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A New Ky Fan Matching Theorem in Noncompact L-Convex Spaces with the Application to Systems of General Quasiequilibrium Problems

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Abstract In this paper, a new Ky Fan matching theorem is established in noncompact L-convex spaces. As applications, a fixed point theorem and equilibrium existence theorems for systems of general quasiequilibrium problems and systems of quasiequilibrium problems in noncompact L-convex spaces are obtained.

Keywords L-convex space; matching; weakly transfer compactly open (closed); fixed point; system of general quasiequilibrium problems; equilibrium.

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1. Introduction

In 1998, Ben-El-Mechaiekh, et al. [1] introduced and studied the abstract convexity concept and the L-convexity structure on topological spaces. Recently, Ding [2] studied the class KKM(X,Y) of mappings and Himmelberg type fixed point theorems. Ding [3] introduced the GLKKM mapping, and obtained some GLKKM theorems, Ky Fan matching theorems, fixed point theorems and a minimax inequality in L-convex spaces. Ding [4] proved a continuous selection theorem, coincidence theorems, fixed point theorems, a minimax inequality and existence theorems of solutions for generalized equilibrium problems in L-convex spaces. Ding [5] presented some KKM theorems, coincidence theorems and some fixed point theorems in L-convex spaces. Liu and Tang [6] established an intersection theorem, fixed point theorem, maximal element theorem, coincidence theorem, minimax inequalities and saddle point theorem in L-convex spaces. In 2007, Fang and Hang [7] introduced some generalized L-KKM type theorems and an existence theorem of equilibrium points for abstract generalized vector equilibrium problems. In 2008, Wen [8] established a new KKM theorem, matching theorem, coincidence theorem, fixed point theorem, maximal element theorem and equilibrium existence theorems for abstract economies and qualitative games in L-convex spaces. In 2009, Wen [9, 10] obtained a new GLKKM theorem, Ky Fan matching theorems, variational inequality, section theorem, coincidence theorem, maximal element theorem and fixed point theorem in L-convex spaces.

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The aim of this paper is to establish a new Ky Fan matching theorem in L-convex spaces. As application, a new fixed point theorem is obtained. Finally, we introduce and study the following system of general quasiequilibrium problem $\mathrm{SGQEP}(T_i, A_i, \psi_i)_{i\in I}$ which includes $\mathrm{QEP}(T, A, f)$ of Noor, et al. [11–13], $\mathrm{GQEP}(T,\ A,\ \psi)$ of Ding [14, 15], $\mathrm{QEP}(A, f)$ of Ding [16] and Lin and Park [17], $\mathrm{SQEP}(T_i, A_i, f_i)_{i\in I}$ of Zheng and Ding [18] and many fundamental mathematical problems, e.g., optimization problems, quasicomplementarity problems, variational inequality problems and others as special cases. Let I be a finite or infinite index set, $\{X_i\}_{i\in I}$ and $\{Y_i\}_{i\in I}$ be two families of nonempty sets. Suppose that for each $i \in I$, $A_i : X := \prod_{j\in I} X_j \to 2^{X_i}$ is a mapping, $T_i : X \to Y_i$ a map, $\psi_i : X \times Y_i \times X \to \overline{\mathbf{R}} := \mathbf{R} \cup \{\pm\infty\}$ a function, $\pi_i : X \to X_i$ the projection of X onto X_i and $A : X \to 2^X$ defined by $A(x) := \prod_{i\in I} A_i(x)$ for each $x \in X$. Then the system of general quasiequilibrium problems $\mathrm{SGQEP}(T_i, A_i, \psi_i)_{i\in I}$ is to find $\hat{x} \in X$ such that

$$\begin{cases} \hat{x}_i := \pi_i(\hat{x}) \in A_i(\hat{x}), & \forall i \in I, \\ \psi_i(\hat{x}, T_i\hat{x}, y) \le 0, & \forall y \in A(\hat{x}), \quad \forall i \in I. \end{cases}$$

