Journal of Mathematical Research & Exposition
Jul., 2011, Vol. 31, No. 4, pp. 721–726
DOI:10.3770/j.issn:1000-341X.2011.04.018
Http://jmre.dlut.edu.cn

ϕ-Derivations on Strongly Double Triangle Subspace Lattice Algebras

Yong Feng PANG[1,*], **Wei YANG**[1], **Hong Ke DU**[2]
1. *Department of Mathematics, School of Science, Xi'an University of Architecture and Technology, Shaanxi* 710055*, P. R. China;*
2. *College of Mathematics and Information Science, Shaanxi Normal University, Shaanxi* 710062*, P. R. China*

Abstract Let $\mathcal{D}=\{\{0\},K,L,M,\mathcal{X}\}$ be a strongly double triangle subspace lattice on a non-zero complex reflexive Banach space $\mathcal{X}$, which satisfies that one of three sums $K+L$, $L+M$ and $M+K$ is closed. It is shown that local ϕ-derivations and ϕ-derivations at zero point on Alg$\mathcal{D}$ are generalized ϕ-derivations.

Keywords generalized ϕ-derivations; local ϕ-derivations; ϕ-derivations at zero point; strongly double triangle subspace lattice.

Document code A
MR(2010) Subject Classification 47B47; 46L40
Chinese Library Classification O177.1

1. Introduction

Let $\mathcal{A}$ be a unital algebra. Recall that a derivation δ is a linear map from $\mathcal{A}$ into $\mathcal{A}$ such that $\delta(AB)=\delta(A)B+A\delta(B)$ for all $A,B\in\mathcal{A}$. A generalized derivation δ is a linear map from $\mathcal{A}$ into $\mathcal{A}$ such that $\delta(AB)=\delta(A)B+A\delta(B)-A\delta(I)B$ for all $A,B\in\mathcal{A}$. Let ϕ be an automorphism on $\mathcal{A}$. A ϕ-derivation η is a linear map from $\mathcal{A}$ into $\mathcal{A}$ such that $\eta(AB)=\eta(A)B+\phi(A)\eta(B)$ for all $A,B\in\mathcal{A}$. A generalized ϕ-derivation η is a linear map from $\mathcal{A}$ into $\mathcal{A}$ such that $\eta(AB)=\eta(A)B+\phi(A)\eta(B)-\phi(A)\eta(I)B$ for all $A,B\in\mathcal{A}$. A local ϕ-derivation η is a linear map from $\mathcal{A}$ into $\mathcal{A}$ if for each $A\in\mathcal{A}$ there is a ϕ-derivation δ_A from $\mathcal{A}$ into $\mathcal{A}$, depending on A, such that $\eta(A)=\delta_A(A)$. A ϕ-derivation at zero point η is a linear map from $\mathcal{A}$ into $\mathcal{A}$ such that $\eta(A)B+\phi(A)\eta(B)=0$ for all $A,B\in\mathcal{A}$ with $AB=0$.

Let $\mathcal{X}$ be a non-zero complex reflexive Banach space with topological dual $\mathcal{X}^*$. If $T\in\mathcal{B}(\mathcal{X})$, then $\mathcal{R}(T)$ denotes the range of T. For a subset E of $\mathcal{X}$, we denote by lin.span$\{E\}$ the linear span of E. If $e^*\in\mathcal{X}^*$, $f\in\mathcal{X}$, then $e^*\otimes f$ denotes the rank one operator $(e^*\otimes f)(x)=e^*(x)f$,

