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Value-Sharing of Meromorphic Functions and Differential Polynomials

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Abstract This paper is devoted to studying the relationship between meromorphic functions $f(z)$ and $g(z)$ when their differential polynomials satisfy sharing condition weaker than sharing one value IM.

Keywords meromorphic function; uniqueness; sharing value; differential polynomial.

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1. Introduction and main results

We assume that the reader is familiar with the usual notations and basic results of the Nevanlinna theory [6, 7, 10]. We use $S(r, f)$ to denote any quantity satisfying $S(r, f) = o(T(r, f))$, as $r \to \infty$, possibly outside of an exceptional set of finite logarithmic measure.

In addition, we also use the following notations. Let a be a finite complex number, and k be a positive integer, we denote by $N_{k)}(r, \frac{1}{f-a})$ the counting function for zeros of $f - a$ with multiplicity no more than k (counting multiplicity), and by $\overline{N}_{k)}(r, \frac{1}{f-a})$ the corresponding one for which multiplicity is not counted. Let $N_{(k}(r, \frac{1}{f-a})$ be the counting function for zeros of $f - a$ with multiplicity at least k (counting multiplicity), and $\overline{N}_{(k}(r, \frac{1}{f-a})$ be the corresponding one for which multiplicity is not counted. We define

$$N_k(r, \frac{1}{f-a}) = \overline{N}(r, \frac{1}{f-a}) + \overline{N}_{(2}(r, \frac{1}{f-a}) + \cdots + \overline{N}_{(k}(r, \frac{1}{f-a}).$$

Clearly, $N_1(r, \frac{1}{f-a}) = \overline{N}(r, \frac{1}{f-a})$.

Suppose now that f and g are two non-constant meromorphic functions. We say that f and g share a CM (IM), if $f - a$ and $g - a$ have the same zeros counting multiplicity (ignoring multiplicity). Let z_0 be the zero of $f - a$ with the multiplicity p and the zero of $g - a$ with the multiplicity q. We denote by $\overline{N}_L(r, \frac{1}{f-a})$ the counting function for the zeros of $f - a$ with $p > q \geq 1$; by $\overline{N}_E^{1)}(r, \frac{1}{f-a})$ the counting function of the zeros of $f - a$ where $p = q = 1$; by $\overline{N}_E^{(2}(r, \frac{1}{f-a})$ the counting function of the zeros of $f - a$ where $p = q \geq 2$. Each point in these

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counting functions is counted only once. In the same way, we can define $\overline{N}_L(r, \frac{1}{g-a})$, $\overline{N}_E^{1)}(r, \frac{1}{g-a})$, $\overline{N}_E^{(2}(r, \frac{1}{g-a})$. We use $\overline{N}_*(r, \frac{1}{f-a})$ to denote the counting function of zeros of $f-a$ but not the zeros of $g-a$, for which multiplicity is not counted, and we can similarly define $\overline{N}_*(r, \frac{1}{g-a})$.

Let $E_{k)}(a, f)$ denote the set of zeros of $f-a$ with multiplicity m no more than k counting m times, and let $\overline{E}_{k)}(a, f)$ denote the set of those zeros of $f-a$ with multiplicity no more than k ignoring multiplicity.

Corresponding to one famous question of Hayman [5], Fang and Hua [2], Yang and Hua [9] obtained the following unicity theorem.

Theorem A *Let $f(z)$ and $g(z)$ be two nonconstant entire functions, and let $n \geq 6$ be a positive integer. If $f^n f'$ and $g^n g'$ share 1 CM, then either $f(z) = c_1 e^{cz}, g(z) = c_2 e^{-cz}$ where c_1, c_2 and c are three constants satisfying $(c_1 c_2)^{n+1} c^2 = -1$, or $f(z) = tg(z)$ for a constant t such that $t^{n+1} = 1$.*

Notice $f^n f' = \frac{1}{n+1}(f^{n+1})'$. Fang [3] considered kth derivative instead of 1st derivative, and proved the following result.