2. Preliminaries

Let X be a nonempty set. We denote by $\langle X\rangle$ and 2^X the family of all nonempty finite subsets of X and the family of all subsets of X, respectively. Let X, Y be two nonempty sets and $F : X \to 2^Y$ a mapping. Then the mapping $F^* : Y \to 2^X$ is defined by $F^*(y) := X \setminus F^{-1}(y)$ for each $y \in Y$. Let X and Y be two topological spaces. We denote by $\mathcal{C}(X, Y)$ the class of single-valued continuous maps of X into Y. Let (X, Γ) be an L-convex space [1–10]. A set $D \subset X$ is said to be L-convex if for each $A \in \langle D\rangle$, $\Gamma(A) \subset D$.

Following [1–10], let X be a nonempty set and (Y, Γ) be an L-convex space. A mapping $G : X \to 2^Y$ is said to be a GLKKM mapping if for each $\{x_1, \dots, x_n\} \in \langle X\rangle$, there exists $\{y_1, \dots, y_n\} \in \langle Y\rangle$ such that for any nonempty subset $\{y_{i_1}, \dots, y_{i_k}\} \subset \{y_1, \dots, y_n\}$, we have $\Gamma(\{y_{i_1}, \dots, y_{i_k}\}) \subset \bigcup_{j=1}^k G(x_{i_j})$.

Definition 2.1 ([9]) *Let X be a nonempty set, Y a topological space and K a nonempty compact subset of Y. A mapping $G : X \to 2^Y$ is said to be weakly transfer compactly open (resp., closed) valued relative to K if the family $\{G(x) \bigcap K\}_{x\in X}$ is transfer open (resp.,closed).*

Definition 2.2 ([9]) *Let X be a nonempty set, Y a topological space, K a nonempty compact subset of Y and $\gamma \in \mathbf{R}$ a real number. A function $f : X \times Y \to \overline{\mathbf{R}} := \mathbf{R} \bigcup\{\pm\infty\}$ is said to be weakly γ-transfer compactly lower semicontinuous (in short, w.γ-t.c.l.s.c) (resp., weakly γ-transfer compactly upper semicontinuous (in short, w.γ-t.c.u.s.c)) relative to K in y if for all $x \in X$ and $y \in K$, $f(x, y) > \gamma$ (resp., $f(x, y) < \gamma$) implies that there exist a relatively open neighborhood $N(y)$ of y in K and $x' \in X$ such that $f(x', z) > \gamma$ (resp., $f(x', z) < \gamma$) for all $z \in N(y)$.*

Lemma 2.1 ([9]) *Let X be a nonempty set, Y a topological space, K a nonempty compact subset of Y and $\gamma \in \mathbf{R}$ a real number. A function $f : X \times Y \to \overline{\mathbf{R}}$ is w.γ-t.c.l.s.c (resp., w.γ-t.c.u.s.c)* *relative to K in y if and only if the mapping $F : X \to 2^Y$ defined by $F(x) := \{y \in Y : f(x, y) > \gamma\}$ (resp., $F(x) := \{y \in Y : f(x, y) < \gamma\}$) for each $x \in X$ is weakly transfer compactly open valued relative to K.*

Lemma 2.2 ([9]) *Let X be a topological space, Y a nonempty set, K a nonempty compact subset of X and $G : X \to 2^Y$ be a mapping such that $G(x) \neq \emptyset$ for each $x \in K$. Then the following conditions are equivalent:*

(a) G has the weakly compactly local intersection property relative to K;

(b) For each $y \in Y$, there exists an open subset O_y of X such that $O_y \cap K \subset G^{-1}(y)$ and $K = \bigcup_{y \in Y}(O_y \cap K)$;

(c) There exists a mapping $F : X \to 2^Y$ such that for each $y \in Y$, $F^{-1}(y)$ is open in X, $F^{-1}(y) \cap K \subset G^{-1}(y)$, and $K = \bigcup_{y \in Y}(F^{-1}(y) \cap K)$;

(d) For each $x \in K$, there exists $y \in Y$ such that $x \in \mathrm{cint}_X G^{-1}(y) \cap K$ and

$$K = \bigcup_{y \in Y}(\mathrm{cint}_X G^{-1}(y) \cap K) = \bigcup_{y \in Y}(G^{-1}(y) \cap K);$$

(e) G^{-1} is weakly transfer compactly open valued relative to K on X.

