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Quasi-Type δ Semigroups with an Adequate Transversal

Shou Feng WANG
School of Mathematics, Yunnan Normal University, Yunnan 650092, P. R. China

Abstract In this paper, some properties of quasi-type δ semigroups with an adequate transversal are explored. In particular, abundant semigroups with a cancellative transversal are characterized. Our results generalize and enrich Saito's results on quasi-orthodox semigroups with an inverse transversal.

Keywords quasi-type δ semigroups; adequate transversals; cancellative transversals.

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1. Introduction

A regular semigroup is called orthodox (inverse) if its idempotents form a subband (subsemilattice) of itself. Suppose that S is a regular semigroup and S° is an inverse subsemigroup of S, then S° is called an inverse transversal if $|V(x) \cap S^\circ| = 1$ for all $x \in S$, where $V(x) = \{a \in S | xax = a, axa = a\}$. The concept of an inverse transversal was introduced in Blyth-McFadden [1]. An analogue of an inverse transversal, which is termed an adequate transversal, was introduced for abundant semigroups in El-Qallali [3] and in the same paper, the author investigated a class of abundant semigroups with an adequate transversal. Furthermore, Guo [5] and Guo-Shum [6] were also devoted to abundant semigroups with an adequate transversal. On the other hand, Yamada [10] introduced and studied a special class of regular semigroups, namely, quasi-orthodox semigroups. In 1987, Saito [9] characterized quasi-orthodox semigroups with an inverse transversal.

In this paper, we shall introduce and investigate the class of quasi-type δ semigroups with an adequate transversal which is a generalization of quasi-orthodox semigroups with an inverse transversal in the range of abundant semigroups. Our results generalize and enrich Saito's results in [9].

2. Preliminaries

Let S be a semigroup and $a, b \in S$. We use $E(S)$ to denote the set of idempotents of S. By

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E-mail address: wsf1004@163.com

$a\mathcal{R}^*b$, we mean that $xa = ya$ if and only if $xb = yb$ for all $x, y \in S^1$. The relation $\mathcal{L}^*$ can be defined dually. Observe that $\mathcal{L}^*$ is a right congruence and $\mathcal{R}^*$ is a left congruence. If $a, b \in \operatorname{Reg} S$, the set of regular elements of S, then $a\mathcal{R}^*b$ if and only if $a\mathcal{R}b$, and $a\mathcal{L}^*b$ if and only if $a\mathcal{L}b$. A semigroup is called abundant if each $\mathcal{L}^*$-class and each $\mathcal{R}^*$-class contains an idempotent. For an abundant semigroup S and $a \in S$, $R_a^*(S)$ and $L_a^*(S)$ denote the $\mathcal{R}^*$-class and $\mathcal{L}^*$-class of S containing a, and a^+ is a typical element in $R_a^*(S) \cap E(S)$ and a^* is a typical element in $L_a^*(S) \cap E(S)$, respectively. An abundant semigroup S is quasi-adequate if $E(S)$ is a subband of S, and S is adequate if $E(S)$ is a subsemilattice of S. Clearly, if S is adequate, then a^* and a^+ are determined by a for any $a \in S$.

Let S and T be two abundant semigroups and φ a morphism from S to T. Then φ is called good if φ preserves the relations $\mathcal{L}^*$ and $\mathcal{R}^*$. In particular, a congruence ρ on S is called good if the natural morphism $\rho^\natural : S \to S/\rho$ is good. Moreover, a congruence ρ on S is called idempotent promoting if for all $x\rho \in E(S/\rho)$, there exists $e \in E(S)$ such that $x\rho e$. Observe that congruences on regular semigroups are idempotent promoting good congruences.

Let S be a quasi-adequate semigroup. In El-Qallali-Fountain [2], the authors defined a relation δ on S by the rule: $a\delta b$ if $b = eaf$ for some $e \in E(a^+)$ and $f \in E(a^*)$, where $E(a^+)$ and $E(a^*)$ denote the $\mathcal{J}$-class in $E(S)$ containing a^+ and a^*, respectively. From El-Qallali-Fountain [2], δ is not a congruence generally, and if δ is a congruence, then δ is good and the quotient semigroup S/δ is adequate. A quasi-adequate semigroup is called a type δ semigroup if δ defined above is a congruence. From [7], every IC quasi-adequate semigroup is a type δ semigroup.

