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Optimal Search Mechanism Analysis of Light Ray Optimization Algorithm

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Abstract Based on Fermat's principle and the automatic optimization mechanism in the propagation process of light, an optimal searching algorithm named light ray optimization is presented, where the laws of refraction and reflection of light rays are integrated into searching process of optimization. In this algorithm, coordinate space is assumed to be the space that is full of media with different refractivities, then the space is divided by grids, and finally the searching path is assumed to be the propagation path of light rays. With the law of refraction, the search direction is deflected to the direction that makes the value of objective function decrease. With the law of reflection, the search direction is changed, which makes the search continue when it cannot keep going with refraction. Only the function values of objective problems are used and there is no artificial rule in light ray optimization, so it is simple and easy to realize. Theoretical analysis and the results of numerical experiments show that the algorithm is feasible and effective.

Keywords Fermat's principle; intelligent optimization algorithm; light ray optimization; optimal search mechanism.

MR(2010) Subject Classification 68W40; 90C59

1. Introduction

With the development of science and improvement of society, more and more practical problems are transformed into optimization problems. The characteristics of these problems are high-dimension, large amount of data, complex-solving and time-consuming [1]. Traditional optimization algorithms cannot meet the need of people in the process of solving this kind of problems because derivative operation and other complex operations are needed. Therefore, more and more mathematicians and engineering experts begin to study new optimization algorithms. The optimization development mode and "Economic nature" in nature enlighten people to solve optimization problems. Some new optimization algorithms are born from the thoughts of natural and physical phenomena. Breaking the mathematical processes of traditional analytical and numerical algorithms, these algorithms solve the optimization problems by simulating the developing process in nature, which is considered as a new effective way to solve the optimization

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problems. In these algorithms, the global convergence is realized by simulating the physical or ecological process in nature and the optimization mechanisms of algorithms themselves. Some common intelligent optimization algorithms are as follows: genetic algorithm (GA) [2, 3], simulated annealing (SA) [4, 5], ant colony optimization (ACO) [6, 7], particle swarm optimization (PSO) [8, 9].

In view of Fermat's principle, the actual path between two points taken by a beam of light is the one which is traversed in the least time [10]. Based on this thought, the optimization search is made by simulating the propagation process of light in inhomogeneous media in light ray optimization (LRO)[11–13]. Divide the searching area into rectangular grids, and then put different media into each grid, that is, let the propagation velocity of light in each grid be the objective function value of some point in this grid. Light rays propagate along the straight line in each gird, and reflection or refraction occurs only when light rays go from one grid to another grid. During the searching process, the search direction is deflected to the direction that makes the value of objective function decrease by simulating refraction phenomenon of light, or the trend of function value is changed by simulating reflection phenomenon of light. Gradient information is not necessary and only a few parameters are needed to be adjusted in LRO, so it is simple and easy to be used. In this paper, some conclusions of optimal search mechanism are obtained by theoretical analysis and research. As a new algorithm, there are still a lot of problems in LRO to be studied.

2. LRO algorithm

2.1. The optimal search mechanism of LRO algorithm

Let us study the following problem

$$\min f(X), \quad X \in M \subset R^2, \tag{1}$$

where $f(X)$ is a positive function, that is, $f(x, y) > 0$ for an arbitrary $(x, y) \in M$, X is a feasible solution, M is the feasible range of $f(X)$, R^2 is a 2-dimensional real number space.

As shown in Figure 1, let X^* be a local minimum point of $f(X)$, and $X^{(0)}$ be an initial point. We can set up an interface between points $X^{(0)}$ and X^*, which separates the search space into two divisions. The light rays go faster in upper plane than in the other one. It can be easily seen that the new iteration point $X^{(2)}$ generated by refraction is closer to the minimum point X^* than the searching point $X^{(1)}$ in unchanged direction $P^{(0)}$ is. In order to make the iteration points generated by the refraction scheme get closer to the minimum point, the more interfaces can be set up, as shown in Figure 2. Generally, the more divisions are divided, the closer the searching point $X^{(k)}$ in the kth iteration is to X^*. Opposite situation will likely happen that an inappropriate selection of initial direction $P^{(0)}$ makes generated points get further from X^*, as shown in Figure 3. As a matter of fact, the dimension of vector space on the plain is 2, so vertical and horizontal searches ought to be taken into consideration when we do not know the exact position of minimum value. In Figure 4, vertical interfaces are established, and then search is made according to the law of refraction. Both vertical and horizontal lines are needed to be used in dividing search space, meanwhile, both vertical and horizontal searches are needed in LRO as shown in Figure 5. If the grid lengths tend to zero, the generated points according to the optical refraction will go towards X^* by continuing searches adjusted vertically and horizontally.

