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Orthogonality of Generalized (θ, ϕ)-Derivations on Ideals

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Abstract In this paper, necessary and sufficient conditions concerning the orthogonality and the composition of a couple of generalized (θ, ϕ)-derivations on a nonzero ideal of a semiprime ring are presented. These results are generalizations of several results of Brešar and Vukman, which are related to a theorem of Posner on the product of two derivations on a prime ring.

Keywords orthogonal; generalized (θ, ϕ)-derivation; semiprime ring.

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1. Introduction

Let R denote an associative ring. Recall that R is prime if $xRy = 0$, $x, y \in R$, implies $x = 0$ or $y = 0$, and R is semiprime if $xRx = 0$, $x \in R$, implies $x = 0$. A ring R is 2-torsion free if for any $x \in R$, $2x = 0$ implies $x = 0$.

An additive mapping d: $R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. An addition mapping $D : R \to R$ is called a generalized derivation if there is a derivation d of R such that $D(xy) = D(x)y + xd(y)$ for all $x, y \in R$. Let θ, ϕ be endomorphisms of R. An additive mapping $d : R \to R$ is called a (θ, ϕ)-derivation if $d(xy) = d(x)\theta(y) + \phi(x)d(y)$ for all $x, y \in R$. Motivated by the definition of (θ, ϕ)-derivation, the notion of generalized derivation was extended. Let θ, ϕ be endomorphisms of R. An addition mapping $D : R \to R$ is called a generalized (θ, ϕ)-derivation if there is a (θ, ϕ)-derivation d of R such that $D(xy) = D(x)\theta(y) + \phi(x)d(y)$ holds for all $x, y \in R$. Two mappings $f, g : R \to R$ are called orthogonal on R if

$$f(x)Rg(y) = 0 = g(y)Rf(x), \quad x, y \in R.$$

In [1], Brešar and Vukman introduced the notion of orthogonality for a pair of derivations (d, g) of a semiprime ring, and they gave several necessary and sufficient conditions for d, g to be orthogonal on a semiprime ring. In [2], Argaç, Nakajima and Albaş introduced the notion of orthogonality for a pair of generalized derivations (D, d), (G, g) of a semiprime ring and gave several necessary and sufficient conditions for (D, d), (G, g) to be orthogonal on a semiprime ring.

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In 2007, Albaş extended the results of Brešar and Vukman to orthogonal generalized derivations on a nonzero ideal of a semiprime ring [3]. Also, in 2007, Gölbaşi and Aydin presented some results cencerning the orthogonality of (θ, ϕ)-derivations and generalized (θ, ϕ)-derivations on a semiprime ring in [4]. Several authors have investigated other properties of prime or semiprime rings with generalized (θ, ϕ)-derivations, see [5, 6].

Motivated by [3] and [4], in this paper we extend these results [2] to orthogonal generalized (θ, ϕ)-derivations on a nonzero ideal of a semiprime ring.

Throughout this paper, R will denote a 2-torsion free semiprime ring, θ and ϕ are automorphisms of R, (D, d) and (G, g) are generalized (θ, ϕ)-derivations of R with associated (θ, ϕ)-derivations d and g such that

$$\begin{aligned} G\theta = \theta G, \quad G\phi = \phi G, \quad D\theta = \theta D, \quad D\phi = \phi D, \\ g\theta = \theta g, \quad g\phi = \phi g, \quad d\theta = \theta d, \quad d\phi = \phi d. \end{aligned} \tag{1}$$

2. Preliminaries

To prove our main results, we need the following lemmas from [4] and [8]. We present them here for convenience.

Lemma 2.1 ([7, Lemma 1]) *Let R be a 2-torsion free semiprime ring with a nonzero ideal I. Then for any $a, b \in R$ the following conditions are equivalent:*

(i) $axb = 0$ for all $x \in I$.

(ii) $bxa = 0$ for all $x \in I$.

(iii) $axb + bxa = 0$ for all $x \in I$.

Moreover, if one of the three conditions is fulfilled and $l(I) = 0$, then $ab = ba$.

