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A Stochastic Restricted s–K Estimator in the Linear Model

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Abstract In this paper, we propose a stochastic restricted s–K estimator in the linear model with additional stochastic linear restrictions by combining the ordinary mixed estimator (OME) with the s–K estimator. It is shown that the proposed estimator is superior to the OME and the s–K estimator under the mean squared error matrix criterion under some conditions. Finally, a numerical example and a Monte Carlo simulation study are given to verify the theoretical results.

Keywords the ordinary mixed estimator; s–K estimator; stochastic restricted s–K estimator; the mean squared error matrix.

MR(2010) Subject Classification 62J05; 62F10

1. Introduction

Consider the following linear regression model

$$Y = X\beta + \varepsilon, \tag{1}$$

where Y is an $n \times 1$ random vector of response variables, X is the known regressor matrix of order $n \times p$ with full column rank, β is a $p \times 1$ vector of unknown parameters, ε is an $n \times 1$ vector of random errors with $\mathrm{E}(\varepsilon) = 0$ and $\mathrm{Cov}(\varepsilon) = \sigma^2 I_n$, and σ^2 is an unknown parameter.

According to the Gauss-Markov theorem, the least squares estimator (LSE) of β is given by

$$\hat{\beta}_{\mathrm{LSE}} = S^{-1}X'Y, \tag{2}$$

where $S = X'X$. The LSE plays an important role in the regression analysis theory. However, it has been shown that the LSE is no longer a good estimator when the problem of multicollinearity in the model is present. To overcome this problem, various biased estimators as one of remedies were put forward in the literature, such as the Stein estimator by Stein [1], the ridge estimator (RE) by Hoerl and Kennard [2] and the Liu estimator (LE) by Liu [3]. Recently, Xu and He [4] proposed a new kind of biased estimator class, which is called the s–K estimator. The LSE, the Stein estimator and the RE are all special cases of the s–K estimator.

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Let Q be the orthogonal matrix such that $Q'X'XQ = \Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_p)$, where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p > 0$ are the ordered eigenvalues of $X'X$. Then the s–K estimator is defined as

$$\hat{\beta}_s(K) = (sS + QKQ')^{-1}X'Y, \tag{3}$$

where $s \geq 1$ and $K = \mathrm{diag}(k_1, k_2, \ldots, k_p)$ are parameters, and $k_i \geq 0$, $i = 1, 2, \ldots, p$.

Denote

$$F_s(K) = (sS + QKQ')^{-1}S. \tag{4}$$

Observing that $F_s(K)$ and S^{-1} are commutative, we can write $\hat{\beta}_s(K)$ as

$$\hat{\beta}_s(K) = F_s(K)\hat{\beta}_{\mathrm{LSE}} = F_s(K)S^{-1}X'Y = S^{-1}F_s(K)X'Y. \tag{5}$$

Another way to combat multicollinearity is through the collection and use of additional information, which can be exact or stochastic restrictions, see Rao and Toutenburg [5]. However, exact restrictions are often appropriate in many applied work such as economic relations, industrial structures, production planning, and so on. While, as pointed out by Arashi and Tabatabaey [6] using stochastic linear restriction, one can accomplish an examination and analysis of one's own thoughts and feelings. Moreover, one may also have prior information from a historical sample which usually makes some relations through stochastic restrictions. Therefore, in addition to model (1), now let us be given some prior information about β in the form of a set of j independent stochastic linear restrictions as follows:

$$r = R\beta + \upsilon, \quad \mathrm{E}(\upsilon) = 0, \quad \mathrm{Cov}(\upsilon) = \sigma^2 W, \tag{6}$$

where r is a $j \times 1$ vector, R is a $j \times p$ matrix with $\mathrm{rank}(R) = j$, υ is a $j \times 1$ vector of disturbances, W is assumed to be known and positive definite. Besides, it is also assumed that the random vector υ is independent of ε.