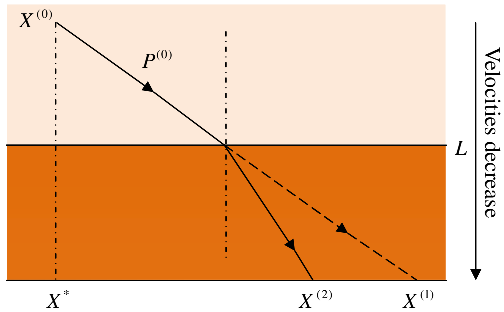


Figure 1 Increase an interface

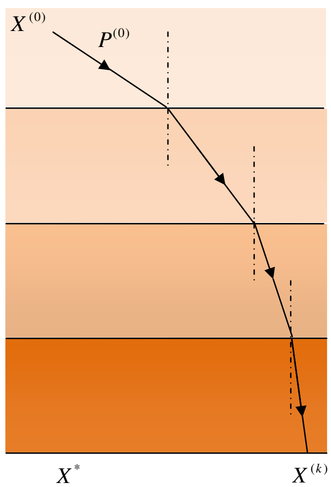


Figure 2 Increase many interfaces

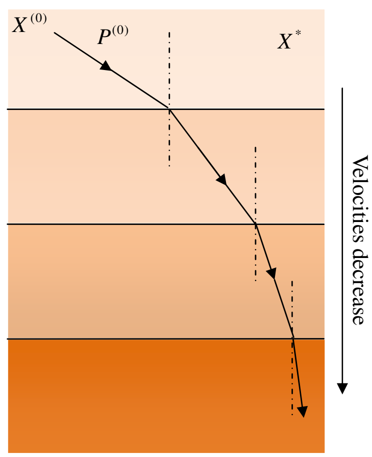


Figure 3 The situation of getting further from the minimum point

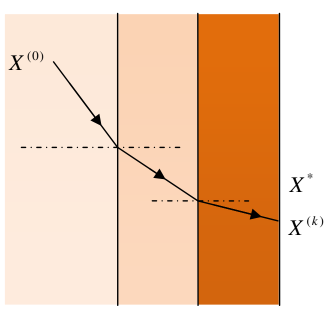


Figure 4 Increase vertical interfaces

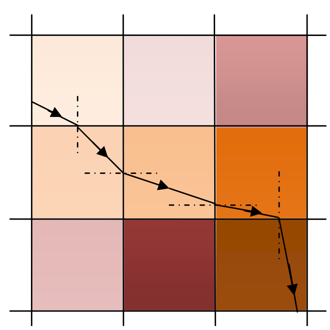


Figure 5 The vertical and horizontal searches

2.2. LRO iterative algorithm

Let us first analyze the relation between directions in the ith iteration and $i+1$st iteration. From $X^{(i)}$ produced by the iteration at ith search step in direction $P^{(i)}$, proceed the iterative search in direction $P^{(i+1)}$, then the next iteration point $X^{(i+1)}$ is obtained. The relation between $P^{(i)}$ and $P^{(i+1)}$ satisfies

1) If $\dfrac{v_{i+1}}{v_i}\sin\alpha_i \le 1$, then refraction occurs as shown in Figure 6:

$$\sin\alpha_{i+1} = \frac{\sin\alpha_i \cdot v_{i+1}}{v_i}, \tag{2}$$

where α_i is the angle of incidence in D_i, α_{i+1} is the angle of refraction in D_{i+1}, v_i is the propagation velocity of light in D_i, which can be set as the value of objective function at point $X^{(i-1)} = (x^{(i-1)}, y^{(i-1)})$ in D_i, v_{i+1} is the propagation velocity of light in D_{i+1}, which can be set as the value of objective function at point $X^{(i)} = (x^{(i)}, y^{(i)})$ in D_{i+1}.

2) If $\frac{v_{i+1}}{v_i}\sin\alpha_i > 1$, then reflection occurs as shown in Figure 7:

$$\alpha_i = \alpha_{i+1}, \tag{3}$$

where α_i is the angle of incidence in D_i, and α_{i+1} is the angle of reflection in D_i.