Received December 21, 2009; Accepted October 3, 2010
Supported by the National Natural Science Foundation of China (Grant No. 10871224), the Natural Science Young Foundation of Shaanxi Province (Grant No. 2010JQ1003), the Natural Science Special Foundation of Education Department of Shaanxi Province (Grant No. 08JK344) and the Basic Research Foundation of Xi'an University of Architecture and Technology (Grant No. JC1009).
* Corresponding author
E-mail address: pangyongfeng75@yahoo.com.cn (Y. F. PANG)

for all $x \in \mathcal{X}$. For any non-empty subset $Y \subseteq \mathcal{X}, Y^\perp$ denotes its annihilator, that is, $Y^\perp = \{f^* \in \mathcal{X}^* : f^*(y) = 0, \forall y \in Y\}$. For any non-empty subset $Z \subseteq \mathcal{X}^*, {}^\perp Z$ denotes its pre-annihilator, that is, ${}^\perp Z = \{x \in \mathcal{X} : f^*(x) = 0, \forall f^* \in Z\}$. Since $\mathcal{X}$ is reflexive, we have ${}^\perp(Y^\perp) = Y$ and $({}^\perp Z)^\perp = Z$ for any closed subspaces $Y \subseteq \mathcal{X}$ and $Z \subseteq \mathcal{X}^*$.

A subspace lattice on $\mathcal{X}$ is a family $\mathcal{L}$ of subspaces of $\mathcal{X}$ which contains $\{0\}$ and $\mathcal{X}$, and is closed under the intersection and closed linear span. That is, for any subfamily $\{L_\gamma\}_{\gamma\in\Gamma}$ of $\mathcal{L}$, we have $\cap_{\gamma\in\Gamma} L_\gamma \in \mathcal{L}$ and $\vee_{\gamma\in\Gamma} L_\gamma \in \mathcal{L}$. For any subspace lattice $\mathcal{L}$ of $\mathcal{X}$, we define Alg$\mathcal{L}$ by

$$\mathrm{Alg}\mathcal{L} = \{T \in \mathcal{B}(\mathcal{X}) : TL \subseteq L, \forall L \in \mathcal{L}\} \text{ and } \mathcal{L}^\perp = \{L^\perp : L \in \mathcal{L}\}.$$

A double triangle subspace lattice on $\mathcal{X}$ is a set $\mathcal{D} = \{\{0\}, K, L, M, \mathcal{X}\}$ of subspaces of $\mathcal{X}$ satisfying $K \cap L = L \cap M = M \cap K = \{0\}$ and $K \vee L = L \vee M = M \vee K = \mathcal{X}$. If one of three sums $K + L$, $L + M$ and $M + K$ is closed, we say that $\mathcal{D}$ is a strongly double triangle subspace lattice. It is known in [1] that Alg$\mathcal{D}$ contains no rank one operators. Alg$\mathcal{D}$ may or may not contain non-zero finite rank operators [2, Theorem 2.1]. Observe that $\mathcal{D}^\perp = \{\{0\}, K^\perp, L^\perp, M^\perp, \mathcal{X}^*\}$ is a double triangle subspace lattice on the reflexive Banach space $\mathcal{X}^*$. As Definition 2.1 in [2], put $K_0 = K \cap (L + M)$, $L_0 = L \cap (M + K)$, $M_0 = M \cap (K + L)$ and $K_p = K^\perp \cap (L^\perp + M^\perp)$, $L_p = L^\perp \cap (M^\perp + K^\perp)$, $M_p = M^\perp \cap (K^\perp + L^\perp)$, respectively. Note that K_p, L_p and M_p play the same role for $\mathcal{D}^\perp$ as K_0, L_0 and M_0 do for $\mathcal{D}$. Each of K_0, L_0, M_0 is an invariant linear manifold of Alg$\mathcal{D}$; each of K_p, L_p, M_p is an invariant linear manifold of Alg$\mathcal{D}^\perp$. By Lemma 2.2 in [2], dimensions of K_0, L_0 and M_0 are the same, denoted by m, where $m = \infty$ indicates that each of the K_0, L_0 and M_0 are infinite-dimensional. Similarly, the dimension of K_p, L_p and M_p are the same, denoted by n (Again $n = \infty$ indicates that each of the K_p, L_p and M_p are infinite-dimensional).