Durbin [7], Theil and Goldberger [8] and Theil [9] proposed the ordinary mixed estimator (OME) by combining the sample model with the stochastic restrictions, which is defined as

$$\hat{\beta}_{\mathrm{OME}} = (S + R'W^{-1}R)^{-1}(X'Y + R'W^{-1}r). \tag{7}$$

Özkale [10] showed that the OME could be rewritten as

$$\hat{\beta}_{\mathrm{OME}} = \hat{\beta}_{\mathrm{LSE}} + S^{-1}R'(W + RS^{-1}R')^{-1}(r - R\hat{\beta}_{\mathrm{LSE}}). \tag{8}$$

For model (1) with the stochastic restrictions (6), Yang and Xu [11] also introduced a stochastic restricted Liu estimator (SRLE) through replacing the LSE in the OME by the LE. Yang and Wu [12] proposed a stochastic restricted k–d class estimator which is a generalization of the k–d class estimator and the SRLE. Some important references on this subject are Li and Yang [13, 14], Yang and Cui [15] and among others.

In this article, we will introduce a stochastic restricted s–K estimator as an alternative method to overcome multicollinearity by combining the OME with the s–K estimator. The new estimator includes the OME and the s–K estimator as special cases. In addition, we will compare the new estimator with the OME and the s–K estimator respectively, in the light of the mean squared error matrix (MSEM) criterion.

The paper is organized as follows. In Section 2, the new estimator is introduced. Some properties of the new estimator are discussed in Section 3. A numerical example and a Monte Carlo simulation study are given in Sections 4 and 5, respectively. Some concluding remarks are presented in Section 6.

2. The new estimator

In this section, we introduce a new stochastic restricted estimator, which is obtained by substituting the s–K estimator for the LSE in the OME. The new estimator is called the stochastic restricted s–K estimator, which is defined as

$$\begin{aligned}\hat{\beta}_s^*(K) &= \hat{\beta}_s(K) + S^{-1}R'(W+RS^{-1}R')^{-1}(r-R\hat{\beta}_s(K))\\ &= S^{-1}F_s(K)X'Y + S^{-1}R'(W+RS^{-1}R')^{-1}(r-RS^{-1}F_s(K)X'Y)\\ &= [S^{-1}-S^{-1}R'(W+RS^{-1}R')^{-1}RS^{-1}]F_s(K)X'Y + S^{-1}R'(W+RS^{-1}R')^{-1}r\\ &= (S+R'W^{-1}R)^{-1}F_s(K)X'Y + S^{-1}R'[W^{-1}-W^{-1}R(S+R'W^{-1}R)^{-1}R'W^{-1}]r\\ &= (S+R'W^{-1}R)^{-1}F_s(K)X'Y + [S^{-1}R'W^{-1}-S^{-1}R'W^{-1}R(S+R'W^{-1}R)^{-1}R'W^{-1}]r\\ &= (S+R'W^{-1}R)^{-1}F_s(K)X'Y + [S^{-1}-S^{-1}R'W^{-1}R(S+R'W^{-1}R)^{-1}]R'W^{-1}r\\ &= (S+R'W^{-1}R)^{-1}F_s(K)X'Y + [S^{-1}-S^{-1}(S+R'W^{-1}R-S)(S+R'W^{-1}R)^{-1}]R'W^{-1}r\\ &= (S+R'W^{-1}R)^{-1}F_s(K)X'Y + (S+R'W^{-1}R)^{-1}R'W^{-1}r\\ &= (S+R'W^{-1}R)^{-1}[F_s(K)X'Y + R'W^{-1}r]. \qquad (9)\end{aligned}$$

Now, we can see that $\hat{\beta}_s^*(K)$ is a general estimator which includes the s–K estimator and the OME as special cases: if $R=0, \hat{\beta}_s^*(K)=\hat{\beta}_s(K)$; if $s=1$ and $K=0$, $\hat{\beta}_s^*(K)=\hat{\beta}_{\text{OME}}$.

For the sake of convenience, we list here some notations and important lemmas needed in the following discussions. For an $n\times n$ symmetric matrix M, $M\geq 0$ means that M is positive semidefinite, and $M>0$ means that M is positive definite.

Note that for any estimator $\hat{\beta}$ of β, its MSEM can be written as

$$\text{MESM}(\hat{\beta}) = \text{E}[(\hat{\beta}-\beta)(\hat{\beta}-\beta)'] = \text{Cov}(\hat{\beta}) + \text{Bias}(\hat{\beta})\text{Bias}(\hat{\beta})',$$

where $\text{Bias}(\hat{\beta}) = \text{E}(\hat{\beta})-\beta$ is bias of $\hat{\beta}$.