Lemma 2.2 ([7, Lemma 3]) *Let R be a semiprime ring with a nonzero ideal I. Suppose that the additive mappings F and H of R satisfy $F(x)IH(x) = 0$ for all $x \in I$, then $F(x)IH(y) = 0$ for all $x, y \in I$.*

Lemma 2.3 ([4, Theorem 1]) *Let R be a 2-torsion free semiprime ring. The (θ, ϕ)-derivations d and g of R satisfying (1) are orthogonal if and only if one of the following conditions holds:*

(i) $dg = 0$.

(ii) $gd = 0$.

(iii) $dg + gd = 0$.

(iv) $d(x)g(x) = 0$ for all $x \in R$.

(v) dg is a (θ^2, ϕ^2)-derivation of R.

Lemma 2.4 ([4, Lemma 4]) *Let R be a 2-torsion free semiprime ring. If the generalized (θ, ϕ)-derivations (D, d) and (G, g) of R satisfying (1) are orthogonal, then*

(i) $D(x)G(y) = G(x)D(y) = 0$ for all $x, y \in R$.

(ii) d and G are orthogonal and $d(x)G(y) = G(y)d(x) = 0$ for all $x, y \in R$.

(iii) g and D are orthogonal and $g(x)D(y) = D(y)g(x) = 0$ for all $x, y \in R$.

(iv) d *and* g *are orthogonal.*

(v) $dG = Gd = 0$, $gD = Dg = 0$ *and* $DG = GD = 0$.

Note that for an ideal I of R, the left and right annihilators of I in R coincide, and $I \cap r(I) = 0$ $(I \cap l(I) = 0)$, where $l(I)$ and $r(I)$ denote the left annihilator and the right annihilator of I, respectively. The following simple fact is also useful.

Lemma 2.5 *Let R be a 2-torsion free semiprime ring with a nonzero ideal I. If $r(I) = 0$ or $l(I) = 0$, then for any automorphism θ of R we have $r(\theta(I)) = l(\theta(I)) = 0$.*

3. Main results

To prove our main result we need the following lemma.

Lemma 3.1 *Let R be a 2-torsion free semiprime ring, I a nonzero ideal such that $l(I) = 0$ and (D, d) and (G, g) generalized (θ,ϕ)-derivations of R satisfying (1). If*

$$D(x)G(y) = G(x)D(y) = 0 \tag{2}$$

for all $x, y \in I$, then (D, d) and (G, g) are orthogonal.

Proof Replacing y by ry in (2), where $r \in R$, we get

$$0 = D(x)G(yr) = D(x)(G(y)\theta(r) + \phi(y)g(r)) = D(x)\phi(y)g(r)$$
$$0 = G(x)D(yr) = G(x)(D(y)\theta(r) + \phi(y)d(r)) = G(x)\phi(y)d(r)$$

for all $x, y \in I$, $r \in R$. Using Lemmas 2.1 and 2.5, we obtain

$$D(x)g(r) = g(r)D(x) = d(r)G(x) = G(x)d(r) = 0 \tag{3}$$

for all $x \in I$, $r \in R$. Substituting xs for x in $g(r)D(x) = 0$, where $s \in R$, gives

$$0 = g(r)D(xs) = g(r)\phi(x)d(s), \quad x \in I,\ r, s \in R.$$

By Lemmas 2.1 and 2.5 this gives $g(r)d(s) = d(s)g(r) = 0$ for all $r, s \in R$. Thus d and g are orthogonal by Lemma 2.3. The orthogonality of d and g and (3) imply that

$$\begin{aligned} &0 = D(sx)g(y) = D(s)\theta(x)g(y), \quad 0 = G(sx)d(y) = G(s)\theta(x)d(y),\\ &0 = g(r)D(sy) = g(r)D(s)\theta(y), \quad 0 = d(r)G(sy) = d(r)G(s)\theta(y) \end{aligned} \tag{4}$$