Theorem B *Let $f(z)$ and $g(z)$ be two nonconstant entire functions, and n, k be two positive integers with $n > 2k+4$. If $[f^n]^{(k)}$ and $[g^n]^{(k)}$ share 1 CM, then either $f(z) = c_1 e^{cz}, g(z) = c_2 e^{-cz}$ where c_1, c_2 and c are three constants satisfying $(-1)^k (c_1 c_2)^n (nc)^{2k} = 1$, or $f(z) = tg(z)$ for a constant t such that $t^n = 1$.*

Recently, Bhoosnurmath and Dyavanal [1] extended Fang's result to the meromorphic functions case.

Theorem C *Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions, and n, k be two positive integers with $n > 3k+8$. If $[f^n]^{(k)}$ and $[g^n]^{(k)}$ share 1 CM, then either $f(z) = c_1 e^{cz}$, $g(z) = c_2 e^{-cz}$ where c_1, c_2 and c are three constants satisfying $(-1)^k (c_1 c_2)^n (nc)^{2k} = 1$, or $f(z) = tg(z)$ for a constant t such that $t^n = 1$.*

Xu [8] improved the above related results by weakening the CM sharing condition, as follows.

Theorem D *Let $f(z)$ and $g(z)$ be two nonconstant meromorphic (resp. entire) functions, and n, k be two positive integers with $n > 3k+8$ (resp. $n > 2k+4$). If $E_{3)}(1, (f^n)^{(k)}) = E_{3)}(1, (g^n)^{(k)})$, then either $f(z) = tg(z)$ for some nth root of unity t or $f(z) = c_1 e^{cz}$, $g(z) = c_2 e^{-cz}$ where c_1, c_2 and c are three constants satisfying $(-1)^k (c_1 c_2)^n (nc)^{2k} = 1$.*

For some general differential polynomials, Zhang et al. [11] obtained the following result.

Theorem E *Let $f(z)$ and $g(z)$ be two nonconstant entire functions; and let n, k and m be three positive integers with $n \geq 3m+2k+5$, and let $P(z) = a_m z^m + a_{m-1} z^{m-1} + \cdots + a_1 z + a_0$ or $P(z) \equiv c_0$, where $a_0 \neq 0, a_1, \ldots, a_{m-1}, a_m \neq 0, c_0 \neq 0$ are complex constants. If $[f^n P(f)]^{(k)} = [g^n P(g)]^{(k)}$ share 1 CM, then*

(i) When $P(z) = a_m z^m + a_{m-1} z^{m-1} + \cdots + a_1 z + a_0$, either $f(z) = tg(z)$ for a constant t such that $t^d = 1$, where $d = (n+m, \ldots, n+m-i, \ldots, n)$, $a_{m-i} \neq 0$ for some $i = 0, 1, \ldots, m$ or f and g satisfy the algebraic equation $R(f, g) \equiv 0$, where $R(w_1, w_2) = w_1^n P(w_1) - w_2^n P(w_2)$;

(ii) When $P(z) \equiv c_0$, either $f(z) = c_1 c_0^{-\frac{1}{n}} e^{cz}$, $g(z) = c_1 c_0^{-\frac{1}{n}} e^{-cz}$ where c_1, c_2 and c are three constants satisfying $(-1)^k (c_1 c_2)^n (nc)^{2k} = 1$, or $f(z) = tg(z)$ for a constant t such that $t^n = 1$.

When $[f^n]^{(k)}$ and $[g^n]^{(k)}$ share 1 IM, Li and Lu [4] obtained the following theorem.

Theorem F *Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions, and n, k be two positive integers with $n > 6k + 14$. If $[f^n]^{(k)}$ and $[g^n]^{(k)}$ share 1 IM, then either $f(z) = c_1 e^{cz}$, $g(z) = c_2 e^{-cz}$ where c_1, c_2 and c are three constants satisfying $(-1)^k (c_1 c_2)^n (nc)^{2k} = 1$, or $f(z) = tg(z)$ for a constant t such that $t^n = 1$.*

In this paper, we will prove the following uniqueness theorems under weaker sharing condition than sharing one value IM.