Derivations and local derivations from some reflexive subalgebras of $\mathcal{B}(\mathcal{X})$ into $\mathcal{B}(\mathcal{X})$ were studied by several papers [3–9]. In [10], we studied ϕ-derivations on some CSL algebras. In [11], we studied derivations and local derivations on strongly double triangle subspace lattice algebras. In [12], authors studied σ-derivable mapping at zero point on nest algebras. In this paper, we consider local ϕ-derivation and ϕ-derivation at zero point between strongly double triangle subspace lattice algebras. We show that every local ϕ-derivation and ϕ-derivation at zero point on Alg$\mathcal{D}$ are generalized ϕ-derivations. We next recall some results which are required in Sections 2 and 3.

Lemma 1.1 ([2, Lemma 2.1]) *Let $\mathcal{D}$ be a double triangle subspace lattice on $\mathcal{X}$. Then the following statements hold*

(i) $K_0 \subseteq K \subseteq^\perp K_p$, $L_0 \subseteq L \subseteq^\perp L_p$ *and* $M_0 \subseteq M \subseteq^\perp M_p$;

(ii) $K_0 \cap L_0 = L_0 \cap M_0 = M_0 \cap K_0 = \{0\}$;

(iii) $K_p \cap L_p = L_p \cap M_p = M_p \cap K_p = \{0\}$;

(iv) $K_0 + L_0 = L_0 + M_0 = M_0 + K_0 = K_0 + L_0 + M_0$;

(v) $K_p + L_p = L_p + M_p = M_p + K_p = K_p + L_p + M_p$.

The presence or absence of finite rank operators is governed by the following theorem.

Theorem 1.1 ([2, Theorem 2.1]) *Let $\mathcal{D}$ be a double triangle subspace lattice on $\mathcal{X}$.*

(i) Every finite rank operator of $\mathrm{Alg}\mathcal{D}$ *has even rank (possibly zero);*

(ii) If $e, f \in X$ and $e^*, f^* \in X^*$ are non-zero vectors satisfying $e \in K_0$, $f \in L_0$, $e + f \in M_0$ and $e^* \in K_p$, $f^* \in L_p$, $e^* + f^* \in M_p$, then $R = e^* \otimes f - f^* \otimes e$ is a rank two operator of $\mathrm{Alg}\mathcal{D}$*. Moreover, every rank two operator of* $\mathrm{Alg}\mathcal{D}$ *has this form for some such vectors e, f, e^*, f^*;*

(iii) $\mathrm{Alg}\mathcal{D}$ *contains a non-zero finite rank operator if and only if* $m \neq 0$ *and* $n \neq 0$;

(iv) Every finite rank operator of $\mathrm{Alg}\mathcal{D}$ *(if there are any) is a finite sum of rank two operators of* $\mathrm{Alg}\mathcal{D}$*.*

Theorem 1.2 ([2, Theorem 2.3]) *Let $\mathcal{D} = \{\{0\}, K, L, M, \mathcal{X}\}$ be a strongly double triangle subspace lattice on $\mathcal{X}$. Then*

(i) $K_0 + L_0 + M_0$ *is dense in* $\mathcal{X}$;

(ii) $K_p + L_p + M_p$ *is dense in* $\mathcal{X}^*$.

Lemma 1.2 ([2, Lemma 2.3]) *If* $\mathrm{Alg}\mathcal{D}$ *contains a rank two operator, then*

(i) $\mathrm{lin.span}\{\mathcal{R}(R) : R \in \mathrm{Alg}\mathcal{D}$ *and* $\mathrm{rank}\, R = 2\} = K_0 + L_0 + M_0$*;*

(ii) $\cap\{\ker R : R \in \mathrm{Alg}\mathcal{D}$ *and* $\mathrm{rank}\, R = 2\} =^{\perp} \{K_p + L_p + M_p\}$.