Now, we introduce the following definitions.

Definition 2.3 *Let X be a nonempty set, (Y, Γ) an L-convex space and $A, B : X \to 2^Y$ two mappings. A is said to be relatively L-convex valued in B if for each $x \in X$ and for each $\{y_1, \ldots, y_n\} \subset B(x)$, $\Gamma(\{y_1, \ldots, y_n\}) \subset A(x)$.*

Remark 2.1 Obviously, A is relatively L-convex valued in A if A is L-convex valued, but A need not be L-convex valued if A is relatively L-convex valued in B.

Definition 2.4 *Let (X, Γ) be an L-convex space and $\gamma \in \mathbf{R}$ a real number. A function $f : X \to \overline{\mathbf{R}}$ is said to be γ-L-quasiconcave(resp., quasiconvex) if the set $\{x \in X : f(x) > \gamma\}$ (resp., $\{x \in X : f(x) < \gamma\}$) is L-convex.*

Remark 2.2 Definition 2.4 generalizes the definition of L-quasiconcave (resp., quasiconvex) in Ding and Park [4].

3. Main results

Theorem 3.1 *Let X a nonempty subset of an L-convex space (Z, Γ), Y be a topological space, K a nonempty compact subset of Z and $A : X \to 2^Y$ a mapping such that*

(1) A is weakly transfer compactly open valued relative to K;

(2) For each $f \in \mathcal{C}(Z, Y)$, there exists $M_f \in \langle X \rangle$ such that $\bigcap_{x \in M_f} \mathrm{cl}_Z(f^{-1}(Y \setminus A(x))) \subset K$;

(3) $A(X) := \bigcup_{x \in X} A(x) = Y$.

Then, for each $f \in \mathcal{C}(Z, Y)$, there exist $\{x_1, \ldots, x_n\} \in \langle X \rangle$ and $x_0 \in \Gamma(\{x_1, \ldots, x_n\})$ such that $f(x_0) \in \bigcap_{i=1}^n A(x_i)$.

Proof Suppose the conclusion is false. Then there exists $f_0 \in \mathcal{C}(Z, Y)$ such that for each

$\{x_1,\dots,x_n\}\in\langle X\rangle$, $f_0(\Gamma(\{x_1,\dots,x_n\}))\subset Y\setminus\bigcap_{i=1}^n A(x_i)$. Define a mapping $F: X\to 2^Y$ by $F(x):=Y\setminus A(x)$ for each $x\in X$. Then $\Gamma(\{x_1,\dots,x_n\})\subset\bigcup_{i=1}^n(f_0^{-1}F)(x_i)$. Define $G: X\to 2^Z$ by $G(x):=(f_0^{-1}F)(x)$ for each $x\in X$. Then $\Gamma(\{x_1,\dots,x_n\})\subset\bigcup_{i=1}^n G(x_i)$. Therefore, G is a GLKKM mapping. Moreover, by (1), A is weakly transfer compactly open valued relative to K, which implies that F is weakly transfer compactly closed valued relative to K. By the continuity of f_0, G is also weakly transfer compactly closed valued relative to K. By (2), there exists $M_{f_0}\in\langle X\rangle$ such that $\bigcap_{x\in M_{f_0}}\mathrm{cl}G(x)\subset K$. In virtue of Theorem 3.1 of Wen [9], $\bigcap_{x\in X}G(x)=\bigcap_{x\in X}(f_0^{-1}F)(x)\neq\emptyset$, thus, $\bigcap_{x\in X}F(x)=Y\setminus\bigcup_{x\in X}A(x)\neq\emptyset$, a contradiction to (3). □

Remark 3.1.1 If A is transfer open valued or transfer compactly open valued, then the condition (1) is satisfied, of course. If $X=Y=Z$ is a compact L-convex space, by letting $K=X=Y=Z$, then the condition (2) holds trivially. Therefore, Theorem 3.1 improves and generalizes Theorem 2.2 of Wen [8], Theorems 2.1 and 2.2 of Wen [10], Lemma 2.1 of Wen [19], Lemma 3.1 of Wen [20], Theorem 2 of Chang and Ma [21] and Theorem 1 of Park [22].