Let T be a semigroup and A, B be two sets. Then the set $A \times T \times B$ forms a semigroup with the following multiplication
$$(i, x, \lambda)(j, y, \mu) = (i, xp_{\lambda j}y, \mu),$$
where $P = (p_{\lambda j})_{B\times A}$ is a $B \times A$-matrix whose entries are elements in T. From Howie [8], this semigroup is called a Rees matrix semigroup over the semigroup T and is denoted by $S = \mathcal{M}(T; A, B; P)$.

Now, we introduce a class of abundant semigroups, namely, quasi-type δ semigroups which contain the class of type δ semigroups as a proper subclass.

Definition 1 *An abundant semigroup S is called a quasi-type δ semigroup if there exist an adequate semigroup T and a good morphism ϕ from S onto T such that $S_\lambda = \{x \in S | x\phi = \lambda\}$ is a Rees matrix semigroup over some cancellative monoid for all $\lambda \in E(T)$.*

Remark 1 In Definition 1, if S is a regular semigroup, then $T = S\phi$ is also regular and so T is an inverse semigroup. In this case, $S_\lambda = \{x \in S | x\phi = \lambda\}$ becomes a regular subsemigroup of S for all $\lambda \in E(T)$. Thus, S_λ is a Rees matrix semigroup over some group for all $\lambda \in E(T)$. In view of Howie [8], S_λ is a completely simple semigroup for all $\lambda \in E(T)$. From Yamada [10], S is called a quasi-orthodox semigroup in this case. Thus, the class of quasi-type δ semigroups is a generalization of quasi-orthodox semigroups in the range of abundant semigroups.

Let S be an abundant semigroup and U an abundant subsemigroup of S. If there exists an idempotent $e \in L_a^*(S) \cap U$ and an idempotent $f \in R_a^*(S) \cap U$ for any $a \in U$, then U is called a $*$-subsemigroup of S. From El-Qallali [3], an adequate $*$-subsemigroup S° of S is called an adequate transversal of S if the following condition is valid: For each $x \in S$, there exist a unique $\bar{x} \in S^\circ$ and $e, f \in E(S)$ such that $x = e\bar{x}f$ and $e\mathcal{L}\bar{x}^+, f\mathcal{R}\bar{x}^*$, where $\bar{x}^+, \bar{x}^* \in E(S^\circ)$. Also from El-Qallali [3], in this case, e and f are determined by x uniquely, so we denote them by e_x and f_x, respectively. Denote $I = \{e_x | x \in S\}$ and $\Lambda = \{f_x | x \in S\}$.

Lemma 1 ([3, 11]) *Let S be an abundant semigroup with an adequate transversal S°. Then*

(1) $I = \{e \in E(S) | (\exists e' \in E(S^\circ)) e\mathcal{L}e'\}$, $\Lambda = \{f \in E(S) | (\exists f' \in E(S^\circ)) f\mathcal{R}f'\}$;

(2) $E(S^\circ) = I \cap \Lambda$;

(3) $\bar{x}^+\mathcal{L}e_x\mathcal{R}^*x\mathcal{L}^*f_x\mathcal{R}\bar{x}^*$ *and* $\bar{x} = \bar{x}^+x\bar{x}^*$ *for any* $x \in S$.

Lemma 2 ([11]) *Let S be an abundant semigroup with an adequate transversal S°. Then there exist a unique $x^\circ \in S^\circ \cap V(x)$ and a unique $x^{\circ\circ} \in S^\circ \cap V(x^\circ)$ such that $\bar{x} = x^{\circ\circ}$ for every $x \in \operatorname{Reg} S$. As a consequence, a subsemigroup T of a regular semigroup S is an adequate transversal of S if and only if T is an inverse transversal of S.*

3. Abundant semigroups with a cancellative transversal

In order to study quasi-type δ semigroups with an adequate transversal, we first consider abundant semigroups with a cancellative transversal. An adequate transversal S° of an abundant semigroup S is called a cancellative trnasversal if S° is a cancellative monoid. In such a case, we denote the identity of S° by e°. Clearly, e° is the unique idempotent in S° in the case. We begin with the following lemma.

