet examples for dilations $M = 3$. Wang [8] also proposed a new construction method of wavelet system for the dilation $M = 3$.

From engineering application viewpoint, the compactly supported wavelet filters are advantageous because of less computation and more accuracy in the signal processing. However, for 3-band symmetric compactly supported wavelet system, it can be easily proved that there did not exist wavelet filters such that the length of lowpass and highpass filters are $(6, 6, 6)$, $(6, 8, 8)$ and (6,9,9), respectively. In the literature [9], Zhou constructed a 3-band wavelet system such that the length of lowpass filter is 6 and the length of highpass filter is 9. Nevertheless, the highpass filter was not symmetric. Therefore, the existence of a symmetric or antisymmetirc wavelet system for the dilation $M = 3$ with length of lowpass and highpass filter (i.e., the length of compact support of the scaling function) being $(6, m, n)$ is becoming a problem. In this paper, we will discuss the inexistence of these kinds of wavelet systems.

2. 3-band wavelet system

A 3-band wavelet system is constituted by a lowpass filter and two highpass filters, where the filter coefficients are $a_{i,0}, a_{i,1}, \ldots, a_{i,K-1}$, $K \in N^+$, $i = 0, 1, 2$, respectively.

Note that

$$A = \begin{pmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,K-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,K-1} \\ a_{2,0} & a_{2,1} & \cdots & a_{2,K-1} \end{pmatrix}$$

is a $3 \times K$ matrix in the real field, which is constituted by a lowpass filter and two highpass filters of a compactly supported 3-band wavelet system.

Definition 1 *A 3-band wavelet system is orthonormal, if the following equations hold.*

$$\sum_k a_{s,k} a_{s',k+3l} = 3\delta_{s,s'}\delta_{0,l}, \quad s, s' = 0, 1, 2; \tag{1}$$

$$\sum_k a_{s,k} = 3\delta_{s,0}, \quad s = 0, 1, 2; \tag{2}$$

$$\sum_l a_{0,k+3l} = 1, \quad k = 0, 1, 2. \tag{3}$$

Definition 2 *A 3-band wavelet system is symmetric (or antisymmetric) if and only if*

$$a_{s,0} = a_{s,K-1}, a_{s,1} = a_{s,K-2}, \ldots, \quad s = 0, 1, 2; \tag{4}$$

or

$$a_{s,0} = -a_{s,K-1}, a_{s,1} = -a_{s,K-2}, \ldots, \quad s = 0, 1, 2. \tag{5}$$

Definition 3 *Polyphase matrix of a 3-band wavelet system is denoted by*

$$\mathbf{C}(z) = \begin{pmatrix} A_{0,0} & A_{0,1} & A_{0,2} \\ A_{1,0} & A_{1,1} & A_{1,2} \\ A_{2,0} & A_{2,1} & A_{2,2} \end{pmatrix} = \begin{pmatrix} \sum_k a_{0,3k} z^k & \sum_k a_{0,3k+1} z^k & \sum_k a_{0,3k+2} z^k \\ \sum_k a_{1,3k} z^k & \sum_k a_{1,3k+1} z^k & \sum_k a_{1,3k+2} z^k \\ \sum_k a_{2,3k} z^k & \sum_k a_{2,3k+1} z^k & \sum_k a_{2,3k+2} z^k \end{pmatrix}.$$

Proposition 1 *Orthonormal condition (1) of the wavelet system is equivalent to*

$$(\mathbf{C}(z))^*\mathbf{C}(z) = I \quad \text{and} \quad \mathbf{C}(z)(\mathbf{C}(z))^* = I, \tag{6}$$

where $(\mathbf{C}(z))^*$ *denotes conjugate transpose of* $\mathbf{C}(z)$.

Now, we have the following lemmas with the preceding definitions.

Lemma 1 *Consider a 3-band wavelet system orthonormal, compactly supported and symmetric. If the length of its lowpass filter is 6, then the lowpass filter must be*

$$\frac{2-\sqrt{6}}{4}, \frac{1}{2}, \frac{2+\sqrt{6}}{4}, \frac{2+\sqrt{6}}{4}, \frac{1}{2}, \frac{2-\sqrt{6}}{4}.$$

