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Fuzzy Regular Relations on Hyperquasigroups

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Abstract Regular relations play in hyperquasigroups theory a role analogous to congruences in semigroup theory. The aim of this paper is to introduce the concept of regular relations on hyperquasigroups and to investigate some related properties. Further, the notion of fuzzy regular relations on hyperquasigroups is introduced and some characterizations are discussed.

Keywords hyperquasigroup; regular relation; fuzzy regular relation.

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1. Introduction

In this section, we describe the motivation and a survey of related works. The theory of algebraic hyperstructures is a well established branch of classical algebraic theory. In the literature, the theory of hyperstructure was first initiated by Marty in 1934 ([16]) when he defined the hypergroup and began to investigate their properties with applications to groups, rational function and algebraic functions. Later on, many people have observed that the theory of hyperstructures also have many applications in both pure and applied sciences, for example, semi-hypergroups are the simplest algebraic hyperstructures which possess the properties of closure and associativity. Some review of the theory of hyperstructures can be found in [1], [8] and [22], respectively. In a recent monograph of Corsini and Leoreanu [2], the authors have collected numerous applications of algebraic hyperstructures, especially those from the last fifteen years to the following subjects: geometry, hypergraphs, binary relations, lattices, fuzzy sets and rough sets, automata, cryptography, codes, median algebras, relation algebras, artificial intelligence and probabilities.

After introducing the concept of fuzzy sets by Zadeh in 1965 ([28]), there are many papers devoted to fuzzifying the classical mathematics into fuzzy mathematics. Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces and so on. This provides sufficient motivations for researchers to review various concepts and results from the realm of abstract algebra to a broader framework of fuzzy setting.

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In 1971, Zadeh introduced the concept of fuzzy relations. Further, fuzzy congruence relations on semigroups appeared in [20] and [21]. In 1999, Xie [26] introduced fuzzy Rees congruences on semigroups.

The fuzzy sets and hyperstructures introduced by Zadeh and Marty, respectively, are now extensively applied to many disciplines. The relationships between the fuzzy sets and algebraic hyperstructures (structures) have been considered by Corsini, Davvaz, Leoreanu, Vougiouklis, Zhan and others. The reader is refereed to [3–15, 17–18, 21–25, 30–34]. In [6, 7], Davvaz defined the concept of fuzzy hyperideals in semihypergroups and investigated fuzzy strong regular relation on semihypergroups. Recently, Davvaz [9] studied fuzzy hyperideals and regular relations in H_v-semigroups and obtained some important results.

The work of this paper is organized as follows. In Section 2, we recall some basic definitions and results of hyperquasigroups. In Section 3, we investigate the regular relations on hyperquasigroups. Finally, the properties of fuzzy regular relations are discussed in Section 4.

2. Preliminaries

A hypergroupoid $(H, \circ)$ is a non-empty set H together with a hyperoperation $\circ$ defined on H, i.e., a mapping $H \times H$ into the family of non-empty subsets of H, where $\rho(H)$ is the set of all the non-empty subsets of H.

If $x \in H$ and A, B are subsets of H, then by $A \circ B, A \circ x$ and $x \circ B$ we mean $A \circ B = \bigcup_{\substack{a\in A\\ b\in B}} a \circ b, A \circ x = A \circ \{x\}, x \circ B = \{x\} \circ B$, respectively.

A hypergroupoid $(H, \circ)$ is called a hypergroup if for all $x, y, z \in H$, the following two conditions hold:

(i) $x \circ (y \circ z) = (x \circ y) \circ z$; (ii) $x \circ H = H \circ x = H$.

The second condition, called the reproducibility condition, means that for any $x, y \in H$, there exist $u, v \in H$ such that $y \in x \circ u$ and $y \in v \circ x$. A hypergroupoid satisfying this condition is called a hyperquasigroup. Thus a hypergroup is a hyperquasigroup with the associative hyperoperation. A non-empty subset K of a hyperquasigroup $(H, \circ)$ is called a sub-hyperquasigroup if $(K, \circ)$ is a hyperquasigroup.