Remark 3.1.2 If the condition (2) in Theorem 3.1 is replaced by that for $f\in\mathcal{C}(Z,Y)$, there exists $M_f\in\langle X\rangle$ such that $\bigcap_{x\in M_f}\mathrm{cl}_Z(f^{-1}(Y\setminus A(x)))\subset K$, then the conclusion of Theorem 3.1 is replaced by that there exist $\{x_1,\dots,x_n\}\in\langle X\rangle$ and $x_0\in\Gamma(\{x_1,\dots,x_n\})$ such that $f(x_0)\in\bigcap_{i=1}^n A(x_i)$, respectively.

Theorem 3.2 *Let X be a topological space, Y a nonempty subset of an L-convex space (Z,Γ), K a nonempty compact subset of Y. Suppose that $s\in\mathcal{C}(Z,X)$ and $A,B: X\to 2^Y\setminus\{\emptyset\}$ such that*

(1) B satisfies one of conditions (a)–(e) in Lemma 2.2;

(2) There exists $M\in\langle Y\rangle$ such that $\bigcap_{y\in M}\mathrm{cl}_Z(s^{-1}B^*(y))\subset K$;

(3) A is relatively L-convex valued in B.

Then, there exists $y_0\in Y$ such that $y_0\in A(s(y_0))$.

Proof By (1), B^{-1} is weakly transfer compactly open valued relative to K. By (2), there exists $M\in\langle Y\rangle$ such that $\bigcap_{y\in M}\mathrm{cl}_Z(s^{-1}(Y\setminus B^{-1}(y)))\subset K$. Since B is nonempty valued, then $X=\bigcup_{y\in Y}B^{-1}(y)$. In virtue of Theorem 3.1 and Remark 3.1.2, there exist $\{y_1,\cdots,y_n\}\in\langle Y\rangle$ and $y_0\in\Gamma(\{y_1,\dots,y_n\})$ such that $s(y_0)\in\bigcap_{i=1}^n B^{-1}(y_i)$, which results in that $\{y_1,\dots,y_n\}\subset B(s(y_0))$. By (3), we have $\Gamma(\{y_1,\dots,y_n\})\subset A(s(y_0))$, and hence, $y_0\in\Gamma(\{y_1,\dots,y_n\})\subset A(s(y_0))$. □

Remark 3.2 Let $X=Y=Z$, $s=I_X$ and $A=B$ be L-convex valued. Then Theorem 3.1 reduces to Theorem 3.5 of Wen [9]. Therefore, Theorem 3.1 unifies, improves and generalizes Theorem 3.5 of Wen [9], Theorem 3 of Park [22], Theorem 3.1 of Kirk, et al. [23], Lemma 2.2 of Zhang [24], Lemma 1 of Wu [25], Theorem 2.3-A of Chowdhury, et al. [26], Theorem 2.4 of Verma [27], Theorems 2, 3, 4, 8 of Park [28], Corollaries 2 and 3 of Chen and Shen [29], Theorem 2 of Horvath [30, p350], Theorem 3.6 of Yuan [31], Corollary 2.3 of Tarafdar [32], Theorem 2.1, Corollaries 2.1–2.3 of Tarafdar [33] and Theorem 4.1 of Watson [34], and so on.