for all $x, y \in I$ and $s, t \in R$. Replacing x, y by rx, sy in (2), respectively, where $r, s \in R$, and applying (4), we get

$$0 = D(rx)G(sy) = D(r)\theta(x)G(s)\theta(y) + \phi(r)d(x)\phi(s)g(y),$$
$$0 = G(rx)D(sy) = G(r)\theta(x)D(s)\theta(y) + \phi(r)g(x)\phi(s)d(y)$$

for all $x, y \in I$, $r, s \in R$. Then Lemma 2.5 and the orthogonality of d and g yield

$$D(r)\theta(x)G(s) = G(r)\theta(x)D(s) = 0, \quad x \in I,\ r, s \in R.$$

Since

$$D(r)\theta(I)RG(s) \subseteq D(r)\theta(I)G(s) = 0,\ G(r)\theta(I)RD(s) \subseteq G(r)\theta(I)D(s) = 0,$$

it follows from Lemma 2.1 that $D(r)RG(s) = G(r)RD(s) = 0$ for all $r, s \in R$, as desired. □

Now we are ready to prove our main results.

Theorem 3.1 *Let R be a 2-torsion free semiprime ring, I a nonzero ideal such that $l(I) = 0$ and (D, d) and (G, g) generalized (θ, ϕ)-derivations of R satisfying (1). Then the following conditions are equivalent:*

(i) (D, d) and (G, g) are orthogonal.

(ii) For all $x, y \in I$, the following relations hold:

(a) $D(x)G(y) + G(x)D(y) = 0$,

(b) $d(x)G(y) + g(x)D(y) = 0$.

(iii) $D(x)G(y) = d(x)G(y) = 0$ for all $x, y \in I$.

(iv) $D(x)G(y) = 0$ for all $x, y \in I$ and $dG = dg = 0$ on I.

(v) (DG, dg) is a generalized (θ^2, ϕ^2)-derivation from I to R and $D(x)G(y) = 0$ for all $x, y \in I$.

Proof (i)⇒(ii), (iii) and (iv) are clear from Lemmas 2.3 and 2.4.

(iii)⇒(i). By $D(x)G(y) = d(x)G(y) = 0$, we get

$$0 = D(rx)G(y) = D(r)\theta(x)G(y), \quad \text{for all } x, y \in I,\ r \in R.$$

Lemmas 2.1 and 2.5 yield $D(r)G(x) = G(x)D(r) = 0$, $x \in I$, $r \in R$. Thus from Lemma 3.1, we obtain (i).

(ii)⇒(iii). From (a) and (b), we get

$$0 = D(xz)G(x) + G(xz)D(x) = D(x)\theta(z)G(x) + G(x)\theta(z)D(x)$$

for all $x, z \in I$. Thus $D(x)\theta(I)G(x) = 0$ by Lemmas 2.1 and 2.5. Then by Lemmas 2.2 and 2.5, We obtain $D(x)\theta(I)G(y) = 0$ and

$$D(x)G(y) = D(x)\theta(I)G(y) = 0, \quad x, y \in I.$$

Hence

$$0 = D(xz)G(y) = \phi(x)d(z)G(y)$$

for all $x, y, z \in I$. Now Lemma 2.5 gives $d(x)G(y) = 0$ for all $x, y \in I$.

(iv)⇒(v). Replacing y by yr in $D(x)G(y) = 0$, where $r \in R$, we obtain

$$0 = D(x)G(yr) = D(x)\phi(y)g(r)$$

for all $x, y \in I$, $r \in R$. Then

$$D(x)g(r) = g(r)D(x) = 0, \quad x \in I,\ r \in R,$$

by Lemmas 2.1 and 2.5. Replacing x by xs in $g(r)D(x) = 0$, where $s \in R$, we get that $d(s)g(r) = 0$ for all $r, s \in R$ by Lemmas 2.1 and 2.5. By Lemma 2.3, d and g are orthogonal. Substituting sx for x in $D(x)g(r) = 0$ we see that $D(s)\theta(x)g(r) = 0$. Thus Lemmas 2.1 and 2.5 give