2. Local ϕ-derivations on $\mathrm{Alg}\mathcal{D}$

Let $\mathcal{D} = \{\{0\}, K, L, M, \mathcal{X}\}$ be a strongly double triangle subspace lattice on $\mathcal{X}$. It is easy to prove that $m \neq 0$ and $n \neq 0$. It follows from Theorem 1.1 that $\mathrm{Alg}\mathcal{D}$ contains non-zero finite rank operators . We may assume that $\mathcal{X} = K + L$. Semi-simplicity follows from Theorem 4 [13]. So there exists a rank two operator in $\mathrm{Alg}\mathcal{D}$ which is not nilpotent. Let ϕ be an isomorphism and η a local ϕ-derivation on $\mathrm{Alg}\mathcal{D}$. In this section, we consider the local ϕ-derivations on $\mathrm{Alg}\mathcal{D}$. By the same method in [10], we also prove the following lemmas.

Lemma 2.1 *(1) $\eta(E) = \eta(E)E + \phi(E)\eta(E)$ for all idempotents E in* $\mathrm{Alg}\mathcal{D}$*;*

(2) Let $A, B, C \in$ $\mathrm{Alg}\mathcal{D}$*. If $AB = BC = 0$, then $\phi(A)\eta(B)C = 0$.*

Lemma 2.2 *Let E and F be idempotents in* $\mathrm{Alg}\mathcal{D}$*. For all A in* $\mathrm{Alg}\mathcal{D}$*, we have $\eta(EAF) = \eta(EA)F + \phi(E)\eta(AF) - \phi(E)\eta(A)F$.*

The following lemmas are important for us to prove our main results.

Lemma 2.3 *Let R and S be rank two operators in* $\mathrm{Alg}\mathcal{D}$*. For all A in* $\mathrm{Alg}\mathcal{D}$*, we have $\eta(RAS) = \eta(RA)S + \phi(R)\eta(AS) - \phi(R)\eta(A)S$.*

Proof Let R be an idempotent in $\mathrm{Alg}\mathcal{D}$. For rank two operator S, by Theorem 1.1, we assume that $S = u^* \otimes v - v^* \otimes u$, where $u \in L_0, v \in M_0, u + v = \beta \in K_0$ and $u^* \in L_p, v^* \in M_p, u^* + v^* = \beta^* \in K_p$. It follows from Lemma 3.2 in [2] that $S^2 = -u^*(v)S$.

Case 1 If $u^*(v) \neq 0$, then $\frac{-1}{u^*(v)}S$ is an idempotent in $\mathrm{Alg}\mathcal{D}$. The consequence follows from Lemma 2.2 and linearity of η.

Case 2 If $u^*(v) = 0$, then there exists a vector $v_1 \in M_0$ such that $u^*(v_1) \neq 0$. Thus, by Lemma 1.1 there exist unique vectors $u_1 \in L_0$ and $\beta_1 \in K_0$ such that $u_1 + v_1 = \beta_1$. Let $S_0 = u^* \otimes v_1 - v^* \otimes u_1$ and $S_1 = u^* \otimes (v + v_1) - v^* \otimes (u + u_1)$. It follows from Theorem 1.1 that we have $S_1, S_0 \in \mathrm{Alg}\mathcal{D}$ and $S = S_1 - S_0$. For operators S_1, S_0, by the result of Case 1 we have

$$\begin{aligned}\eta(RAS) =& \eta(RAS_1) - \eta(RAS_0)\\ =&(\eta(RA)S_1 + \phi(R)\eta(AS_1) - \phi(R)\eta(A)S_1) -\\ &(\eta(RA)S_0 + \phi(R)\eta(AS_0) - \phi(R)\eta(A)S_0)\\ =&\eta(RA)S + \phi(R)\eta(AS) - \phi(R)\eta(A)S.\end{aligned}$$

By the same method, we have $\eta(RAS) = \eta(RA)S + \phi(R)\eta(AS) - \phi(R)\eta(A)S$ for all rank two operators R in $\mathrm{Alg}\mathcal{D}$.

Now we prove our main result.