Proof According to the symmetry of the 3-band wavelet system, its lowpass filter with length 6 must be $a_{0,0}$, $a_{0,1}$, $a_{0,2}$, $a_{0,2}$, $a_{0,1}$, $a_{0,0}$. Also, because orthonormal 3-band wavelet system satisfies the above two equations (1) and (3), the following conclusion holds

$$\begin{cases} a_{0,0} + a_{0,2} = 1, \\ a_{0,1} + a_{0,1} = 1, \\ a_{0,0}^2 + a_{0,1}^2 + a_{0,2}^2 + a_{0,2}^2 + a_{0,1}^2 + a_{0,0}^2 = 3, \\ a_{0,0}a_{0,2} + a_{0,1}^2 + a_{0,2}a_{0,0} = 0. \end{cases}$$

The lemma 1 is proved by solving this equations. □

Lemma 2 *Consider a 3-band wavelet system orthonormal and compactly supported. If the lenth of its lowpass filter and two highpass filters is* $(6, m, n)(m, n \in N^+)$ *and* $a_{1,m-1} \neq 0$, $a_{2,n-1} \neq 0$, *then* $(m, 3) \neq 1$, $(n, 3) \neq 1$, *where* $(m, 3)$ *denotes remainder of* m *dividing 3.*

We refer to Lemma 4 of [7] for the proof of the above lemma.

Lemma 3 *Symmetric (or antisymmetric), orthonormal and compactly suppored 3-band wavelet systems satisfy the following properties:*

(i) If the length of filter is $3L + 3$,

$$\begin{cases} A_{i,0}(z) = z^L A_{i,2}(z^{-1}), \\ A_{i,1}(z) = z^L A_{i,1}(z^{-1}), \\ 2A_{i,0}(z)A_{i,2}(z) + (A_{i,1})^2 = 3z^L, \end{cases} \tag{7}$$

where $i = 0, 1$, *and*

$$\begin{cases} A_{2,0}(z) = -z^L A_{2,2}(z^{-1}), \\ A_{2,1}(z) = z^L A_{2,1}(z^{-1}), \\ 2A_{2,0}(z)A_{2,2}(z) + (A_{2,1})^2 = 3z^L. \end{cases}$$

(ii) If the length of filter is $3L + 2$,

$$\begin{cases} A_{i,0}(z) = z^L A_{i,1}(z^{-1}), \\ A_{i,2}(z) = z^{L-1} A_{i,2}(z^{-1}), \\ 2A_{i,0}(z)A_{i,2}(z) + z(A_{i,1})^2 = 3z^L, \end{cases} \tag{8}$$

where $i = 0, 1$, and

$$\begin{cases} A_{2,0}(z) = -z^L A_{2,2}(z^{-1}), \\ A_{2,1}(z) = z^{L-1} A_{2,1}(z^{-1}), \\ 2A_{2,0}(z)A_{2,2}(z) + (A_{2,1})^2 = 3z^L. \end{cases}$$

We refer to Lemma 5 of [7] for the proof of the above lemma.

From Lemma 1, we get an explicit lowpass filter with length 6. In order to get the 3-band wavelet system whose lowpass filter length is 6, we must get the other two highpass filters such that they are orthonormal, compactly supported and one of the high-pass filter is symmetric while another is asymmetric. However, it is not easy to get the corresponding two highpass filters of the 3-band wavelet system from the lowpass filters. We give the main conclusion of this paper in next section.

3. The inexistence of 3-band $(6, m, n)$ wavelet systems

Theorem 1 *The 3-band wavelet systems which are symmetric (or asymmetric), orthonormal and compactly supported do not exist, if the length of their filters is* $(6, m, n)$ $(m, n \in N^+)$ *and* $a_{1,m-1} \neq 0$, $a_{2,n-1} \neq 0$.

Proof If $m \leq 4$, the theorem holds obviously.

In the following, we always assume $L \in N^+$.

(i) If $m = 3L + 3$, with equation (7), we have

$$A_{0,0}(z)A_{1,0}(z^{-1}) + A_{0,1}(z)A_{1,1}(z^{-1}) + A_{0,2}(z)A_{1,2}(z^{-1}) = 0; \tag{9}$$

$$A_{1,0}(z)A_{1,0}(z^{-1}) + A_{1,1}(z)A_{1,1}(z^{-1}) + A_{1,2}(z)A_{1,2}(z^{-1}) = 3. \tag{10}$$

By applying first two equations in (7) of Lemma 3, the equations (9) and (10) will become

$$A_{0,0}(z)(z^L A_{1,0}(z^{-1})) + A_{0,1}(z)A_{1,1}(z) + A_{0,2}(z)A_{1,0}(z) = 0;$$

$$2A_{1,0}(z)(z^L A_{1,0}(z^{-1})) + (A_{1,1}(z))^2 = 3z^L.$$

Eliminating the item of $A_{1,1}$ gives

$$(A_{0,0}(z)(z^L A_{1,0}(z^{-1})) + A_{0,2}(z)A_{1,0}(z))^2 = 3z^L(A_{0,1}(z))^2 - 2(A_{0,1}(z))^2 A_{1,0}(z)(z^L A_{1,0}(z^{-1})).$$