$$D(s)g(r) = 0 \tag{5}$$

for all $r,s\in R$. Since $dG(x)=dg(y)=0$, $x,y\in I$, we have

$$0=dG(xy)=d(G(x)\theta(y)+\phi(x)g(y))=G\phi(x)d\theta(y)+d\phi(x)g\theta(y)$$

for all $x,y\in I$. Then the orthogonality of d and g implies

$$G\phi(x)d\theta(y)=0. \tag{6}$$

Thus using (5) and (6), we obtain

$$\begin{aligned}DG(xy)&=DG(x)\theta^2(y)+G\phi(x)d\theta(y)+D\phi(x)g\theta(y)+\phi^2(x)dg(y)\\&=DG(x)\theta^2(y)+\phi^2(x)dg(y)\end{aligned}$$

for all $x,y\in I$.

(v)⇒(iii). Since (DG,dg) and (dg,dg) are both generalized (θ^2,ϕ^2)-derivations from I to R,

$$G\phi(x)d\theta(y)+D\phi(x)g\theta(y)=0, \tag{7}$$

$$g\phi(x)d\theta(y)+d\phi(x)g\theta(y)=0 \tag{8}$$

for all $x,y\in I$. Substituting yr for y in $D(x)g(y)=0$ we have

$$0=D(x)G(yr)=D(x)\phi(y)g(r)$$

for all $x,y\in I$, $r\in R$. Thus Lemmas 2.1 and 2.5 yield $D(x)g(r)=0$, in which replacing r by rs we have $D(x)\phi(r)g(s)=0$, for all $x\in I$, $r,s\in R$. Since ϕ is an automorphism, we have

$$D(x)Rg(s)=0,\quad r,s\in R. \tag{9}$$

Substituting $z\phi(x)$ for $\phi(x)$ in (7), where $z\in I$, and applying (8) and (9), we get

$$0=G(z\phi(x))d\theta(y)+D(z\phi(x))g\theta(y)=G(z)\theta\phi(x)d\theta(y)$$

for all $x,y,z\in I$. By Lemmas 2.1 and Lemma 2.5, we get

$$d\theta(y)G(z)=0,\quad y,z\in I. \tag{10}$$

Replacing $\theta(y)$ by $x\theta(y)$ in (10), we obtain

$$0=d(x\theta(y))G(z)=(d(x)\theta^2(y)+\phi(x)d\theta(y))G(z)=d(x)\theta^2(y)G(z)$$

for all $x,y\in I$. Therefore, Lemmas 2.1 and 2.5 give $d(x)G(z)=0$ for all $x,z\in I$. □

Notice that (D,d) and (G,g) are symmetric in Theorem 3.1. The theorem is still true if we change (iii), (iv) and (v) of Theorem 3.1 to

(iii′) $G(x)D(y)=g(x)D(y)=0$ for all $x,y\in I$;

(iv′) $G(x)D(y)=0$ for all $x,y\in I$ and $gD=gd=0$ on I;

(v′) (GD,gd) is a generalized (θ^2,ϕ^2)-derivation from I to R and $G(x)D(y)=0$ for all $x,y\in I$.

Theorem 3.2 *Let R be a 2-torsion free semiprime ring, I a nonzero ideal such that $l(I)=0$ and (D,d) and (G,g) generalized (θ,ϕ)-derivations of R satisfying (1). Then the following conditions are equivalent:*

(i) (DG, dg) *is a generalized* (θ^2, ϕ^2)*-derivation from* I *to* R.

(ii) (GD, gd) *is a generalized* (θ^2, ϕ^2)*-derivation from* I *to* R.