Theorem 2.1 *Let $\mathcal{D}$ be a strongly double triangle subspace lattice on a Banach space $\mathcal{X}$ and ϕ be an isomorphism on* $\mathrm{Alg}\mathcal{D}$. *Suppose that η is a local ϕ-derivation of* $\mathrm{Alg}\mathcal{D}$. *Then η is a generalized ϕ-derivation; particularly, if $\eta(I) = 0$, then η is a ϕ-derivation.*

Proof Let S and R be rank two operators in $\mathrm{Alg}\mathcal{D}$. It follows from Proposition 3.1 in [14] that there is a rank two operator T in $\mathrm{Alg}\mathcal{D}$ such that $\phi(T) = R$. Let A, B be in $\mathrm{Alg}\mathcal{D}$. Then TA and BS are either rank two operators or zero in $\mathrm{Alg}\mathcal{D}$. It follows from Lemma 2.3 that we have

$$\eta(TABS) = \eta((TA)BS) = \eta(TAB)S + \phi(TA)\eta(BS) - \phi(TA)\eta(B)S,$$

$$\eta(TABS) = \eta(T(AB)S) = \eta(TAB)S + \phi(T)\eta(ABS) - \phi(T)\eta(AB)S.$$

It follows from $\phi(T) = R$ that $R\eta(ABS) = R[\eta(AB)S + \phi(A)\eta(BS) - \phi(A)\eta(B)S]$. By Lemma 2.1 in [14], we get $\eta(ABS) = \eta(AB)S + \phi(A)\eta(BS) - \phi(A)\eta(B)S$. Let C be in $\mathrm{Alg}\mathcal{D}$. Replacing B by C and S by BS, respectively, we have $\eta(ACBS) = \eta(AC)BS + \phi(A)\eta(CBS) - \phi(A)\eta(C)BS$. Taking $C = I$, we have $\eta(ABS) = \eta(A)BS + \phi(A)\eta(BS) - \phi(A)\eta(I)BS$. Combining above two equations, we have $\eta(AB)S = [\eta(A)B + \phi(A)\eta(B) - \phi(A)\eta(I)B]S$. It follows from Lemma 2.1 in [14] that we have $\eta(AB) = \eta(A)B + \phi(A)\eta(B) - \phi(A)\eta(I)B$.

3. ϕ-derivations at zero point on $\mathrm{Alg}\mathcal{D}$

Let η be a ϕ-derivation at zero point on $\mathrm{Alg}\mathcal{D}$. In this section, we consider the ϕ-derivations at zero point on $\mathrm{Alg}\mathcal{D}$. Let $E^{\perp}$ be $I - E$ for every idempotent E in $\mathrm{Alg}\mathcal{D}$.

Lemma 3.1 *$\phi(E)\eta(I) = \eta(I)E$ for all idempotents E in* $\mathrm{Alg}\mathcal{D}$.

Proof Since $EE^{\perp} = 0 = E^{\perp}E$, we obtain that $\eta(E)E^{\perp} + \phi(E)\eta(E^{\perp}) = 0$ and $\eta(E^{\perp})E + \phi(E^{\perp})\eta(E) = 0$. It follows from the linearity of η and ϕ that $\eta(E) - \eta(E)E + \phi(E)\eta(I) - \phi(E)\eta(E) = 0$. Therefore we have

$$\phi(E)\eta(I) = \eta(E)E + \phi(E)\eta(E) - \eta(E)$$

$$= \eta(E)E + \eta(E^{\perp})E + \phi(E)\eta(E) + \phi(E^{\perp})\eta(E) - \eta(E)$$
$$= \eta(E + E^{\perp})E + \phi(E + E^{\perp})\eta(E) - \eta(E)$$
$$= \eta(I)E + \phi(I)\eta(E) - \eta(E) = \eta(I)E.$$

Lemma 3.2 *$\eta(AE) = \eta(A)E + \phi(A)\eta(E) - \phi(A)\eta(I)E$ for all $A, E \in \mathrm{Alg}\mathcal{D}$, where E is an idempotent.*

Proof It follows from $AEE^{\perp} = 0 = AE^{\perp}E$ that we have $\eta(AE)E^{\perp} + \phi(AE)\eta(E^{\perp}) = 0$ and $\eta(AE^{\perp})E + \phi(AE^{\perp})\eta(E) = 0$. By the linearity of η and ϕ, we have $\eta(AE) - \eta(AE)E + \phi(AE)\eta(I) - \phi(AE)\eta(E) = 0$ and $\eta(A)E - \eta(AE)E + \phi(A)\eta(E) - \phi(AE)\eta(E) = 0$.