Theorem 3.3 *Let I be a finite or infinite index set, $\{(X_i, \Gamma_i)\}_{i\in I}$ be a family of L-convex spaces, $\{Y_i\}_{i\in I}$ a family of nonempty sets. Suppose that for each $i \in I$, $A_i : X := \prod_{j\in I} X_j \to 2^{X_i} \setminus \{\emptyset\}$ is a mapping, $T_i : X \to Y_i$ a map, $\psi_i : X \times Y_i \times X \to \overline{\mathbf{R}}$ a function, $\pi_i : X \to X_i$ the projection of X onto X_i, K a nonempty compact subset of X and $A : X \to 2^X$ defined by $A(x) := \prod_{i\in I} A_i(x)$ for each $x \in X$ satisfying*

(1) For each $i \in I$ and for each $x \in X$, $y \mapsto \psi_i(x, T_i x, y)$ is 0-L-quasiconcave;

(2) For each $i \in I$, $f_i(x, y) := \psi_i(x, T_i x, y)$ is w.0-t.c.l.s.c. relative to K in x;

(3) A is L-convex valued;

(4) A satisfies one of conditions (a)–(e) in Lemma 2.2;

(5) $D := \{x \in X : x \in A(x)\}$ is compactly closed;

(6) There exists $M \in \langle X\rangle$ such that $\bigcap_{x\in M} \mathrm{cl}_X A^*(x) \subset K$ and $\bigcap_{x\in M} \mathrm{cl}_X(\{x \in D : \max_{i\in I}\psi_i(x, T_i x, y) \le 0\}) \subset K$;

(7) For each $i \in I$ and for each $x \in X$, $\psi_i(x, T_i x, x) \le 0$.

Then there exists $\hat{x} \in X$ such that

$$\begin{cases} \hat{x}_i := \pi_i(\hat{x}) \in A_i(\hat{x}), & \forall i \in I, \\ \psi_i(\hat{x}, T_i\hat{x}, y) \le 0, & \forall y \in A(\hat{x}), \quad \forall i \in I. \end{cases}$$

Proof For each $i \in I$, define a mapping $P_i : X \to 2^X$ by $P_i(x) := \{y \in X : \psi_i(x, T_i x, y) > 0\}$ for each $x \in X$. Then, by (1), P_i is L-convex valued, and for each $y \in X$, $P_i^{-1}(y) = \{x \in X : \psi_i(x, T_i x, y) > 0\}$, so that P_i^{-1} is weakly transfer compactly open valued relative to K by (2) and Lemma 2.1.

We claim that there exists $\hat{x} \in D$ such that $A(\hat{x}) \bigcap (\bigcup_{i\in I} P_i(\hat{x})) = \emptyset$. Otherwise, define a mapping $G : X \to 2^X$ by

$$G(x) := \begin{cases} A(x), & x \in X \setminus D, \\ A(x) \bigcap (\bigcup_{i\in I} P_i(x)), & x \in D. \end{cases}$$

Note that A is also nonempty valued. Then G is nonempty valued. Moreover, since P_i is L-convex valued for each $i \in I$, $\bigcup_{i\in I} P_i(x)$ is L-convex for each $x \in X$. Therefore, G is L-convex valued by (3). By the definition of G, for each $y \in X$, we have

$$\begin{aligned} G^{-1}(y) :=& \{x \in X : y \in G(x)\} \\ =& \{x \in X \setminus D : y \in A(x)\} \bigcup \{x \in D : y \in A(x) \bigcap (\bigcup_{i\in I} P_i(x))\} \\ =& ((X \setminus D) \cap A^{-1}(y)) \bigcup (D \bigcap A^{-1}(y) \bigcap (\bigcup_{i\in I} P_i^{-1}(y))) \\ =& A^{-1}(y) \bigcap ((X \setminus D) \bigcup (\bigcup_{i\in I} P_i^{-1}(y))) \end{aligned}$$

and

$$\begin{aligned} G^*(y) :=& X \setminus G^{-1}(y) = X \setminus (A^{-1}(y) \bigcap ((X \setminus D) \bigcup (\bigcup_{i\in I} P_i^{-1}(y)))) \\ =& A^*(y) \bigcup (D \bigcap (\bigcap_{i\in I} P_i^*(y))). \end{aligned}$$