(iii) D *and* g *are orthogonal, and* G *and* d *are orthogonal.*

Proof (i)⇒(iii). Since (DG, dg) and (dg, dg) are both generalized (θ^2, ϕ^2)-derivations from I to R, we have

$$G\phi(x)d\theta(y) + D\phi(x)g\theta(y) = 0, \tag{11}$$

$$d\phi(x)g\theta(y) + g\phi(x)d\theta(y) = 0 \tag{12}$$

for all $x, y \in I$. Replacing $\phi(x)$ by $r\phi(x)$ in (12), where $r \in R$, we get

$$0 = d(r\phi(x))g\theta(y) + g(r\phi(x))d\theta(y) = d(r)\theta\phi(x)g\theta(y) + g(r)\theta\phi(x)d\theta(y) \tag{13}$$

for all $x, y \in I$ and $r \in R$. Substituting $s\theta(y)$ for $\theta(y)$ in (13) for any $s \in R$ yields

$$\begin{aligned} 0 &= d(r)\theta\phi(x)g(s\theta(y)) + g(r)\theta\phi(x)d(s\theta(y)) \\ &= d(r)\theta\phi(x)(g(s)\theta^2(y) + \phi(s)g\theta(y)) + g(r)\theta\phi(x)(d(s)\theta^2(y) + \phi(s)d\theta(y)) \end{aligned}$$

for all $x, y \in I$ and $r, s \in R$. Noticing (13), we obtain

$$d(r)\theta\phi(x)\phi(s)g\theta(y) + g(r)\theta\phi(x)\phi(s)d\theta(y) = 0.$$

Then

$$(d(r)\theta\phi(x)g(s) + g(r)\theta\phi(x)d(s))\theta^2(y) = 0 \tag{14}$$

for all $x, y \in I$, $r, s \in R$. In particular, if $r = s$ in (14), then Lemmas 2.1 and 2.5 yield

$$d(r)\theta\phi(I)g(r) = 0, \quad r \in R.$$

Combining Lemmas 2.1, 2.2 and 2.5 gives $d(r)g(s) = 0$. Applying Lemma 2.3, we obtain that d and g are orthogonal. Replacing $\phi(x)$ by $r\phi(x)$ in (11) and applying the orthogonality of d and g, we obtain that

$$0 = D(r\phi(x))g\theta(y) + G(r\phi(x))d\theta(y) = D(r)\theta\phi(x)g\theta(y) + G(r)\theta\phi(x)d\theta(y) \tag{15}$$

for all $x, y \in I$, $r \in R$. Substituting $g\theta(y)\theta\phi(x)$ for $\theta\phi(x)$ in (15) and applying the orthogonality of d and g gives

$$G(r)g\theta(y)\theta\phi(x)d\theta(y) + D(r)g\theta(y)\theta\phi(x)g\theta(y) = D(r)g\theta(y)\theta\phi(x)g\theta(y)$$

for all $x, y \in I$, $r \in R$. Noticing

$$D(r)g\theta(y)\theta\phi(I)RD(r)g\theta(y) \subseteq D(r)g\theta(y)\theta\phi(I)g\theta(y) = 0$$

and Lemma 2.1, we have $D(r)g\theta(y)RD(r)g\theta(y) = 0$. Since R is semiprime, we have

$$D(r)g\theta(y) = 0, \quad x, y \in I, \ r \in R. \tag{16}$$

Replacing $\theta(y)$ by $\theta(y)s$ in (16), we get

$$0 = D(r)g(\theta(y)s) = D(r)g\theta(y)\theta(s) + D(r)\phi\theta(y)g(s) = D(r)\phi\theta(y)g(s)$$

for all $y \in I$, $r, s \in R$. Then Lemmas 2.1 and 2.5 imply $g(s)D(r) = D(r)g(s) = 0$. Hence

$$0 = g(ts)D(r) = g(t)\theta(s)D(r)$$

for all $r, s, t \in R$. Then $g(t)RD(r) = D(r)Rg(t) = 0$. Similarly,

$$G(r)d(t) = d(t)G(r) = d(t)RG(r) = G(r)Rd(t) = 0$$

for all $r, t \in R$. Hence, D and g are orthogonal, and G and d are also orthogonal.