Note that $\phi(AE) = \phi(A)\phi(E)$. Combining above two equations, we get $\eta(AE) = \eta(A)E + \phi(A)\eta(E) - \phi(A)\phi(E)\eta(I)$. By Lemma 3.1, we have $\eta(AE) = \eta(A)E + \phi(A)\eta(E) - \phi(A)\eta(I)E$.

Now we prove our main result.

Theorem 3.1 *Let $\mathcal{D}$ be a strongly double triangle subspace lattice on a Banach space $\mathcal{X}$ and ϕ be an isomorphism on $\mathrm{Alg}\mathcal{D}$. Suppose that η is a ϕ-derivation at zero point on $\mathrm{Alg}\mathcal{D}$. Then η is a generalized ϕ-derivation; particularly, if $\eta(I) = 0$, then η is a ϕ-derivation.*

Proof We complete the proof by the following several steps.

Claim 1 $\eta(ABR) = \eta(AB)R + \phi(AB)\eta(R) - \phi(AB)\eta(I)R$ for any rank two operator $R \in \mathrm{Alg}\mathcal{D}$ and any operator $A, B \in \mathrm{Alg}\mathcal{D}$. We assume that $R = u^* \otimes v - v^* \otimes u$, where $u \in L_0$, $v \in M_0$, $u + v = \beta \in K_0$ and $u^* \in L_p$, $v^* \in M_p$, $u^* + v^* = \beta^* \in K_p$. It follows from Lemma 3.2 in [2] that we have $R^2 = -u^*(v)R$.

Case 1 If $u^*(v) \neq 0$, then $\frac{-1}{u^*(v)}R$ is an idempotent in $\mathrm{Alg}\mathcal{D}$. The consequence follows from Lemma 3.2 and linearity of η.

Case 2 If $u^*(v) = 0$, then there exists a vector $v_1 \in M_0$ such that $u^*(v_1) \neq 0$. Hence there exist unique vectors $u_1 \in L_0$ and $\beta_1 \in K_0$ such that $u_1 + v_1 = \beta_1$ by Lemma 1.1. Let $R_0 = u^* \otimes v_1 - v^* \otimes u_1$ and $R_1 = u^* \otimes (v + v_1) - v^* \otimes (u + u_1)$. It follows from Theorem 1.1 that we have $R = R_1 - R_0$ and R_1, $R_0 \in \mathrm{Alg}\mathcal{D}$. For operators R_1, R_0, by the result of Case 1 we have

$$\begin{aligned}\eta(ABR) =& \eta(ABR_1) - \eta(ABR_0)\\ =& (\eta(AB)R_1 + \phi(AB)\eta(R_1) - \phi(AB)\eta(I)R_1) -\\ & (\eta(AB)R_0 + \phi(AB)\eta(R_0) - \phi(AB)\eta(I)R_0)\\ =& \eta(AB)R + \phi(AB)\eta(R) - \phi(AB)\eta(I)R.\end{aligned}$$

Claim 2 $\eta(AB) = \eta(A)B + \phi(A)\eta(B) - \phi(A)\eta(I)B$ for all operators A, B in $\mathrm{Alg}\mathcal{D}$.