Theorem 1 *Let $f(z)$ and $g(z)$ be two nonconstant meromorphic (resp. entire) functions, and n, k be two positive integers with $n > 9k + 14$ (resp. $n > 5k + 7$). If $\overline{E}_{l)}(1, (f^n)^{(k)}) = \overline{E}_{l)}(1, (g^n)^{(k)})$ with positive integer l, then either $f(z) = tg(z)$ for some nth root of unity t or $f(z) = c_1 e^{cz}$, $g(z) = c_2 e^{-cz}$ where c_1, c_2 and c are three constants satisfying $(-1)^k (c_1 c_2)^n (nc)^{2k} = 1$.*

Theorem 2 *Let $f(z)$ and $g(z)$ be two nonconstant entire functions, n, k and m be three positive integers with $n > 5m + 5k + 7$, and $P(z)$ be defined as in Theorem E. If $\overline{E}_{l)}(1, [f^n P(f)]^{(k)}) = \overline{E}_{l)}(1, [g^n P(g)]^{(k)})$ with positive integer l, then two conclusions of Theorem E also hold.*

2. Preliminary lemmas

Firstly, we recall a few lemmas that play important roles in the reasoning.

Lemma 1 ([10]) *Let $f(z)$ be a nonconstant meromorphic function, k be a positive integer, and c be a nonzero finite complex number. Then*

$$T(r, f) \leq \overline{N}(r, f) + N_{k+1}(r, \frac{1}{f}) + \overline{N}(r, \frac{1}{f^{(k)} - c}) - N_0(r, \frac{1}{f^{(k+1)}}) + S(r, f).$$

Here $N_0(r, \frac{1}{f^{(k+1)}})$ is the counting function which only counts these points such that $f^{(k+1)} = 0$ but $f(f^{(k)} - c) \neq 0$.

Lemma 2 *Let $f(z)$ be a nonconstant meromorphic function, and k be a positive integer. Then*

$$\overline{N}_{(2}(r, \frac{1}{f^{(k)} - 1}) \leq (k+1)\overline{N}(r, f) + N_k(r, \frac{1}{f}) + \overline{N}(r, \frac{1}{f}) + S(r, f). \tag{2.1}$$

Proof Firstly, we have

$$\begin{aligned}
\overline{N}(r, \frac{1}{f^{(k)}}) &\leq \overline{N}(r, \frac{f}{f^{(k)}}) + \overline{N}(r, \frac{1}{f}) \\
&\leq T(r, \frac{f^{(k)}}{f}) + \overline{N}(r, \frac{1}{f}) + S(r, f) \\
&\leq N(r, \frac{f^{(k)}}{f}) + \overline{N}(r, \frac{1}{f}) + S(r, f) \\
&\leq k\overline{N}(r, f) + N_k(r, \frac{1}{f}) + \overline{N}(r, \frac{1}{f}) + S(r, f).
\end{aligned}$$

Next, it is easy to see

$$\begin{aligned}\overline{N}_{(2}(r,\frac{1}{f^{(k)}-1}) &\le \overline{N}(r,\frac{f^{(k)}}{f^{(k+1)}})\\ &\le T(r,\frac{f^{(k+1)}}{f^{(k)}})+S(r,f)\\ &\le N(r,\frac{f^{(k+1)}}{f^{(k)}})+S(r,f)\\ &\le \overline{N}(r,f)+\overline{N}(r,\frac{1}{f^{(k)}})+S(r,f). \qquad (2.2)\end{aligned}$$

Combining (2.2) with the estimation for $\overline{N}(r,\frac{1}{f^{(k)}})$ yields (2.1). □