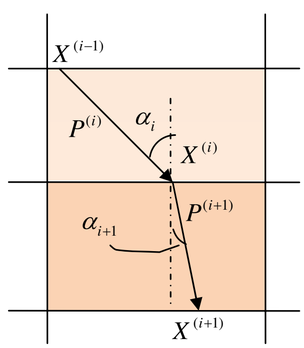


Figure 6 The direction updating based on refraction

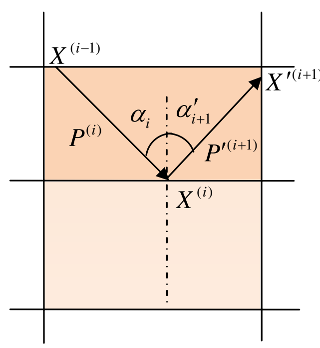


Figure 7 The direction updating based on reflection

According to the above analysis, the procedure of LRO is given as follows:

Step 1. Select a vertical grid length and a horizontal one as well, and divide the search space.

Step 2. Let the velocity of light rays traveling in each division be a value of objective function at some point in the division.

Step 3. An initial point $X^{(0)}$, an initial vector $P^{(0)}$ are appropriately given.

Step 4. Compute the next iteation point.

Step 5. If the stopping criterion is satisfied, go to Step 7, else go to Step 6.

Step 6. If the condition of total reflection is satisfied, compute the next searching direction according to law of reflection, else compute the next searching direction according to law of refraction, go to Step 4.

Step 7. Stop optimal search and output extreme value.

3. Related theorems of LRO algorithm

Theorem 1 *Let $f(x,y)$ be a positive continuous function. For any initial point $(x^{(0)},y^{(0)})$ and initial direction $P^{(0)}\neq(0,\pm1),(\pm1,0)$, if reflection does not occur, then variable quantity in the horizontal direction $|x^{(n)}-x^{(0)}|>h$ after finite iterations n according to LRO algorithm when the grid lengths h and τ are sufficiently small.*

Proof According to LRO iterative algorithm, the trend of two components of iteration point does not change when refraction occurs on horizontal and vertical grids, that is, if refraction occurs in the ith iteration and

$$x^{(i)}>x^{(i-1)}(x^{(i)}<x^{(i-1)}),\quad y^{(i)}>y^{(i-1)}(y^{(i)}<y^{(i-1)}),$$

then

$$x^{(i+1)}>x^{(i)}(x^{(i+1)}<x^{(i)}),\quad y^{(i+1)}>y^{(i)}(y^{(i+1)}<y^{(i)}).$$

Since reflection does not occur, all $x^{(i)}-x^{(i-1)}$ have the same sign. Let us assume that refractions occur in vertical direction. After k iterations, variable quantity in the horizontal direction is

$$|x^{(k)}-x^{(0)}|=|\sum_{i=1}^{k}(x^{(i)}-x^{(i-1)})|=\sum_{i=1}^{k}|x^{(i)}-x^{(i-1)}|=\sum_{i=1}^{k}\tau\tan\alpha_i=\tau\sum_{i=1}^{k}\tan\alpha_i. \tag{4}$$

1) When the search proceeds to the direction in which function increases

Since $\alpha_i>\alpha_{i-1}$, $\tan\alpha_i>\tan\alpha_{i-1}$. Then

$$|x^{(k)}-x^{(0)}|=\tau\sum_{i=1}^{k}\tan\alpha_i>k\tau\tan\alpha_1.$$

So, let $n=[\frac{h}{\tau\tan\alpha_1}]+1$. Then we have

$$|x^{(n)}-x^{(0)}|>h. \tag{5}$$

2) When the search proceeds to the direction in which function decreases

As $f(x,y)$ is continuous, $\forall\varepsilon>0$, $\exists a,b>0$, such that when

$$|x-x'|<a,\quad |y-y'|<b,$$

we have

$$|f(x,y)-f(x',y')|<\varepsilon.$$

Especially, for

$$\varepsilon_i=\frac{1}{N}f(x^{(i-1)},y^{(i-1)}),\quad i=1,2,\ldots,k,$$

$\exists\tau_i,h_i>0$, such that when

$$|x^{(i)}-x^{(i-1)}|<\tau_i,\quad |y^{(i)}-y^{(i-1)}|<h_i,$$

we have

$$|f(x^{(i)},y^{(i)})-f(x^{(i-1)},y^{(i-1)})|<\frac{1}{N}f(x^{(i-1)},y^{(i-1)}).$$

Therefore,

$$f(x^{(i)},y^{(i)}) > f(x^{(i-1)},y^{(i-1)}) - \frac{1}{N}f(x^{(i-1)},y^{(i-1)}),$$

$$\frac{\sin\alpha_i}{\sin\alpha_{i-1}} = \frac{f(x^{(i)},y^{(i)})}{f(x^{(i-1)},y^{(i-1)})} > \frac{N-1}{N}. \tag{6}$$