Let R be a rank two operator in $\mathrm{Alg}\mathcal{D}$. Then BR is rank two operator or zero operator. It follows from the result of Case 1 that we have

$$\eta(ABR) = \eta((AB)R) = \eta(AB)R + \phi(AB)\eta(R) - \phi(AB)\eta(I)R,$$

$$\eta(ABR) = \eta(A(BR)) = \eta(A)BR + \phi(A)\eta(BR) - \phi(A)\eta(I)BR,$$

$$\eta(BR) = \eta(B)R + \phi(B)\eta(R) - \phi(B)\eta(I)R.$$

By an elementary calculation, we have

$$\begin{aligned}\eta(AB)R =&\eta(ABR) - (\phi(AB)\eta(R) - \phi(AB)\eta(I)R)\\ =&\eta(A)BR + \phi(A)\eta(BR) - \phi(A)\eta(I)BR - \phi(AB)\eta(R) + \phi(AB)\eta(I)R\\ =&\eta(A)BR + \phi(A)\eta(B)R + \phi(A)\phi(B)\eta(R) - \phi(A)\phi(B)\eta(I)R-\\ &\phi(A)\eta(I)BR - \phi(AB)\eta(R) + \phi(AB)\eta(I)R\\ =&\eta(A)BR + \phi(A)\eta(B)R - \phi(A)\eta(I)BR\\ =&(\eta(A)B + \phi(A)\eta(B) - \phi(A)\eta(I)B)R.\end{aligned}$$

By Lemma 2.1 in [14], we have $\eta(AB) = \eta(A)B + \phi(A)\eta(B) - \phi(A)\eta(I)B$.

References

[1] LONGSTAFF W E. *Strongly reflexive lattices* [J]. J. London Math. Soc. (2), 1975, **11**(4): 491–498.
[2] LAMBROU M S, LONGSTAFF W E. *Finite rank operators leaving double triangles invariant* [J]. J. London Math. Soc. (2), 1992, **45**(1): 153–168.
[3] KADISON R V. *Local derivations* [J]. J. Algebra, 1990, **130**(2): 494–509.
[4] LARSON D R, SOUROUR A R. *Local Derivations and Local Automorphisms of* $\mathscr{B}(X)$ [M]. Proc. Sympos. Pure Math., 51, Part 2, Amer. Math. Soc., Providence, RI, 1990.
[5] LI Pengtong, MA Jipu. *Derivations, local derivations and atomic Boolean subspace lattices* [J]. Bull. Austral. Math. Soc., 2002, **66**(3): 477–486.
[6] LU Fangyan. *Derivations of CDC algebras* [J]. J. Math. Anal. Appl., 2006, **323**(1): 179–189.
[7] LU Shijie, LU Fangyan. *Non-Selfadjoint Operator Algebras* [M]. Beijing: Science Press, 2004. (in Chinese)
[8] ZHU Jun, XIONG Changping. *Generalized derivable mappings at zero point on some reflexive operator algebras* [J]. Linear Algebra Appl., 2005, **397**: 367–379.
[9] ZHANG Jianhua, JI Guoxing, CAO Huaixin. *Local derivations of nest subalgebras of von Neumann algebras* [J]. Linear Algebra Appl., 2004, **392**: 61–69.
[10] PANG Yongfeng, JI Guoxing, YANG Wei. *Local ϕ-derivations on certain CSL algebras* [J]. J. Shaanxi Normal Univ. Nat. Sci. Ed., 2008, **36**(4): 8–11. (in Chinese)
[11] PANG Yongfeng, YANG Wei. *Derivations and local derivations on strongly double triangle subspace lattices* [J]. Linear and Multilinear Algebra, 2010, **58**: 855-862.
[12] ZHU Lingyun, ZHANG Jianhua. *σ-derivable mapping at the zero point on nest algebras* [J]. J. Yunnan Normal University, 2008, **28**: 9–12. (in Chinese)
[13] LONGSTAFF W E. *Remarks on semi-simple reflexive algrbras. Conference on Automatic continuity and Banach algebras* [C]. Canberra, January 1989, **28**: 273–287.
[14] PANG Yongfeng, JI Guoxing. *Algebraic isomorphisms and strongly double triangle subspace lattices* [J]. Linear Algebra Appl., 2007, **422**(1): 265–273.