(iii)⇒(i). By the hypothesis and Lemma 2.1, we get that $G(r)d(s) = D(r)g(s) = 0$, for all $r, s \in R$. Hence

$$DG(xy) = DG(x)\theta^2(y) + \phi^2(x)dg(y), \quad x, y \in I.$$

Substituting rt for r in $G(r)d(s) = 0$, we obtain $0 = G(rt)d(s) = \phi(r)g(t)d(s)$, for all $r, s, t \in R$. Then Lemmas 2.3 and 2.5 yield $dg = 0$. Therefore, (DG, dg) is a generalized (θ^2, ϕ^2)-derivation from I to R.

Similarly, (ii)⇔(iii). □

Corollary 3.1 *Let R be a 2-torsion free semiprime ring, I be a nonzero ideal such that $l(I) = 0$. If (DG, dg) is a generalized (θ^2, ϕ^2)-derivation from I to R, then (DG, dg) is a generalized (θ^2, ϕ^2)-derivation of R and $dg = 0$.*

Proof By Theorem 3.2, D and g are orthogonal, and G and d are also orthogonal. Applying Lemma 2.1 gives $G(r)d(s) = D(r)g(s) = 0$. Then

$$G\phi(r)d\theta(s) + D\phi(r)g\theta(s) = 0$$

for all $r, s \in R$. Substituting rt for r in $G(r)d(s) = 0$, we get

$$0 = G(rt)d(s) = \phi(r)g(t)d(s)$$

for all $r, s, t \in R$. Applying Lemmas 2.3 and 2.5, we obtain $dg = 0$. Then Lemma 2.3 gives

$$g\phi(r)d\theta(s) + d\phi(r)g\theta(s) = 0$$

for all $r, s \in R$. Hence $DG(rs) = DG(r)\theta^2(s) + \phi^2 dg(s)$ for all $r, s \in R$. □

Corollary 3.2 *Let R be a 2-torsion free semiprime ring, I be a nonzero ideal such that $l(I) = 0$ and (D, d) be a generalized (θ, ϕ)-derivation of R. If (D^2, d^2) is a generalized (θ^2, ϕ^2)-derivation from I to R, then $d = 0$.*

Proof According to Corollary 3.1 and Lemma 2.3, d and d are orthogonal. Therefore, the semiprimeness of R implies that $d(R) = 0$. □

Corollary 3.3 *Let R be a 2-torsion free semiprime ring, I be a nonzero ideal such that $l(I) = 0$ and (D, d) be a generalized (θ, ϕ)-derivation of R. If $D(x)D(y) = 0$ for all $x, y \in I$, then $D = d = 0$.*

Proof By the hypothesis we have

$$0 = D(x)D(yr) = D(x)\phi(y)d(r)$$

for all $x, y \in I,\ r \in R$. Whence

$$d(r)D(x) = 0 \tag{17}$$

for all $x \in I$ and $r \in R$ by Lemmas 2.1 and 2.5. Replacing x by xs in (17), where $s \in R$, we get

$$0 = d(r)D(xs) = d(r)\phi(x)d(s)$$

for all $x \in I$, $r, s \in R$. Then from Lemmas 2.1 and 2.5 it follows $d(r)d(s) = 0$ for all $r, s \in R$. Applying the Lemma 2.3, we get that d and d are orthogonal. Therefore, by the semiprimeness of R, we obtain $d(R) = 0$. Thus

$$0 = D(rx)D(y) = D(r)\theta(x)D(y)$$

for all $x, y \in I$, $r \in R$. Then

$$D(r)D(y) = 0 \tag{18}$$

by Lemmas 2.1 and 2.5. Substituting sy for y in (18) and noticing that $d = 0$ leads to

$$0 = D(r)D(sy) = D(r)D(s)\theta(y)$$

for all $y \in I$, $r, s \in R$. Then Lemma 2.5 implies $D(r)D(s) = 0$. Noticing $d = 0$, we get

$$0 = D(rt)D(s) = D(r)\theta(t)D(s), \quad r, s, t \in R.$$

Hence, we obtain $D = 0$ by the semiprimeness of R. □

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