Let

$$\tau = \min(\tau_1,\tau_2,\dots,\tau_k), \quad h = \min(h_1,h_2,\dots,h_k),$$

$$p = [\frac{h}{\tau\tan\alpha_1}] + 3, \quad s = \frac{N-1}{N}.$$

Since $\sin\alpha_{i-1} < 1$,

$$p^2 - \frac{2p-1}{\sin^2\alpha_{i-1}} < p^2 - 2p + 1 = (p-1)^2,$$

$$\frac{p^2 - \frac{2p-1}{\sin^2\alpha_{i-1}}}{(p-1)^2} < 1.$$

Since $s^2 \to 1$ when $N \to \infty$, $\exists N^* > 0$, *s.t.* when $N > N^*$,

$$s^2 > \frac{p^2 - \frac{2p-1}{\sin^2\alpha_{i-1}}}{(p-1)^2},$$

$$(p-1)^2 s^2 \sin^2\alpha_{i-1} - (p-1)^2 > p^2\sin^2\alpha_{i-1} - p^2,$$

$$\frac{1-\sin^2\alpha_{i-1}}{1-s^2\sin^2\alpha_{i-1}} > \frac{(p-1)^2}{p^2}.$$

Hence

$$\frac{\cos\alpha_{i-1}}{\cos\alpha_i} = \frac{\sqrt{1-\sin^2\alpha_{i-1}}}{\sqrt{1-\sin^2\alpha_i}} > \frac{\sqrt{1-\sin^2\alpha_{i-1}}}{\sqrt{1-s^2\sin^2\alpha_{i-1}}} > \sqrt{\frac{(p-1)^2}{p^2}} = \frac{p-1}{p}.$$

And then

$$\frac{\tan\alpha_i}{\tan\alpha_{i-1}} = \frac{\sin\alpha_i}{\sin\alpha_{i-1}} \cdot \frac{\cos\alpha_{i-1}}{\cos\alpha_i} > \frac{p-1}{p} \cdot \frac{\sin\alpha_i}{\sin\alpha_{i-1}} > \frac{p-1}{p} \cdot \frac{N-1}{N},$$

$$\tan\alpha_i > \frac{p-1}{p} \cdot \frac{N-1}{N} \cdot \tan\alpha_{i-1} > \cdots > (\frac{p-1}{p} \cdot \frac{N-1}{N})^{i-1} \cdot \tan\alpha_1, \tag{7}$$

$$\begin{aligned}|x^{(k)} - x^{(0)}| &= \tau\sum_{i=1}^{k}\tan\alpha_i > \tau\sum_{i=1}^{k}(\frac{p-1}{p} \cdot \frac{N-1}{N})^{i-1} \cdot \tan\alpha_1 \\ &= \tau \cdot \tan\alpha_1 \cdot \frac{1 - (\frac{p-1}{p} \cdot \frac{N-1}{N})^k}{-\frac{p-1}{p} \cdot \frac{N-1}{N}} \\ &= \tau \cdot \tan\alpha_1 \cdot \frac{pN}{N+p-1} \cdot [1 - (\frac{p-1}{p} \cdot \frac{N-1}{N})^k]. \end{aligned} \tag{8}$$

As $\frac{pN}{N+p-1} \to p$ when $N \to \infty$, $\exists N^{**} > 0$ such that when $N > N^{**}$,

$$\frac{pN}{N+p-1} > p - 1.$$

Since $1-(\frac{p-1}{p}\cdot\frac{N-1}{N})^k \to 1$ when $k\to\infty$, $\exists k^*>0$ such that when $k>k^*$,

$$1-(\frac{p-1}{p}\cdot\frac{N-1}{N})^k > \frac{p-2}{p-1}.$$

Let $N=\max(N^*,N^{**})+1$ and $n=k^*+1$. Then

$$\begin{aligned}|x^{(n)}-x^{(0)}| &= \tau\cdot\tan\alpha_1\cdot\frac{pN}{N+p-1}\cdot[1-(\frac{p-1}{p}\cdot\frac{N-1}{N})^n] > \tau\cdot\tan\alpha_1\cdot(p-1)\cdot\frac{p-2}{p-1}\\ &= \tau\cdot\tan\alpha_1\cdot(p-2)=\tau\cdot\tan\alpha_1\cdot([\frac{h}{\tau\tan\alpha_1}]+1)>h.\end{aligned}\tag{9}$$

□

The corresponding theorem is as follows:

Theorem 2 *Let $f(x,y)$ be a positive continuous function. For any initial point $(x^{(0)},y^{(0)})$ and initial direction $P^{(0)}\neq(0,\pm1),(\pm1,0)$, if reflection does not occur, then variable quantity in vertical direction $|y^{(n)}-y^{(0)}|>\tau$ after finite iterations n according to LRO algorithm when the grid lengths h and τ are sufficiently small.*

Theorems 1 and 2 show that refraction occurs in two directions in LRO algorithm, that is the setting of rectangular grids is meanful. In general, initial search direction $P^{(0)}\neq(0,\pm1),(\pm1,0)$. This is because if $P^{(0)}=(0,\pm1),(\pm1,0)$, the sine of the angle of incidence $\sin\alpha_i=0$, which leads to $\frac{v_{i+1}}{v_i}\sin\alpha_i=0\le1$. Refraction occurs and reflection does not occur, and the sine of the angle of refraction $\sin\alpha_{i+1}=0$. Iteration points extend to infinity because search directions are not changed in iterations.

Theorem 3 *Let $f(x,y)$ be a positive continuous function. In the ith iteration, $X^{(i)}$ is on the horizontal grid and iteration point $X^{(i+1)}$ is on the vertical grid. $X'^{(i+1)}$ is the iteration point when refraction does not occur and the direction is not changed. $f(X^{(i+1)})<f(X'^{(i+1)})$ when grid lengths h and τ are sufficiently small.*

Proof It can be guaranteed that monotonicity of the value of contour line is satisfied when grid lengths h and τ are sufficiently small. In Figure 8, the values of contour lines line$_1$, line$_2$, line$_3$ and line$_4$ are $f(X^{(i-1)})$, $f(X^{(i)})$, $f(X'^{(i+1)})$ and $f(X^{(i+1)})$, respectively. When the value of function decreases, the value of line$_1$ is more than that of line$_2$. Because

$$\frac{\sin\alpha_i}{\sin\alpha_{i+1}}=\frac{v_i}{v_{i+1}},\quad v_i>v_{i+1},$$

we get

$$\alpha_{i+1}<\alpha_i=\alpha'_{i+1},\ \overline{X^{(i+1)}F}=\frac{\overline{X^{(i)}F}}{\tan(\alpha_{i+1})},\ \overline{X'^{(i+1)}F}=\frac{\overline{X^{(i)}F}}{\tan(\alpha'_{i+1})},\ \overline{X^{(i+1)}F}>\overline{X'^{(i+1)}F}.$$

$f(X^{(i+1)})<f(X'^{(i+1)})$ because of the monotonicity of contour line.

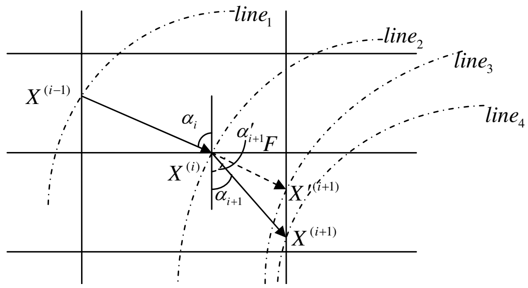


Figure 8 The function value decreases

Similarily, in Figure 9, the values of contour lines $line_1$, $line_2$, $line_3$ and $line_4$ are $f(X^{(i-1)})$, $f(X^{(i)})$, $f(X^{(i+1)})$ and $f(X'^{(i+1)})$, respectively. the value of $line_1$ is less than that of $line_2$ when the value of function increases. Since

$$\frac{\sin\alpha_i}{\sin\alpha_{i+1}} = \frac{v_i}{v_{i+1}}, \quad v_i < v_{i+1},$$

we have

$$\alpha_{i+1} > \alpha_i = \alpha'_{i+1}, \ \overline{X^{(i+1)}F} = \frac{\overline{X^{(i)}F}}{\tan(\alpha_{i+1})}, \ \overline{X'^{(i+1)}F} = \frac{\overline{X^{(i)}F}}{\tan(\alpha'_{i+1})}, \ \overline{X^{(i+1)}F} < \overline{X'^{(i+1)}F}.$$

$f(X^{(i+1)}) < f(X'^{(i+1)})$ because of the monotonicity of contour line. □

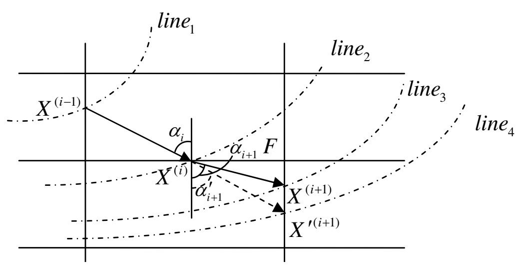


Figure 9 The function value increases

The corresponding theorem is as follows:

Theorem 4 *Let $f(x,y)$ be a positive continuous function. In the ith iteration, $X^{(i)}$ is on the vertical grid and iteration point $X^{(i+1)}$ is on the horizontal grid. $X'^{(i+1)}$ is the iteration point when refraction does not occur and the direction is not changed. $f(X^{(i+1)}) < f(X'^{(i+1)})$ when grid lengths h and τ are sufficiently small.*

Theorems 3 and 4 show that search direction will deflect to the direction that is relatively small by one refraction in LRO algorithm. If the next directional value decreases, then the decrease of function value of iteration point after refraction is larger, that is, the decrease is accelerated. If the next directional value increases, then the increase of function value of iteration point after refraction is smaller, that is, the increase is reduced.

Theorem 5 *Let $f(x,y)$ be a positive continuous function. α is the angle between the vertical* *and diagonal lines, β is the angle between the horizontal and diagonal lines, and*

$$\max\{\sin\alpha, \sin\beta\} < \frac{10}{11}.$$

If the value of $f(x, y)$ increases constantly after the mth iteration and

$$\lim_{i\to\infty}\frac{f(x^{(i)}, y^{(i)})}{c^i} = \infty (c > 1), f$$

then there exists a limited positive integer $N > m$ such that reflection occurs in the Nth iteration when grid lengths h and τ are sufficiently small.

Proof It is assumed that reflection does not occur after the mth iteration. As shown in Figure 10, suppose refraction occurs in vertical direction, we have

$$\frac{\sin\alpha_m}{f(x^{(m)}, y^{(m)})} = \cdots = \frac{\sin\alpha_{n_1-1}}{f(x^{(n_1-1)}, y^{(n_1-1)})} = \frac{\sin\alpha_{n_1}}{f(x^{(n_1)}, y^{(n_1)})}. \tag{10}$$

The above refraction process occurs in vertical direction constantly and then turns to horizontal direction, and we get

$$\frac{\sin(\frac{\pi}{2}-\alpha_{n_1})}{f(x^{(n_1)}, y^{(n_1)})} = \cdots \frac{\sin(\alpha_{n_2-1})}{f(x^{(n_2-1)}, y^{(n_2-1)})} = \frac{\sin(\alpha_{n_2})}{f(x^{(n_2)}, y^{(n_2)})}. \tag{11}$$

Then the process turns to vertical direction, we have

$$\frac{\sin(\frac{\pi}{2}-\alpha_{n_2})}{f(x^{(n_2)}, y^{(n_2)})} = \cdots \frac{\sin(\alpha_{n_3-1})}{f(x^{(n_3-1)}, y^{(n_3-1)})} = \frac{\sin(\alpha_{n_3})}{f(x^{(n_3)}, y^{(n_3)})}. \tag{12}$$

After the kth refraction turn, we get

$$\frac{\sin(\frac{\pi}{2}-\alpha_{n_k})}{f(x^{(n_k)}, y^{(n_k)})} = \cdots \frac{\sin(\alpha_{n_{k+1}-1})}{f(x^{(n_{k+1}-1)}, y^{(n_{k+1}-1)})} = \frac{\sin(\alpha_{n_{k+1}})}{f(x^{(n_{k+1})}, y^{(n_{k+1})})}. \tag{13}$$

According to (10)–(13), we have

$$\sin\alpha_{n_{k+1}} = \frac{\sin\alpha_m}{f(x^{(m)}, y^{(m)})}\cdot f(x^{(n_{k+1})}, y^{(n_{k+1})})\cdot\cot\alpha_{n_1}\cot\alpha_{n_2}\cdots\cot\alpha_{n_k}. \tag{14}$$

According to (10) and the condition that the value of function increases constantly after the mth iteration, we get

$$0 < \alpha_m < \alpha_{n_1-1} < \alpha < \frac{\pi}{2}. \tag{15}$$

As $\lim_{h,\tau\to 0}\frac{f(x^{(n_1)}, y^{(n_1)})}{f(x^{(n_1-1)}, y^{(n_1-1)})} = 1$, there exist sufficiently small h_1, τ_1 that make

$$1 < \frac{f(x^{(n_1)}, y^{(n_1)})}{f(x^{(n_1-1)}, y^{(n_1-1)})} < 1.1. \tag{16}$$

From (10), we get

$$\sin\alpha_{n_1} = \sin\alpha_{n_1-1}\cdot\frac{f(x^{(n_1)}, y^{(n_1)})}{f(x^{(n_1-1)}, y^{(n_1-1)})}. \tag{17}$$