Example 2.1 (i) Let $(G, \cdot)$ be a group and ρ an equivalence relation on G. In G/ρ, the set of all equivalence classes, we consider the hyperoperation $\odot$ such that

$$\overline{x} \odot \overline{y} = \{\overline{z} | z \in \overline{x} \cdot \overline{y}\},$$

where $\overline{x}$ denotes the class of element x. Then $(G, \odot)$ is a hyperquasigroup which is not always a hypergroup.

(ii) On the set $\mathbb{Z}_{mn}$, consider the hyperoperation $\oplus$ defined by setting

$$\overline{0} \oplus \overline{m} = \{\overline{0}, \overline{m}\} \quad \text{and} \quad \overline{x} \oplus \overline{y} = \overline{x + y}, \quad \text{for all} \quad (\overline{x}, \overline{y}) \in \mathbb{Z}_{mn}^2 \backslash \{(\overline{0}, \overline{m})\}.$$

Then $(\mathbb{Z}_{mn}, \oplus)$ becomes a hyperquasigroup which is not a hypergroup.

Definition 2.2 ([4]) *Let $(H, \circ)$ be a hyperquasigroup. A fuzzy set F of H is a fuzzy subhyperquasigroup of H if it satisfies the inequality:*

(i) $\min\{F(x), F(y)\} \leq \inf_{z \in x \circ y} F(z)$, *for all $x, y \in H$;*

(ii) For all $x, a \in H$, there exists $y \in H$ such that $x \in a \circ y$ and $\min\{F(a), F(x)\} \leq F(y)$;

(iii) For all $x, a \in H$, there exists $z \in H$ such that $x \in z \circ a$ and $\min\{F(a), F(x)\} \leq F(z)$.

3. Regular relations

Let ρ be an equivalence relation on a hyperquasigroup H. If A and B are non-empty subsets of H, we write $A\overline{\rho}B$ to denote that for every $a \in A$, there exists $b \in B$ such that $a\rho b$ and for every $b \in B$, there exists $a \in A$ such that $a\rho b$. We write $A\overline{\overline{\rho}}B$ if for every $a \in A$ and every $b \in B$, one has $a\rho b$.

In what follows, let H denote a hyperquasigroup unless otherwise specified.

Definition 3.1 ([2]) *An equivalence relation ρ on H is called regular to the right if for every $(x, y) \in H \times H$, one has $x\rho y \Rightarrow x \circ c\overline{\rho}y \circ c$, for all $c \in H$. We say that ρ is strongly regular to the right if for every $(x, y) \in H \times H$, the implication $x\rho y \Rightarrow x \circ c\overline{\overline{\rho}}y \circ c$, for all $c \in H$, is valid. Analogously, we define the regularity (strong regularity) of an equivalence relation to the left. A regular (strong regular) equivalence relation to the right and to the left is called regular (strongly regular).*

Regular equivalence relations play in hyperqusigroup theory, a role analogous to congruences in semigroup theory.

Theorem 3.2 *Let H be a hyperquasigroup and ρ an equivalence relation on H. If ρ is a regular relation, a hyperquasigroup structure turns out to be defined on H/ρ, the set of an equivalence classes, with respect to the hyperproduct $\rho(x) \otimes \rho(y) = \{\rho(z) | z \in x \circ y\}$; Conversely, if ρ is an equivalence relation such that the preceding hyperoperation turns out to be well-defined, i.e., if $(H/\rho, \otimes)$ is a hyperquasigroup, then ρ is a regular relation.*

Proof Let $\rho(x)$, $\rho(y) \in H/\rho$ and $x' \in \rho(x)$, $y' \in \rho(y)$. From the regularity of ρ, we have $x' \circ y'\overline{\rho}x \circ y$. Thus, for every $a \in x' \circ y'$, there exists $b \in x \circ y$ such that $a\rho b$, that is, $\rho(a) = \rho(b)$. Then it is proved that for every $\rho(a) \in \rho(x') \otimes \rho(y')$, there exists $b \in x \circ y$ such that $\rho(a) = \rho(b) \in \rho(x) \otimes \rho(y)$, i.e., $\rho(x') \otimes \rho(y') \subseteq \rho(x) \otimes \rho(y)$. Similarly, we get $\rho(x) \otimes \rho(y) \subseteq \rho(x') \otimes \rho(y')$. Thus, $\rho(x) \otimes \rho(y) = \rho(x') \otimes \rho(y')$, which implies $(H/\rho, \otimes)$ is a hypergroupoid.