Therefore, considering $2A_{0,0}(z)A_{0,2}(z) + (A_{0,1})^2 = 3z^L$ in (7) yields

$$(A_{0,0}(z)(z^L A_{1,0}(z^{-1})) - A_{0,2}(z)A_{1,0}(z))^2 = 3z^L((A_{0,1}(z))^2 - 2zA_{1,0}(z)A_{1,0}(z^{-1})). \tag{11}$$

According to Lemma 1, we denote $\alpha = \frac{2-\sqrt{6}}{4}$, $\beta = \frac{1}{2}$, $\gamma = \frac{2+\sqrt{6}}{4}$. Then $A_{0,0} = \alpha + \gamma z$, $A_{0,1} = \beta + \beta z$, $A_{0,2} = \gamma + \alpha z$, and $A_{1,0} = \sum_k a_{1,3k} z^k = \sum_k x_k z^k$. Substituting $A_{0,0}$, $A_{0,1}$, $A_{0,2}$, $A_{1,0}$ into equation (11) gives

$$(\gamma \sum_{i=0}^{L} x_i z^i + \alpha z \sum_{i=0}^{L} x_i z^i - \alpha \sum_{i=0}^{L} x_{L-i} z^i - \gamma z \sum_{i=0}^{L} x_{L-i} z^i)^2$$

$$= \frac{1}{3}(\beta^2 z^L + 2\beta^2 z^{L+1} + \beta^2 z^{L+2} - 2\sum_{j=0}^{L}(\sum_{i=0}^{L} x_j x_{L-i} z^{1+i+j})).$$

Now, let us consider a more general situation

$$\begin{aligned}&(\gamma\sum_{i=k}^{L-k} x_i z^i + \alpha z\sum_{i=k}^{L-k} x_i z^i - \alpha\sum_{i=k}^{L-k} x_{L-i} z^i - \gamma z\sum_{i=k}^{L-k} x_{L-i} z^i)^2 \\ &= \frac{1}{3}(\beta^2 z^L + 2\beta^2 z^{L+1} + \beta^2 z^{L+2} - 2\sum_{j=k}^{L-k}(\sum_{i=k}^{L-k} x_j x_{L-i} z^{1+i+j})), \quad 0 \le k < L. \qquad (12)\end{aligned}$$

If $2k+1 < L$, equation (12) can be written as the following form,

$$\begin{aligned}&((\gamma x_k - \alpha x_{L-k})z^k + C_1 z^{k+1} + \cdots + C_{L-2k+1} z^{L-k+1})^2 \\ &= \frac{1}{3}(-2x_k x_{L-k} z^{2k+1} + D_1 z^{2k+2} + \cdots + D_{2L-4k} z^{2L-2k+1}),\end{aligned}$$

where C_i, D_j, $1 \le i \le L-2k+1$, $1 \le j \le 2L-4k$, are all coefficients constituted by α, β, γ, x_i, $0 \le i \le L$.

Hence, we have $\gamma x_k - \alpha x_{L-k} = 0$ by comparing the coefficients of z. Similarly, we have $x_k x_{L-k} = 0$ by considering $\gamma x_k - \alpha x_{L-k} = 0$ and comparing the coefficients of z again. Therefore, $x_k = x_{L-k} = 0$. Substituting $x_k = x_{L-k} = 0$ into equation (12), we conclude that the equation (12) becomes

$$\begin{aligned}&(\gamma\sum_{i=k+1}^{L-k-1} x_i z^i + \alpha z\sum_{i=k+1}^{L-k-1} x_i z^i - \alpha\sum_{i=k+1}^{L-k-1} x_{L-i} z^i - \gamma z\sum_{i=k+1}^{L-k-1} x_{L-i} z^i)^2 \\ &= \frac{1}{3}(\beta^2 z^L + 2\beta^2 z^{L+1} + \beta^2 z^{L+2} - 2\sum_{j=k+1}^{L-k-1}(\sum_{i=k+1}^{L-k-1} x_j x_{L-i} z^{1+i+j})), \quad 0 \le k < L, \qquad (13)\end{aligned}$$

which is the same as (12) after changing k into $k+1$.