By (4), A^{-1} is weakly transfer compactly open valued relative to K. By (5), $X \setminus D$ is compactly open. Note that P_i^{-1} is also weakly transfer compactly open valued relative to K for each $i \in I$. Then, G^{-1} is weakly transfer compactly open valued relative to K. Since $D \bigcap (\bigcap_{i\in I} P_i^*(y)) = \{x \in D : \psi_i(x, T_i x, y) \le 0, \forall i \in I\} = \{x \in D : \max_{i\in I} \psi_i(x, T_i x, y) \le 0\}$ for each $y \in X$, by (6), there exists $M \in \langle X \rangle$ such that $\bigcap_{x\in M} \mathrm{cl}_X(G^*(x)) \subset K$. In virtue of Theorem 3.2, there exists $x_0 \in X$ such that $x_0 \in G(x_0)$.

On the other hand, if $x_0 \in D$, $x_0 \in G(x_0) = A(x_0) \bigcap (\bigcup_{i\in I} P_i(x_0)) \subset \bigcup_{i\in I} P_i(x_0)$, then there exists $i_0 \in I$ such that $x_0 \in P_{i_0}(x_0)$, hence, $\psi_{i_0}(x_0, T_{i_0}x_0, x_0) > 0$, which contradicts the condition (7). If $x_0 \in X \setminus D$, $x_0 \in G(x_0) = A(x_0)$, then $x_0 \in D$, which is also a contradiction. Therefore, there exists $\hat{x} \in D$ such that $A(\hat{x}) \bigcap (\bigcup_{i\in I} P_i(\hat{x})) = \emptyset$. Namely, there exists $\hat{x} \in X$ such that

$$\begin{cases} \hat{x}_i := \pi_i(\hat{x}) \in A_i(\hat{x}), & \forall i \in I; \\ \psi_i(\hat{x}, T_i\hat{x}, y) \le 0, & \forall y \in A(\hat{x}), \quad \forall i \in I. \ \Box \end{cases}$$

As a special case of Theorem 3.3, we have the equilibrium existence theorem for the system of quasiequilibrium problems SQEP(T_i, A_i, f_i).

Theorem 3.4 *Let I be a finite or infinite index set, $\{(X_i, \Gamma_i)\}_{i\in I}$ be a family of L-convex spaces, $\{Y_i\}_{i\in I}$ a family of nonempty sets. Suppose that for each $i \in I$, $A_i : X := \prod_{j\in I} X_j \to 2^{X_i} \setminus \{\emptyset\}$ is a mapping, $T_i : X \to Y_i$ a map, and $f_i : X \times Y_i \to \overline{\mathbf{R}}$ a function, $\pi_i : X \to X_i$ the projection of X onto X_i and $A : X \to 2^X$ defined by $A(x) := \prod_{i\in I} A_i(x)$ for each $x \in X$ satisfying*

(1) For each $i \in I$ and for each $x \in X$, $y \mapsto f_i(x, T_i x) - f_i(y, T_i x)$ is 0-L-quasiconcave;

(2) For each $i \in I$, $\phi_i(x, y) := f_i(x, T_i x) - f_i(y, T_i x)$ is 0-t.c.l.s.c. in x;

(3) A is L-convex valued;

(4) A satisfies one of conditions (a)–(e) in Lemma 2.2;

(5) $D := \{x \in X : x \in A(x)\}$ is compactly closed;

(6) There exists $M \in \langle X \rangle$ such that $\bigcap_{x\in M} \mathrm{cl}_X(A^*(x)) \subset K$ and $\bigcap_{x\in M} \mathrm{cl}_X(\{x \in D : \max_{i\in I}(f_i(x, T_i x) - f_i(y, T_i x)) \le 0\}) \subset K$;

Then there exists $\hat{x} \in X$ such that

$$\begin{cases} \hat{x}_i := \pi_i(\hat{x}) \in A_i(\hat{x}), & \forall i \in I, \\ f_i(x, T_i x) \le f_i(y, T_i x), & \forall y \in A(\hat{x}), \quad \forall i \in I. \end{cases}$$

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