Since H is a hyperquasigroup, we have $x \circ H = H$, for all $x \in H$. That is, for any $y \in H, \exists a \in H$ such that $y \in x \circ a$. Thus $\rho(x) \otimes \rho(a) = \{\rho(y) | y \in x \circ a\}$, which implies $\rho(x) \otimes H/\rho = H/\rho$. Similarly, we get $H/\rho \otimes \rho(x) = H/\rho$. Thus, $\rho(x) \otimes H/\rho = H/\rho \otimes \rho(x) = H/\rho$, for all $\rho(x) \in H/\rho$. This proves $(H/\rho, \otimes)$ is a hyperquasigroup.

Conversely, let $x\rho y$. Then $\rho(x) = \rho(y)$ and for every $c \in H$, we have $\rho(x) \otimes \rho(c) = \rho(y) \otimes \rho(c)$. For every $a \in x \circ c$, since $\rho(a) \in \rho(x) \otimes \rho(c) = \rho(y) \otimes \rho(c)$, there exits $b \in y \circ c$ such that $\rho(a) = \rho(b)$, and so $a\rho b$, i.e., ρ is regular to the right. Similarly, we can see ρ is regular to the left.

Let H and H' be two hyperquasigroups. A map $f : H \to H'$ is called an inclusion homomorphism if it satisfies the condition $f(x\circ y)\subseteq f(x)\circ f(y)$, for all $x,y\in H$; f is called a strong homomorphism if $f(x\circ y)=f(x)\circ f(y)$, for all $x,y\in H$. A strong homomorphism is called an epimorphism if f is surjective.

Let f be a strong homomorphism from a hyperquasigroup H into a hyperquasigroup H'. The relation $f^{-1}f$ is an equivalence ρ on H($a\rho b$ if and only if $f(a)=f(b)$) known as the kernel of f. The natural mapping associated with ρ is $g:H\to H/\ker f$, where $g(a)=\rho(a)$, for all $a\in H$.

Theorem 3.3 *Let $f:H\to H'$ be a strong homomorphism of hyperquasigroups. Then $\rho=\ker f=\{(a,b)|f(a)=f(b)\}$ is a regular relation and there exists a strong homomorphism $g:H/\rho\to H'$ such that $g\varphi=f$, where φ is the natural mapping associated with ρ.*

Proof Let $a\rho b$. Then for any $c\in H$, we have $f(a\circ c)=f(a)\circ f(c)=f(b)\circ f(c)=f(b\circ c)$. Therefore, for every $x\in a\circ c$, there exists $y\in b\circ c$ such that $f(x)=f(y)$, which implies, $x\rho y$, i.e., ρ is regular to the right. Similarly, we can see ρ is regular to the left. Hence ρ is a regular relation.

Now, let $\rho(a)\in H/\rho$. Define $g:H/\rho\to H'$ by $g(\rho(a))=f(a)$. Note that if $b\in\rho(a)$, then $f(a)=f(b)$, which implies, g is well-defined. We prove that g is a strong homomorphism. For any $\rho(a),\rho(b)\in H/\rho$, then

$$\begin{aligned}&g(\rho(a)\otimes\rho(b))=g(\{\rho(z)|z\in\rho(a)\circ\rho(b)\})=\{f(z)|z\in\rho(a)\circ\rho(b)\}\\&\quad=f(\bigcup_{\substack{x\in\rho(a)\\ y\in\rho(b)}}x\circ y)=\bigcup_{\substack{x\in\rho(a)\\ y\in\rho(b)}}f(x\circ y)\\&\quad=\bigcup_{\substack{x\in\rho(a)\\ y\in\rho(b)}}f(a)\circ f(b)=g(\rho(a))\circ g(\rho(b)),\end{aligned}$$

which implies, g is a strong homomorphism. Finally, for any $a\in H$, we have $g(\varphi(a))=g(\rho(a))=f(a)$, i.e., $g\varphi=f$.