If L is even, the index k in equation (12) can be increased to $\frac{L}{2}$ after repeating this process. Hence

$$((\gamma x_{\frac{L}{2}} - \alpha x_{\frac{L}{2}})z^{\frac{L}{2}} + (\alpha x_{\frac{L}{2}} - \gamma x_{\frac{L}{2}})z^{\frac{L+2}{2}})^2 = \frac{1}{3}(\beta^2 z^L + 2\beta^2 z^{L+1} + \beta^2 z^{L+2} - 2x_{\frac{L}{2}} x_{\frac{L}{2}} z^{L+1}).$$

By comparing the coefficients of z,

$$(\gamma - \alpha)^2 x_{\frac{L}{2}}^2 = \frac{1}{3}\beta^2; \quad x_{\frac{L}{2}}^2 = \frac{1}{2}\beta^2.$$

This indicates $(\gamma-\alpha)^2 = \frac{2}{3}$. However, it contradicts $(\gamma-\alpha)^2 = ((\frac{2+\sqrt{6}}{4}) - (\frac{2-\sqrt{6}}{4}))^2 = \frac{3}{2}$.

If L is odd, the index k in equation (12) can be increased to $\frac{L-1}{2}$. Hence

$$\begin{aligned}&((\gamma x_{\frac{L-1}{2}} - \alpha x_{\frac{L+1}{2}})z^{\frac{L-1}{2}} + (\gamma x_{\frac{L+1}{2}} + \alpha x_{\frac{L-1}{2}} - \alpha x_{\frac{L-1}{2}} - \gamma x_{\frac{L+1}{2}})z^{\frac{L+1}{2}} + (\alpha x_{\frac{L+1}{2}} - \gamma x_{\frac{L-1}{2}})z^{\frac{L+3}{2}})^2 \\ &= ((\gamma x_{\frac{L-1}{2}} - \alpha x_{\frac{L+1}{2}})z^{\frac{L-1}{2}} + (\alpha x_{\frac{L+1}{2}} - \gamma x_{\frac{L-1}{2}})z^{\frac{L+3}{2}})^2 \\ &= \frac{1}{3}((\beta^2 - 2x_{\frac{L-1}{2}} x_{\frac{L+1}{2}})z^L + (2\beta^2 - 2x_{\frac{L-1}{2}} x_{\frac{L-1}{2}} - 2x_{\frac{L+1}{2}} x_{\frac{L+1}{2}})z^{L+1} + (\beta^2 - 2x_{\frac{L+1}{2}} x_{\frac{L-1}{2}})z^{L+2}).\end{aligned}$$

By comparing the coefficients of z, we find

$$\gamma x_{\frac{L-1}{2}} - \alpha x_{\frac{L+1}{2}} = 0; \qquad (14)$$

$$\beta^2 - 2x_{\frac{L-1}{2}}x_{\frac{L+1}{2}} = 0; \tag{15}$$

$$\beta^2 - x_{\frac{L-1}{2}}x_{\frac{L-1}{2}} - x_{\frac{L+1}{2}}x_{\frac{L+1}{2}} = 0. \tag{16}$$

From equations (14) to (16), we have $x_{\frac{L-1}{2}} = x_{\frac{L-1}{2}} = 0$ and that means $x_i = 0, i = 0, 1, \ldots, L$. Also, this contradicts $\sum_{i=0}^{L} x_i^2 = 3$ included in (1).

Therefore, the solutions do not exist if $m = 3L + 3$.

(ii) Similarly, if $m = 3L + 2$, by (6), we have equation (9) and (10). By applying equation (8) in Lemma 3, the above two equations (7) and (8) yield

$$A_{0,0}(z)(z^L A_{1,0}(z^{-1})) + A_{0,1}(z)A_{1,0}(z) + zA_{0,2}(z)A_{1,2}(z) = 0;$$

$$2A_{1,0}(z)(z^L A_{1,0}(z^{-1})) + z(A_{1,2}(z))^2 = 3z^L.$$

Eliminating the item of $A_{1,2}$ gives

$$(z^L A_{0,0}(z)A_{1,0}(z^{-1}) + A_{0,1}(z)A_{1,0}(z))^2 = (3z^L - 2z^L A_{1,0}(z)A_{1,0}(z^{-1}))z(A_{0,2}(z))^2. \tag{17}$$