Lemma 3 *Let S be an abundant semigroup with a cancellative transversal S° and $x, y \in S$. Then*

(1) *I is a left zero band and Λ is a right zero band, respectively;*

(2) $\overline{xy} = \bar{x}f_xe_y\bar{y}, e_{xy} = e_x$ and $f_{xy} = f_y$;

(3) $\lambda j \in S^\circ$ *for all* $j \in I$ *and* $\lambda \in \Lambda$.

Proof (1) From Lemma 1 (1), every element in I is $\mathcal{L}$-related to e° and so I is a left zero band. Dually, Λ is a right zero band.

(2) Since $x = e_x\overline{x}f_x, y = e_y\overline{y}f_y$ and $xy = e_{xy}\overline{xy}f_{xy}$, by (1), we have

$$e_x\mathcal{L}\overline{x}^+ = e^\circ = \overline{y}^*\mathcal{R}f_y, xy = e_xyf_y = e_xe_{xy}\overline{xy}f_{xy}f_y = e_x\overline{xy}f_y.$$

Observe that $\overline{xy}^+ = \overline{xy}^* = e^\circ$, $e_x\mathcal{L}\overline{xy}^+ = e^\circ = \overline{xy}^*\mathcal{R}f_y$. By the definition of an adequate transversal, $e_x = e_{xy}$ and $f_y = f_{xy}$. Thus by (1),

$$\overline{xy} = e^\circ\overline{xy}e^\circ = (e^\circ e_{xy})\overline{xy}(f_{xy}e^\circ) = e^\circ xye^\circ = (e^\circ e_x)\bar{x}f_xe_y\bar{y}(f_ye^\circ) = e^\circ\bar{x}f_xe_y\bar{y}e^\circ = \bar{x}f_xe_y\bar{y}.$$

(3) If $j \in I$ $\lambda \in \Lambda$, then by Lemma 1 (1), $j\mathcal{L}e^\circ\mathcal{R}\lambda$. Observe that $j = je^\circ e^\circ$ and $\lambda = e^\circ e^\circ\lambda$, it follows that $\bar{j} = \bar{\lambda} = e^\circ$ and $e_j = j, f_\lambda = \lambda$. Thus by (1) and (2), $\lambda j = e^\circ\lambda je^\circ = \bar{\lambda}f_\lambda e_j\bar{j} = \overline{\lambda j} \in S^\circ$. $\square$

Proposition 1 *Let S be an abundant semigroup with an adequate transversal S°. Then S° is a cancallative transversal if and only if S is a Rees matrix semigroup over some cancallative monoid.*

Proof Assume that S° is a cancallative monoid. Let $M = \mathcal{M}(S^\circ; I, \Lambda; P)$ be the Rees matrix semigroup over S° with $P = (p_{\lambda j})_{\Lambda\times I}$ and $p_{\lambda j} = \lambda j$ for all $j \in I$ and $\lambda \in \Lambda$ by Lemma 3 (3). Define

$$\varphi : M \to S,\ (i, x, \lambda) \mapsto ix\lambda.$$

Then φ is an isomorphism from M onto S. In fact, for any $s \in S$, we have $(e_s, \overline{s}, f_s) \in M$ and $(e_s, \overline{s}, f_s)\varphi = e_s\overline{s}f_s = s$. Let $(i, x, \lambda), (j, y, \mu) \in M$. If $ix\lambda = jy\mu$, then by Lemma 3 (1) and the fact that $e^\circ \in I \cap \Lambda = E(S^\circ)$, we have

$$x = e^\circ xe^\circ = e^\circ ix\lambda e^\circ = e^\circ jy\mu e^\circ = e^\circ ye^\circ = y.$$

In view of Lemma 3 (2) and the proof of Lemma 3 (3), $i = e_i = e_{ix\lambda} = e_{jy\mu} = e_j = j$. Dually, $\lambda = \mu$. Moreover,

$$[(i, x, \lambda)(j, y, \mu)]\varphi = (i, xp_{\lambda j}y, \mu)\varphi = ixp_{\lambda j}y\mu = ix\lambda jy\mu = (i, x, \lambda)\varphi(j, y, \mu)\varphi.$$