From (15)–(17), we have

$$\sin\alpha_m < \sin\alpha_{n_1} < 1.1\sin\alpha.$$

As $\max\{\sin\alpha, \sin\beta\} < \dfrac{10}{11}$, $1.1\sin\alpha < 1$,

$$0 < \alpha_m < \alpha_{n_1} < arc\sin(1.1\sin\alpha) < \frac{\pi}{2}. \tag{18}$$

Similarly, according to (18) and (11)

$$0 < \frac{\pi}{2} - \arcsin(1.1\sin\alpha) < \frac{\pi}{2} - \alpha_{n_1} < \alpha_{n_2-1} < \beta < \frac{\pi}{2},$$

there exist sufficiently small h_2, τ_2 that make

$$1 < \frac{f(x^{(n_2)}, y^{(n_2)})}{f(x^{(n_2-1)}, y^{(n_2-1)})} < 1.1.$$

We have

$$\frac{\pi}{2} - \arcsin(1.1\sin\alpha) < \alpha_{n_2} < \arcsin(1.1\sin\beta),$$

similarly,

$$\frac{\pi}{2} - \arcsin(1.1\sin\beta) < \alpha_{n_3} < \arcsin(1.1\sin\alpha),$$
$$\frac{\pi}{2} - \arcsin(1.1\sin\alpha) < \alpha_{n_4} < \arcsin(1.1\sin\beta).$$

Let

$$\bar{\alpha} = \min\{\alpha_m, \frac{\pi}{2} - \arcsin(1.1\sin\alpha), \frac{\pi}{2} - \arcsin(1.1\sin\beta)\},$$
$$\bar{\beta} = \max\{\arcsin(1.1\sin\alpha), \arcsin(1.1\sin\beta)\}. \tag{19}$$

Then for $\forall k$, we get

$$0 < \bar{\alpha} < \alpha_{n_k} < \bar{\beta} < \frac{\pi}{2}, \quad \cot\bar{\alpha} > \cot\alpha_{n_k} > \cot\bar{\beta}.$$

From (14),

$$\sin\alpha_{n_{k+1}} > \frac{\sin\alpha_m}{f(x^{(m)}, y^{(m)})} \cdot f(x^{(n_{k+1})}, y^{(n_{k+1})}) \cdot \cot^k\bar{\beta} = \frac{\sin\alpha_m}{f(x^{(m)}, y^{(m)})} \cdot \frac{f(x^{(n_{k+1})}, y^{(n_{k+1})})}{\tan^k\bar{\beta}}. \tag{20}$$

According to (19) and $\alpha + \beta = \frac{\pi}{2}$,

$$\bar{\beta} > \max\{\arcsin(\sin\alpha), \arcsin(\sin\beta)\} = \max\{\alpha, \beta\} \geq \frac{\pi}{4}.$$

Therefore,

$$\tan\bar{\beta} > 1, \quad \lim_{k\to\infty} \frac{f(x^{(n_{k+1})}, y^{(n_{k+1})})}{\tan^{n_{k+1}}\bar{\beta}} = \infty.$$

$\tan^{n_{k+1}}\bar{\beta} > \tan^k\bar{\beta}$ as $n_{k+1} > k$, then

$$\frac{f(x^{(n_{k+1})}, y^{(n_{k+1})})}{\tan^k\bar{\beta}} > \frac{f(x^{(n_{k+1})}, y^{(n_{k+1})})}{\tan^{n_{k+1}}\bar{\beta}}, \quad \lim_{k\to\infty} \frac{f(x^{(n_{k+1})}, y^{(n_{k+1})})}{\tan^k\bar{\beta}} = \infty.$$

There exists a positive integer N such that

$$\frac{f(x^{(n_{N+1})}, y^{(n_{N+1})})}{\tan^N\bar{\beta}} > \frac{f(x^{(m)}, y^{(m)})}{\sin\alpha_m}.$$

Let

$$h = \min\{h_1, h_2, \ldots, h_N\}, \quad \tau = \min\{\tau_1, \tau_2, \ldots, \tau_N\}.$$

According to (20),

$$\sin\alpha_{n_{N+1}} > \frac{\sin\alpha_m}{f(x^{(m)}, y^{(m)})} \cdot \frac{f(x^{(n_{N+1})}, y^{(n_{N+1})})}{\tan^N\bar{\beta}} > 1 \tag{21}$$

which is a contradiction. The assumption is not true and the theorem is proved. □

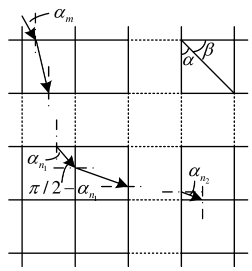


Figure 10 The search path of LRO algorithm

Theorem 5 shows that, the search along the directions that make the value of function increase will not keep going, and search direction will be changed by reflection algorithm in LRO.