Therefore, by denoting α, β, γ, x_k, $k = 0, 1, \ldots, m-1$, $A_{1,0}$ as we have done above and applying the expressions of $A_{0,0}$, $A_{0,1}$, $A_{0,2}$, $A_{1,0}$, the equation (17) yields

$$\begin{aligned}
&(\alpha \sum_{i=0}^{L} x_{L-i}z^i + \gamma z \sum_{i=0}^{L} x_{L-i}z^i + \beta \sum_{i=0}^{L} x_i z^i + \beta z \sum_{i=0}^{L} x_i z^i)^2 \\
&= (\gamma^2 z + 2\alpha\gamma z^2 + \alpha^2 z^3)(\frac{1}{3}z^L - 2\sum_{j=0}^{L}(\sum_{i=0}^{L} x_j x_{L-i} z^{i+j})).
\end{aligned}$$

Consider a more general situation

$$\begin{aligned}
&(\alpha \sum_{i=k}^{L-k} x_{L-i}z^i + \gamma z \sum_{i=k}^{L-k} x_{L-i}z^i + \beta \sum_{i=k}^{L-k} x_i z^i + \beta z \sum_{i=k}^{L-k} x_i z^i)^2 \\
&= (\gamma^2 z + 2\alpha\gamma z^2 + \alpha^2 z^3)(\frac{1}{3}z^L - 2\sum_{j=k}^{L-k}(\sum_{i=k}^{L-k} x_j x_{L-i} z^{i+j})).
\end{aligned}$$

If $2k < L$, it can be written as the following form,

$$\begin{aligned}
&((\alpha x_{L-k} + \beta x_k)z^k + C_1 z^{k+1} + \cdots + C_{L-2k+1} z^{L-k+1})^2 \\
&= -2\gamma^2 x_k x_{L-k} z^{2k+1} + D_1 z^{2k+2} + \cdots + D_{2L-4k+2} z^{2L-2k+3},
\end{aligned}$$

where C_i, D_j, $1 \le i \le L - 2k + 1$, $1 \le j \le L - 4k + 2$, are all coefficients constituted by α, β, γ, x_i, $0 \le i \le L$.

Hence, we have $\alpha x_{L-k} + \beta x_k = 0$ by comparing the coefficients of z. Similarly, we have $\gamma^2 x_k x_{L-k} = 0$ by considering $\alpha x_{L-k} + \beta x_k = 0$ and comparing the coefficients of z again. Therefore, $x_k = x_{L-k} = 0$. Substituting $x_k = x_{L-k} = 0$ into equation (18), we conclude that the equation (18) becomes

$$(\alpha \sum_{i=k+1}^{L-k-1} x_{L-i}z^i + \gamma z \sum_{i=k+1}^{L-k-1} x_{L-i}z^i + \beta \sum_{i=k+1}^{L-k-1} x_i z^i + \beta z \sum_{i=k+1}^{L-k-1} x_i z^i)^2$$

$$= (\gamma^2 z + 2\alpha\gamma z^2 + \alpha^2 z^3)(\frac{1}{3}z^L - 2\sum_{j=k+1}^{L-k-1}(\sum_{i=k+1}^{L-k-1} x_j x_{L-i} z^{i+j})),$$

which is the same as (18) after changing k into $k+1$.

If L is even, the index k can be increased to $\frac{L-1}{2}$ in (18) after repeating this process. Hence,

$$((\alpha x_{\frac{L}{2}} + \beta x_{\frac{L}{2}})z^{\frac{L}{2}} + (\gamma x_{\frac{L}{2}} + \beta x_{\frac{L}{2}})z^{\frac{L+2}{2}})^2$$
$$= (\gamma^2 z + 2\alpha\gamma z^2 + \alpha^2 z^3)(\frac{1}{3}z^L - 2x_{\frac{L}{2}} x_{\frac{L}{2}} z^L).$$

By comparing the coefficients of z, we have

$$-2\alpha^2 x_{\frac{L}{2}} x_{\frac{L}{2}} = 0$$

which means that $x_i = 0$, $i = 0, 1, \ldots, L$. This contradicts $\sum_{i=0}^{L} x_i^2 = 3$ included in (1).

If L is odd, since $2k < L$, we have $x_k = x_{L-k} = 0$, $x_i = 0$, $i = 0, 1, \ldots, L$. This also contradicts $\sum_{i=0}^{L} x_i^2 = 3$.

Therefore, the solutions do not exist if $m = 3L + 2$.

All in all, symmetric (or antisymmetric), orthonormal and compactly suppored 3-band wavelet systems do not exist, if the length of their filters is $(6, m, n)(m, n \in N^+)$. $\square$

4. Conclusions

In this paper, we first review the 3-band wavelet systems theories. Then, we prove that symmetric (or antisymmetric), orthonormal and compactly suppored 3-band wavelet systems do not exist, if the length of their filters is $(6, m, n)$ $(m, n \in N^+)$.

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