Conversely, let $S = \mathcal{M}(C; A, B; P)$ for some cancallative monoid C and S° an adequate transversal of S. Assume that $(a, x, b), (c, y, d) \in E(S^\circ)$. We have

$$xp_{ba}x = x, yp_{dc}y = y, (c, yp_{da}x, b) = (c, y, d)(a, x, b) = (a, x, b)(c, y, d) = (a, xp_{bc}y, d).$$

This implies that $p_{ba}x, p_{dc}y \in E(C)$ and so $p_{ba}x = p_{dc}y$ is the identity of C. Therefore, $a = c$, $b = d$ and $y = yp_{ba}x = yp_{da}x = xp_{bc}y = xp_{dc}y = x$. This shows that $(a, x, b) = (c, y, d)$. Thus S° contains only one idempotent whence S° is a cancallative monoid. □

In the end of this section, we observe that a Rees matrix semigroup over some cancallative monoid which is also abundant may have no adequate transversal.

Example 1 Let C be the set of all natural numbers. Then C is a cacellative monoid with the ordinary multiplication. Assume that $A = B = \{1, 2\}$ and P is a 2×2-matrix such that $p_{12} = p_{21} = 2$ and $p_{11} = p_{22} = 1$. It can be proved that $M = \mathcal{M}(C; A, B; P)$ is an abundant semigroup (the $\mathcal{R}^*$-classes of M are $\{1\}\times C\times B$ and $\{2\}\times C\times B$, the $\mathcal{L}^*$-classes of M are $A\times C\times\{1\}$ and $A\times C\times\{2\}$) and $E(M) = \{(1, 1, 1), (2, 1, 2)\}$. However, M contains no adequate transversal. In fact, if M has an adequate transversal M°, then M° is a cancellative monoid by Proposition 1. Without loss of generality, we assume $(1, 1, 1) \in M^\circ$. Then $M^\circ \subseteq \{(1, x, 1)|x \in C\}$. Now, we let $m = (2, 1, 2)$ and $m = e_m\bar{m}f_m, \bar{m} = (1, y, 1) \in M^\circ$. This yields that $e_m = f_m = m$. Thus, $m = (2, 4y, 2)$. A contradiction.

4. Quasi-type δ semigroups with an adequate transversal

In this section, we investigate the class of quasi-type δ semigroups with an adequate transversal by using some congruences on these semigroups. Let S be an abundant semigroup with an adequate transversal S°. Define a relation on S:

$$\mu = \{(x,y) \in S \times S | (e\bar{x})^* = (e\bar{y})^* \text{ for all } e \in E^1(S^\circ)\}.$$

Now, we have the main result of this paper.

Theorem 1 *Let S be an abundant semigroup with an adequate transversal S°. If S is a quasi-type δ semigroup, then μ is a congruence on S. Conversely, if μ is an idempotent promoting good congruence on S, then S is a quasi-type δ semigroup.*

Proof If S is a quasi-type δ semigroup, there exist an adequate semigroup T and a good morphism ϕ from S onto T such that $S_\lambda = \{x \in S | x\phi = \lambda\}$ is a Ree's matrix semigroup over some cancellative monoid for all $\lambda \in E(T)$. Let $x \in S$ and $x = e_x \bar{x} f_x$. Then $e_x \mathcal{L} \bar{x}^+, f_x \mathcal{R} \bar{x}^*$ and so $e_x\phi \mathcal{L} \bar{x}^+\phi, f_x\phi \mathcal{R} \bar{x}^*\phi$. Since T is adequate, we have $e_x\phi = \bar{x}^+\phi$ and $f_x\phi = \bar{x}^*\phi$. By Lemma 1 (3),

$$\bar{x}\phi = (\bar{x}^+ x \bar{x}^*)\phi = \bar{x}^+\phi \cdot x\phi \cdot \bar{x}^*\phi = e_x\phi \cdot x\phi \cdot f_x\phi = (e_x x f_x)\phi = x\phi. \tag{4.1}$$