4. Numerical experiments

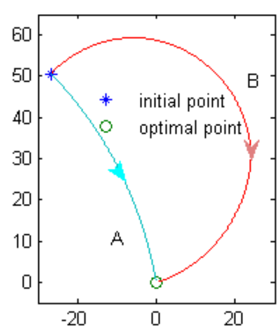
Seven standard test functions in literature [14] are chosen to test the performance of LRO algorithm. As the paper mainly analyzed some basic theory of LRO used for two dimensional optimization problems, the chosen test functions are set to be of two dimensions. The size of grid and precision are both 0.1, and the maximum iteration times is 10000. Run LRO independently 50 times for different test functions. If the maximum iteration number is reached and the approximate minimum point in a certain precision range is not found in an experiment, then it is considered to be a failed experiment. The average iteration times (AVEIT), maximum iteration times (MAXIT), minimum iteration times (MINIT) over successful runs and the success rate (RATE) are listed in Table 1.

Function	AVEIT	MAXIT	MINIT	RATE(%)
Sphere	1657	5450	506	100
Rosenbrock	2434	8277	105	100
Six-hump Camel-Back	216	1172	19	100
Goldstein-Price	2125	8733	45	60
Branin	186	636	30	100
Schwefel 2.22	2708	9445	165	100
Schwefel 1.2	1817	3246	530	100

Table1 The results of numerical experimets

As shown in Table 1, the success rate of Goldstein-Price function is lower. Through the analysis of the property of this function, it can be seen that it is a highly oscillatory function at the adjacency of the global minimum point. As the media in the the same grid are assumed to be uniform in LRO, a rather large error can be caused when a test function is highly oscillatory. Therefore, the optimization effect is not ideal and success rate is low. According to this situation, the authors set the size of grid 0.01 and MAXIT 50000. 50 times experiments were carried out for Goldstein-Price function, which made the success rate be increased to 100% from 60%. The corresponding iteration times increases, so selecting the appropriate gird size according to precision range is a key of LRO algorithm.

The maximum and minimum iteration times for each text function have larger difference because of the differences of initial points and directions. As 50 points are chosen randomly in the domain of each function, some points are far away from the minimum point and others are close to it. Moreover, the iterations times have larger difference even for the same initial point and the different inital directions. The Figure 11 shows the searching paths A and B of two experiments for Sphere function with iteration times 771 and 1427, respectively. As the direction in LRO is needed to be adjusted slowly, the iteration times is more when the initial direction deflected its way from pointing to minimum point. The situation of Figure 12 may occur, where a certain iteration times adjusting searching direction are required to find the appropriate minimum point in the precision range. The convergence rate of LRO is slow in some cases for the above reasons, but LRO can jump out local minimum point easily and has great ability of global optimization.



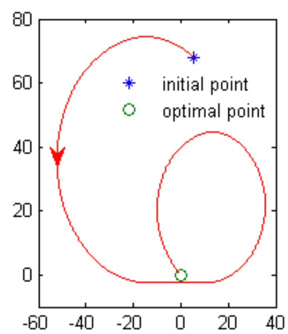


Figure 11 The searching paths A and B Figure 12 A particular situation

5. Conclusion

LRO algorithm is an intelligent optimization algorithm based on Fermat's principle. Derivative operation and other complex operations are not needed in this algorithm. LRO, which completely simulates the refraction and reflection phenomena of light, is an algorithm following the natural law. A large number of numerical experiments show that LRO will find the appropriate minimum point in a certain precision range for different test functions, initial points, initial directions. The performance study shows LRO is effective and is a very potential global optimization algorithm. Theoretical analysis and some conclusions of the optimal search mechanism of LRO are given in this paper. These theoretical results, which have been proved, are presented in form of theorems. In Theorems 1 and 2, we have proved that refractions in LRO algorithm alternatively occur in both horizontal and vertical directions. In Theorems 3 and 4, we have proved that the decrease of function is accelerated and the increase of function is reduced by refractions in algorithm. In Theorem 5, we have proved that reflection will inevitably occur if the search along the directions that make the value of function increase keeps going. These theoretical results lay a foundation for convergence proof, and provide theoretical basis for improvement and applications of LRO.

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