3. Proofs of Theorems

Proof of Theorem 1 Consider $F(z)=f^n$ and $G(z)=g^n$. Since $\overline{E}_{l)}(1,(f^n)^{(k)})=\overline{E}_{l)}(1,(g^n)^{(k)})$, it means that $\overline{E}_{l)}(1,F^{(k)})=\overline{E}_{l)}(1,G^{(k)})$. Clearly, from the standard Valiron-Mohon'ko theorem, we have

$$T(r,F)=nT(r,f)+S(r,f),\quad T(r,G)=nT(r,g)+S(r,g). \qquad (3.1)$$

We set

$$H(z)=\Big(\frac{F^{(k+2)}}{F^{(k+1)}}-2\frac{F^{(k+1)}}{F^{(k)}-1}\Big)-\Big(\frac{G^{(k+2)}}{G^{(k+1)}}-2\frac{G^{(k+1)}}{G^{(k)}-1}\Big). \qquad (3.2)$$

Suppose that $H(z)\not\equiv 0$. Clearly, $m(r,H)=S(r,f)+S(r,g)$. Firstly, a simple computation on local expansions shows that $H(z_0)=0$ if z_0 is a common simple zero of $F^{(k)}-1$ and $G^{(k)}-1$. Then we have

$$\overline{N}_E^{1)}(r,\frac{1}{F^{(k)}-1})\le N(r,\frac{1}{H})\le N(r,H)+S(r,f)+S(r,g). \qquad (3.3)$$

The poles of $H(z)$ only come from the zeros of $F^{(k+1)}$ and $G^{(k+1)}$, the poles of f and g, the zeros of $F^{(k)}-1$ and $G^{(k)}-1$ with different multiplicity, the zeros of $F^{(k)}-1$ which are not the zeros of $G^{(k)}-1$, and the zeros of $G^{(k)}-1$ but not the zeros of $F^{(k)}-1$. By analysis, we deduce that

$$\begin{aligned}N(r,H)\le&\overline{N}(r,f)+\overline{N}(r,g)+\overline{N}(r,\frac{1}{F})+\overline{N}(r,\frac{1}{G})+\overline{N}_L(r,\frac{1}{F^{(k)}-1})+\overline{N}_L(r,\frac{1}{G^{(k)}-1})+\\ &\overline{N}_*(r,\frac{1}{F^{(k)}-1})+\overline{N}_*(r,\frac{1}{G^{(k)}-1})+\overline{N}_0(r,\frac{1}{F^{(k+1)}})+\overline{N}_0(r,\frac{1}{G^{(k+1)}})+\\ &S(r,f)+S(r,g), \qquad (3.4)\end{aligned}$$

where $N_0(r,\frac{1}{F^{(k+1)}})$ only counts those zeros of $F^{(k+1)}$ but not the zeros of $F(F-1)$, and $\overline{N}_0(r,\frac{1}{F^{(k+1)}})$ denotes the corresponding reduced counting function. At the same time, obviously

$$\begin{aligned}\overline{N}(r,\frac{1}{F^{(k)}-1})=&\overline{N}_E^{1)}(r,\frac{1}{F^{(k)}-1})+\overline{N}_E^{(2}(r,\frac{1}{F^{(k)}-1})+\overline{N}_L(r,\frac{1}{F^{(k)}-1})+\\ &\overline{N}_L(r,\frac{1}{G^{(k)}-1})+\overline{N}_*(r,\frac{1}{F^{(k)}-1}).\end{aligned}$$

Combining this with (3.3) and (3.4) gives

$$\overline{N}(r,\frac{1}{F^{(k)}-1})$$

$$\le \overline{N}(r,f)+\overline{N}(r,g)+\overline{N}(r,\frac{1}{F})+\overline{N}(r,\frac{1}{G})+2\overline{N}_L(r,\frac{1}{F^{(k)}-1})+2\overline{N}_L(r,\frac{1}{G^{(k)}-1})+$$
$$2\overline{N}_*(r,\frac{1}{F^{(k)}-1})+\overline{N}_*(r,\frac{1}{G^{(k)}-1})+\overline{N}_0(r,\frac{1}{F^{(k+1)}})+\overline{N}_0(r,\frac{1}{G^{(k+1)}})+$$
$$\overline{N}_E^{(2}(r,\frac{1}{F^{(k)}-1})+S(r,f)+S(r,g). \tag{3.5}$$