Lemma 3.4 *Let $f_1:H\to H_1$ and $f_2:H\to H_2$ be the epimorphisms of hyperquasigroups such that $f_1^{-1}f_1\subseteq f_2^{-1}f_2$. Then there exists a unique epimorphism $\theta:H_1\to H_2$ such that $\theta f_1=f_2$.*

Proof For any $a_1\in H$, there exists $a\in H$ such that $f_1(a_1)=a$. Define $\theta:H_1\to H_2$ by $\theta(a_1)=f_2(a)$. If $f_1(b)=a_1(b\in H)$, we have $(a,b)\in f_1^{-1}f_1\subseteq f_2^{-1}f_2$, so that $f(a)=f(b)$. This proves that θ is well-defined.

For any $a\in H,(\theta f_1)(a)=\theta(f_1(a))=\theta(a_1)=f_2(a)$. Hence $\theta f_1=f_2$. We prove that θ is an epimorphism. In fact, for any $a,b\in H$, we have $\theta(f_1(a)\circ f_1(b))=\theta(f_1(a\circ b))=f_2(a\circ b)=f_2(a)\circ f_2(b)=\theta(f_1(a))\circ\theta(f_2(b))$. Clearly, θ is surjective and the uniqueness is evident. This completes the proof. □

Theorem 3.5 *If ρ_1 and ρ_2 are regular relations on H such that $\rho_1\subseteq\rho_2$, then there exists an epimorphism $H/\rho_1\to H/\rho_2$.*

Proof Let $\pi_1:H\to H/\rho_1$ and $\pi_2:H\to H/\rho_2$ be canonical homomorphisms. Since $\rho_1=\pi_1^{-1}\pi$

and $\rho_2 = \pi_2^{-1}\pi$. By Lemma 3.4, the proof is completed. $\square$

Proposition 3.6 *Let A be a sub-hyperquasigroup of H. Define the Rees relation on H as follows: $a\rho b \Leftrightarrow a = b$ or ($a \in A$ and $b \in A$). Then ρ is a regular relation on H.*

Proof Clearly, ρ is an equivalence relation. Let $x\rho y$ and $a \in H$. If both x and y belong to A, then $x \circ a \subseteq A$ and $y \circ a \subseteq A$. Thus, for every $u \in x \circ a$, and $v \in y \circ a$, we have $u\rho v$. If $x = y$, then $x \circ a = y \circ a$, we have $u\rho u$ for every $u \in x \circ a$. This proves ρ is regular to the right. Similarly, we can see that ρ is regular to the left. Therefore, ρ is a regular relation.

4. Fuzzy regular relations

In 1971, Zadeh in [29] introduced the concept of fuzzy relations. This has been further investigated by Samhan [20], Xie [26, 27] and Davvaz [7] et al. In this section, we discuss fuzzy regular relations on hyperquasigroups.

A function $\mu : A \times A \to [0, 1]$ is called a fuzzy relation on A. A fuzzy relation μ on A is called fuzzy reflexive if $\mu(x, x) = 1$, for all $x \in A$, and μ is called fuzzy symmetric if $\mu(x, y) = \mu(y, x)$, for all $x, y \in A$, and μ is called fuzzy transitive if $\mu(x, y) \geq \sup\{\min\{\mu(x, z), \mu(y, z)\}|z \in A\}$, for all $x, y \in A$. A fuzzy relation μ on a set A is called a fuzzy equivalence relation on A if it is fuzzy reflexive, fuzzy symmetric and fuzzy transitive.