Now, let $\lambda \in E(T)$ and $x \in S_\lambda$. Then $\bar{x}^+ \mathcal{L} e_x \mathcal{R}^* x \mathcal{L}^* f_x \mathcal{R} \bar{x}^*$ by Lemma 1 (3). Since ϕ is good, T is adequate and $x\phi = \lambda \in E(T)$, we have

$$\bar{x}^+\phi = e_x\phi = x\phi = f_x\phi = \bar{x}^*\phi = \lambda,$$

which shows that $\bar{x}^+, \bar{x}^*, e_x, f_x \in S_\lambda$. In view of the equality (4.1), $\bar{x} \in S_\lambda$. Thus, $S^\circ \cap S_\lambda$ is an adequate transversal of S_λ. Since S_λ is a Rees matrix semigroup over some cancellative monoid, by Proposition 1, $S^\circ \cap S_\lambda$ is a cancellative monoid.

To show that μ is a congruence on S, we first prove the following fact

$$(\forall x, y \in S) \ \ \overline{xy}^* = (\bar{x}\bar{y})^*. \tag{4.2}$$

In fact, by the equality (4.1), we have $x\phi = \bar{x}\phi$, $y\phi = \bar{y}\phi$ and $(xy)\phi = \overline{xy}\phi$. This implies that $\overline{xy}\phi = (\bar{x}\bar{y})\phi$. Observe that $\overline{xy}\mathcal{L}^*\overline{xy}^*$, $\bar{x}\bar{y}\mathcal{L}^*(\bar{x}\bar{y})^*$ and ϕ is good, it follows that $\overline{xy}^*\phi\mathcal{L}^*\overline{xy}\phi = (\bar{x}\bar{y})\phi\mathcal{L}^*(\bar{x}\bar{y})^*\phi$. Since T is adequate, $\overline{xy}^*\phi = (\bar{x}\bar{y})^*\phi$. Denote $\lambda = \overline{xy}^*\phi = (\bar{x}\bar{y})^*\phi \in E(T)$. Then $\overline{xy}^*, (\bar{x}\bar{y})^* \in S_\lambda \cap S^\circ$. In view of the fact that $S_\gamma \cap S^\circ$ is a cancellative monoid, $\overline{xy}^* = (\bar{x}\bar{y})^*$.

Clearly, μ is an equivalence on S. Now, let $x, y, z \in S$ and $x\mu y$. Then for any $e \in E^1(S^\circ)$, we have $(e\bar{x})^* = (e\bar{y})^*$ whence $\overline{ex}^* = \overline{ey}^*$. Since $\mathcal{L}^*$ is a right congruence, we have $(\overline{ex}\bar{z})^* = (\overline{ey}\bar{z})^*$. Therefore, $(\overline{exz})^* = (\overline{eyz})^*$ and so

$$(e\overline{xz})^* = (\bar{e}\ \overline{xz})^* = (\overline{exz})^* = (\overline{eyz})^* = (\bar{e}\ \overline{yz})^* = (e\overline{yz})^*.$$

Thus, $xz\mu yz$. This shows that μ is a right congruence. On the other hand, we have

$$(e\overline{zx})^* = (\bar{e}\ \overline{zx})^* = (\overline{ezx})^* = (\overline{ez}\bar{x})^* = (\overline{ez}^*\bar{x})^* = ((\bar{e}\bar{z})^*\bar{x})^* = ((e\bar{z})^*\bar{x})^*.$$

Dually, $(e\overline{zy})^* = ((e\bar{z})^*\bar{y})^*$. Observe that $(e\bar{z})^* \in E(S^\circ)$, $((e\bar{z})^*\bar{x})^* = ((e\bar{z})^*\bar{y})^*$ by the fact $x\mu y$. This implies that $(e\overline{zx})^* = (e\overline{zy})^*$ and so $zx\mu zy$. Thus, μ is a left congruence on S.