Since $\overline{E}_{l)}(1,F^{(k)})=\overline{E}_{l)}(1,G^{(k)})$, it follows

$$\overline{N}_L(r,\frac{1}{F^{(k)}-1})+\overline{N}_*(r,\frac{1}{F^{(k)}-1})\le \overline{N}_{(2}(r,\frac{1}{F^{(k)}-1}).$$

By Lemma 2.2, we have

$$\overline{N}_L(r,\frac{1}{F^{(k)}-1})+\overline{N}_*(r,\frac{1}{F^{(k)}-1})\le (k+1)\overline{N}(r,f)+N_k(r,\frac{1}{F})+\overline{N}(r,\frac{1}{F})+S(r,f). \tag{3.6}$$

The similar inequality also holds for G. Consider the following relation

$$\overline{N}(r,\frac{1}{G^{(k)}-1})+\overline{N}_L(r,\frac{1}{G^{(k)}-1})+\overline{N}_E^{(2}(r,\frac{1}{G^{(k)}-1})$$
$$\le N(r,\frac{1}{G^{(k)}-1})\le T(r,G)+k\overline{N}(r,g)+S(r,g).$$

By Lemma 2.1 and (3.5), we have

$$\begin{aligned}T(r,F)+T(r,G)\le&\overline{N}(r,f)+\overline{N}(r,g)+N_{k+1}(r,\frac{1}{F})+N_{k+1}(r,\frac{1}{G})+\overline{N}(r,\frac{1}{F^{(k)}-1})+\\&\overline{N}(r,\frac{1}{G^{(k)}-1})-N_0(r,\frac{1}{F^{(k+1)}})-N_0(r,\frac{1}{G^{(k+1)}})+S(r,f)+S(r,g)\\\le&2\overline{N}(r,f)+(k+2)\overline{N}(r,g)+N_{k+1}(r,\frac{1}{F})+N_{k+1}(r,\frac{1}{G})+\overline{N}(r,\frac{1}{F})+\\&\overline{N}(r,\frac{1}{G})+2\overline{N}_L(r,\frac{1}{F^{(k)}-1})+\overline{N}_L(r,\frac{1}{G^{(k)}-1})+2\overline{N}_*(r,\frac{1}{F^{(k)}-1})+\\&T(r,G)+\overline{N}_*(r,\frac{1}{G^{(k)}-1})+S(r,f)+S(r,g).\end{aligned} \tag{3.7}$$

It follows from this and (3.6) that

$$\begin{aligned}T(r,F)\le&(2k+4)\overline{N}(r,f)+(2k+3)\overline{N}(r,g)+N_{k+1}(r,\frac{1}{F})+N_{k+1}(r,\frac{1}{G})+2N_k(r,\frac{1}{F})+\\&N_k(r,\frac{1}{G})+3\overline{N}(r,\frac{1}{F})+2\overline{N}(r,\frac{1}{G})+S(r,f)+S(r,g).\end{aligned}$$

Similarly, we have

$$\begin{aligned}T(r,G)\le&(2k+4)\overline{N}(r,g)+(2k+3)\overline{N}(r,f)+N_{k+1}(r,\frac{1}{F})+N_{k+1}(r,\frac{1}{G})+2N_k(r,\frac{1}{G})+\\&N_k(r,\frac{1}{F})+3\overline{N}(r,\frac{1}{G})+2\overline{N}(r,\frac{1}{F})+S(r,f)+S(r,g).\end{aligned}$$