Definition 4.1 *A fuzzy equivalence relation μ on H is called a fuzzy strongly regular relation to the right if $\mu(x, y) \leq \inf\{\mu(a, b)|a \in x \circ c, b \in y \circ c\}$, for all $x, y, c \in H$, and μ is called a fuzzy strongly regular relation to the left if $\mu(x, y) \leq \inf\{\mu(a, b)|a \in c \circ x, b \in c \circ y\}$, for all $x, y, c \in H$. It is called a fuzzy strongly regular relation if $\min\{\mu(x, y), \mu(z, t)\} \leq \inf\{\mu(a, b)|a \in x \circ z, b \in y \circ t\}$, for all $x, y, z, t \in H$.*

Theorem 4.2 *Let μ be a fuzzy equivalence relation on H. Then the following conditions are equivalent:*

(i) μ is a fuzzy strongly regular relation;

(ii) μ is a fuzzy strongly regular relation to the right and to the left.

Proof Let μ be a fuzzy strongly regular relation. For any $x, y, a \in H$. Since μ is fuzzy reflexive, then $\mu(a, a) = 1$. By the hypothesis, we have

$$\mu(x, y) = \min\{1, \mu(x, y)\} = \min\{\mu(a, a), \mu(x, y)\} \leq \inf_{\substack{\alpha \in a \circ x \\ \beta \in a \circ y}}\{\mu(\alpha, \beta)\}.$$

This proves that μ is a fuzzy strongly regular relation to the left. Similarly, we can obtain that μ is a fuzzy strongly regular relation to the right.

Conversely, assume that the condition (ii) holds. Let $a, b, x, y \in H$. Then we have

$$\begin{aligned}\min\{\mu(x, y), \mu(a, b)\} &\leq \min\{\inf_{\alpha \in x \circ a, \beta \in y \circ a}\{\mu(\alpha, \beta)\}, \inf_{\gamma \in y \circ a, \delta \in y \circ b}\{\mu(\gamma, \delta)\}\} \\ &= \inf_{\substack{\alpha, \delta \in x \circ a \\ \beta, \gamma \in y \circ a}}\{\min\{\mu(\alpha, \beta), \mu(\gamma, \delta)\}\}\end{aligned}$$

$$\le \sup_{\beta_0 \in y \circ a} \{\min\{\mu(\alpha, \beta_0), \mu(\beta_0, \delta)\}\}$$
$$\le \sup_{\beta_0 \in H} \{\min\{\mu(\alpha, \beta_0), \mu(\beta_0, \delta)\}\}$$
$$\le \mu(\alpha, \delta), \forall \alpha \in x \circ a, \delta \in y \circ b.$$

Thus, $\min\{\mu(x,y), \mu(a,b)\} \le \inf_{\substack{\alpha \in x \circ a \\ \delta \in y \circ b}} \{\mu(\alpha, \delta)\}$. This proves that μ is a fuzzy strongly regular relation.

Definition 4.3 *A fuzzy equivalence relation μ on H is called a fuzzy regular relation to the right if for all $x, y, c \in H$, and for all $a \in x \circ c$, there exists $b \in y \circ c$ such that $\mu(x,y) \le \mu(a,b)$; μ is called a fuzzy relation to the left if for all $x, y, c \in H$, and for all $a \in c \circ x$, there exists $b \in c \circ y$ such that $\mu(x,y) \le \mu(a,b)$. It is called a fuzzy regular relation if for all $x, y, z, t \in H$, and for all $a \in x \circ z$, there exists $b \in y \circ t$ such that $\min\{\mu(x,y), \mu(z,t)\} \le \mu(a,b)$.*

Theorem 4.4 *Let μ be a fuzzy equivalence relation on H. Then the following are equivalent:*

(i) μ is a fuzzy regular relation;

(ii) μ is a fuzzy relation to the right and to the left.

Proof It is similar to the proof of Theorem 4.2.

The following two Propositions are easy and we omit the details.

Proposition 4.5 *Let ρ be an equivalence relation on H. Then the following are equivalent:*

(i) ρ is a regular relation to the left;

(ii) The characteristic function χ_ρ of ρ is a fuzzy regular relation to the left.

Proposition 4.6 *Let ρ be an equivalence relation on H. Then the following are equivalent:*

(i) ρ is a regular relation to the right;

(ii) The characteristic function χ_ρ of ρ is a fuzzy regular relation to the right.