Conversely, if μ is an idempotent promoting good congruence on S, then S/μ is abundant and $E((S/\mu) = \{x\mu | x \in E(S)\}$. If $x \in E(S)$, then by Lemma 2, there exist a unique $x^\circ \in S^\circ \cap V(x)$

and a unique $x^{\circ\circ} \in S^\circ \cap V(x^\circ)$ such that $\bar{x} = x^{\circ\circ}$. Observe that $\bar{\bar{x}} = \bar{x}$, it follows that

$$x\mu\bar{x} = x^{\circ\circ}\mu(x^{\circ\circ})^2 = x^{\circ\circ}(x^\circ x^{\circ\circ})(x^{\circ\circ}x^\circ)x^{\circ\circ} = (x^{\circ\circ})^2(x^\circ)^2(x^{\circ\circ})^2\mu(x^{\circ\circ})(x^\circ)^2(x^{\circ\circ}) \in E(S^\circ).$$

Now, let $x\mu, y\mu \in E(S/\mu)$. Then $x\mu = e\mu$ and $y\mu = f\mu$ for some $e, f \in E(S^\circ)$. This implies that S/μ is adequate.

Consider the natural morphism $\mu^\natural : S \to S/\mu, x \mapsto x\mu$. Then $\mu^\natural$ is good. For $\lambda \in E(S/\mu)$, denote $S_\lambda = \{x \in S | x\mu^\natural = \lambda\}$. Observe that $\bar{\bar{x}} = \bar{x}$, $x\mu\bar{x}$ for any $x \in S$. Therefore, $\bar{x} \in S_\lambda$ for any $x \in S_\lambda$. On the other hand, for $x \in S_\lambda$, by Lemma 1 (3) and the fact that $\mu^\natural$ is good, we have $\bar{x}^+\mu\mathcal{L}e_x\mu\mathcal{R}^*x\mu\mathcal{L}^*f_x\mu\mathcal{R}\bar{x}^*\mu$. Since S/μ is adequate, $\bar{x}^+\mu = e_x\mu = x\mu = f_x\mu = \bar{x}^*\mu = \lambda$, which implies that $\bar{x}^+, e_x, f_x, \bar{x}^* \in S_\lambda$. Thus, $S_\lambda \cap S^\circ$ is an adequate transversal of S_λ. Now, if $g, h \in E(S_\lambda \cap S^\circ)$, then $g, h \in E(S^\circ)$ and $g\mu h$ whence $\bar{g} = g, \bar{h} = h$ and $g^* = h^*$. Observe that $g = g^*$ and $h = h^*$, it follows that $g = h$. This implies that $S_\lambda \cap S^\circ$ contains only one idempotent and so is a cancellative monoid. By Proposition 1, S_λ is a Rees matrix semigroup over a cancellative monoid. Thus, S is a quasi-type δ semigroup. □

Let S be a quasi-type δ semigroup with an adequate transversal S°. By Theorem 1, μ is congruence on S. However, μ may not be an idempotent promoting good congruence. This can be illustrated by the following example.

Example 2 (Example 2.4 in [4]) Let A be the infinite cyclic semigroup with generator a and let B be the infinite cyclic monoid with generator b and identity e. Let $S = A \cup B \cup \{1\}$ and define a product on S which extends those on A and B and has 1 as the identity by putting $a^m b^n = b^{m+n}, b^n a^m = a^{n+m}$ for integers $m > 0$ and $n \geq 0$ where $b^0 = e$. Then S is an adequate monoid and so a quasi-type δ semigroup with an adequate transversal S trivially. However, the quotient semigroup S/μ is isomorphic to $R_2 \cup \{1\}$ where R_2 is the two element right zero semigroup. This implies that S/μ is not adequate whence μ is not an idempotent promoting good congruence.

If S is a quasi-orthodox semigroup with an inverse transversal S°, then by Lemma 2, S° is an adequate transversal of S. In this case, μ is always an idempotent promoting good congruence. Thus, we have the following theorem as a consequence of Remark 1 and Theorem 1.

Theorem 2 ([9]) *Let S be a regular semigroup with an inverse transversal S°. Then S is a quasi-orthodox semigroup if and only if μ is a congruences on S.*

Remark 2 By symmetry, we can replace μ with

$$\nu = \{(x, y) \in S \times S | (\bar{x}e)^+ = (\bar{y}e)^+ \text{ for any } e \in E^1(S^\circ)\}$$

in Theorems 1 and 2.

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