Therefore, we can obtain

$$\begin{aligned}T(r,F)+T(r,G)\le&(4k+7)[\overline{N}(r,f)+\overline{N}(r,g)]+2[N_{k+1}(r,\frac{1}{F})+N_{k+1}(r,\frac{1}{G})]+3[N_k(r,\frac{1}{G})+\\&N_k(r,\frac{1}{F})]+5[\overline{N}(r,\frac{1}{G})+\overline{N}(r,\frac{1}{F})]+S(r,f)+S(r,g).\end{aligned} \tag{3.8}$$

Note that

$$N_{k+1}(r,\frac{1}{F})=(k+1)\overline{N}(r,\frac{1}{f}),\quad N_k(r,\frac{1}{F})=k\overline{N}(r,\frac{1}{f}),\quad \overline{N}(r,\frac{1}{F})=\overline{N}(r,\frac{1}{f}),$$
$$N_{k+1}(r,\frac{1}{G})=(k+1)\overline{N}(r,\frac{1}{g}),\quad N_k(r,\frac{1}{G})=k\overline{N}(r,\frac{1}{g}),\quad \overline{N}(r,\frac{1}{G})=\overline{N}(r,\frac{1}{g}),$$

and from (3.1), we get

$$n(T(r,f)+T(r,g))\le(9k+14)(T(r,f)+T(r,g))+S(r,f)+S(r,g).$$

It is impossible since $n>9k+14$.

If f and g are entire, we can deduce that

$$n(T(r,f)+T(r,g))\le(5k+7)(T(r,f)+T(r,g))+S(r,f)+S(r,g),$$

which contradicts $n>5k+7$.

Thus, it remains to treat the case $H(z)\equiv 0$. Integrating twice results in

$$\frac{1}{F^{(k)}-1}=A\frac{1}{G^{(k)}-1}+B, \tag{3.9}$$

where $A\ne 0, B$ are two constants. It follows that $F^{(k)}$ and $G^{(k)}$ share 1 CM, that is $[f^n]^{(k)}$ and $[g^n]^{(k)}$ share 1 CM. By Theorem C, we obtain the conclusion of Theorem 1. This completes the proof of Theorem 1. □

Proof of Theorem 2 Consider $F(z)=f^nP(f)$ and $G(z)=g^nP(g)$. Thus, $\overline{E}_{l)}(1,F^{(k)})=\overline{E}_{l)}(1,G^{(k)})$. Clearly, from the standard Valiron-Mohon'ko theorem, we have

$$T(r,F)=(n+m)T(r,f)+S(r,f),\quad T(r,G)=(n+m)T(r,g)+S(r,g). \tag{4.1}$$

Suppose now $H(z)\not\equiv 0$. By using the argument similar to that of (3.8), since f and g are entire, we can get

$$\begin{aligned}T(r,F)+T(r,G)\le&2[N_{k+1}(r,\frac{1}{F})+N_{k+1}(r,\frac{1}{G})]+3[N_k(r,\frac{1}{G})+N_k(r,\frac{1}{F})]+\\&5[\overline{N}(r,\frac{1}{G})+\overline{N}(r,\frac{1}{F})]+S(r,f)+S(r,g).\end{aligned} \tag{4.2}$$

Notice

$$N_{k+1}(r,\frac{1}{F})\le(k+1)\overline{N}(r,\frac{1}{f})+N(r,\frac{1}{P(f)})\le(m+k+1)T(r,f)$$
$$N_k(r,\frac{1}{F})\le(k+m)T(r,f),\quad \overline{N}(r,\frac{1}{F})\le(1+m)T(r,f),$$

and similar inequalities hold for G, it follows from (4.1) and (4.2) that

$$(n+m)(T(r,f)+T(r,g))\le(6m+5k+7)(T(r,f)+T(r,g))+S(r,f)+S(r,g).$$

It is impossible since $n>5m+5k+7$. Thus, we just need to treat the case $H(z)\equiv 0$. By integrating twice, we conclude that $[f^nP(f)]^{(k)}$ and $[g^nP(g)]^{(k)}$ share 1 CM. Then by using Theorem E, we complete the proof of Theorem 2. □

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