Definition 4.7 *Let μ be a fuzzy sub-hyperquasigroup of H. Then, we can assign a fuzzy relation θ_μ on H defined as follows:*

$$\theta_\mu(x,y) = \begin{cases} \min\{\mu(x), \mu(y)\}, & \text{if } x \ne y, \\ 1, & \text{otherwise.} \end{cases}$$

Theorem 4.8 *Let μ be a fuzzy sub-hyperquasigroup of H. Then θ_μ is a fuzzy regular relation on H.*

Proof By the definition of θ_μ, it is obvious that θ_μ is reflexive and symmetric. For all $x, y \in H$, we consider the following two cases:

(i) If $x = y$, then

$$\sup\{\min\{\theta_\mu(x,z), \theta_\mu(z,x)\} | z \in H\} = \sup\{\theta_\mu(x,z) | z \in H\} = \theta_\mu(x,x) = 1.$$

(ii) If $x \ne y$, then

$$\sup\{\min\{\theta_\mu(x,z), \theta_\mu(z,y)\} | z \in H\} = \max\{\sup\{\min\{\theta_\mu(x,z), \theta_\mu(z,y)\} | z \in H - \{x,y\}\},$$

$$\begin{aligned}&\min\{\theta_\mu(x,x),\theta_\mu(x,y)\},\min\{\theta_\mu(x,y),\theta_\mu(y,y\}\}\\&\quad=\max\{\theta_\mu(x,y),\sup\{\min\{\mu(x),\mu(z),\mu(z),\mu(y)\}|z\in H-\{(x,y)\}\}\}\\&\quad\le\max\{\theta_\mu(x,y),\sup\{\min\{\mu(x),\mu(y)\}|z\in H-\{(x,y)\}\}\}\\&\quad=\max\{\theta_\mu(x,y),\theta_\mu(x,y)\}=\theta_\mu(x,y).\end{aligned}$$

By (i) and (ii), θ_μ is fuzzy transitive.

Furthermore, θ_μ is a fuzzy regular relation to the left. In fact, for $x,y,c\in H$, if $c\circ x=c\circ y$, then for all $a\in c\circ x$, we have $\theta_\mu(a,a)\ge\theta_\mu(x,y)$.

If $c\circ x\ne c\circ y$, then $x\ne y$. Thus,

$$\begin{aligned}\theta_\mu(x,y)=\min\{\mu(x),\mu(y)\}&\le\min\{\inf\{\mu(a)|a\in c\circ x\},\inf\{\mu(b)|b\in c\circ y\}\}\\&=\inf\{\min\{\mu(a),\mu(b)\}|a\in c\circ x,b\in c\circ y\}\\&\le\min\{\mu(a),\mu(b)\},\ \text{for all } a\in c\circ x \text{ and } b\in c\circ y\\&\le\theta_\mu(a,b),\ \text{for all } a\in c\circ x \text{ and } b\in c\circ y.\end{aligned}$$

Similarly, we can also prove that θ_μ is a fuzzy regular relation to the right.

Let θ be a fuzzy regular relation on H. For any $a\in H$, we define a fuzzy set $\widetilde{\theta_a}$ of H by $\widetilde{\theta_a}(x)=\theta(a,x)$, for all $x\in H$.

One can easily obtain the following result:

Lemma 4.9 *Let $a,b\in H$. Then $\widetilde{\theta_a}=\widetilde{\theta_b}\Leftrightarrow\theta(a,b)=1$.*

We shall say that $\widetilde{\theta_a}$ is a fuzzy class related to a of H, and we denote by H/θ the set $\{\widetilde{\theta_a}|a\in H\}$ of all fuzzy classes of H.

Define the hyperoperation $\odot$ on H/θ as follows: $\widetilde{\theta_a}\odot\widetilde{\theta_b}=\{\widetilde{\theta_c}|c\in a\circ b\}$. From the above discussion, we can obtain the following result:

Theorem 4.10 *$(H/\theta,\odot)$ is a hyperquasigroup.*

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