Lemma 2.1 *Let $M>0$, α be some vector. Then $M-\alpha\alpha'>0$ if and only if $\alpha'M^{-1}\alpha<1$.*

Proof See Farebrother [16]. □

Lemma 2.2 *Let $\hat{\beta}_1=A_1Y$, $\hat{\beta}_2=A_2Y$ be two homogeneous linear estimators of β such that $D=A_1A_1'-A_2A_2'>0$. Then*

$$\mathit{MSEM}(\hat{\beta}_1) - \mathit{MSEM}(\hat{\beta}_2) = \sigma^2 D + b_1b_1' - b_2b_2' > 0$$

if and only if

$$b_2'(\sigma^2 D + b_1 b_1')^{-1} b_2 < 1,$$

where $b_i = Bias(\hat{\beta}_i) = (A_i X - I)\beta$, $i = 1, 2$.

Proof Lemma 2.2 is a consequence of Lemma 2.1.

Lemma 2.3 *Suppose that* M, N *are* $n \times n$ *matrices and* $M > 0$, $N \geq 0$. *Then* $M > N \Longleftrightarrow \lambda_{\max}(NM^{-1}) < 1$.

Proof See Wang et al. [17]. □

3. The superiority of the new estimator

The expectation, the bias and the covariance matrix of the stochastic restricted s–K estimator are given, respectively, by

$$\begin{aligned} \mathrm{E}(\hat{\beta}_s^*(K)) &= (S + R'W^{-1}R)^{-1}(F_s(K)S + R'W^{-1}R)\beta \\ &= (S + R'W^{-1}R)^{-1}F_s(K)S\beta + (S + R'W^{-1}R)^{-1}(S + R'W^{-1}R - S)\beta \\ &= (S + R'W^{-1}R)^{-1}F_s(K)S\beta + [I - (S + R'W^{-1}R)^{-1}S]\beta \\ &= \beta + (S + R'W^{-1}R)^{-1}(F_s(K) - I)S\beta, \end{aligned} \tag{10}$$

$$b_1 := \mathrm{Bias}(\hat{\beta}_s^*(K)) = \mathrm{E}(\hat{\beta}_s^*(K)) - \beta = (S + R'W^{-1}R)^{-1}(F_s(K) - I)S\beta, \tag{11}$$

$$\mathrm{Cov}(\hat{\beta}_s^*(K)) := \sigma^2 B, \tag{12}$$

where

$$B = (S + R'W^{-1}R)^{-1}(A + R'W^{-1}R)(S + R'W^{-1}R)^{-1}, \quad A = F_s(K)SF_s(K)'. \tag{13}$$

From (11) and (12), we can obtain that

$$\mathrm{MSEM}(\hat{\beta}_s^*(K)) = \mathrm{Cov}(\hat{\beta}_s^*(K)) + \mathrm{Bias}(\hat{\beta}_s^*(K))\mathrm{Bias}(\hat{\beta}_s^*(K))' = \sigma^2 B + b_1 b_1'. \tag{14}$$

Similarly, we can get

$$\mathrm{MSEM}(\hat{\beta}_s(K)) := \sigma^2 C + b_2 b_2', \tag{15}$$

$$\mathrm{MSEM}(\hat{\beta}_{\mathrm{OME}}) = \sigma^2 (S + R'W^{-1}R)^{-1}, \tag{16}$$

where

$$C = F_s(K)S^{-1}F_s'(K), \quad b_2 = (F_s(K) - I_p)\beta. \tag{17}$$

In order to compare $\hat{\beta}_s^*(K)$ with $\hat{\beta}_s(K)$ and $\hat{\beta}_{\mathrm{OME}}$ in the MSEM sense, we now investigate the differences

$$\begin{aligned} \Delta_1 &= \mathrm{MSEM}(\hat{\beta}_{\mathrm{OME}}) - \mathrm{MSEM}(\hat{\beta}_s^*(K)) = \sigma^2 (S + R'W^{-1}R)^{-1} - \sigma^2 B - b_1 b_1' \\ &:= \sigma^2 D_1 - b_1 b_1', \end{aligned} \tag{18}$$

and

$$\Delta_2 = \mathrm{MSEM}(\hat{\beta}_s(K)) - \mathrm{MSEM}(\hat{\beta}_s^*(K)) = \sigma^2 C + b_2 b_2' - \sigma^2 B - b_1 b_1'$$

$$:= \sigma^2 D_2 + b_2 b_2' - b_1 b_1', \tag{19}$$

where

$$D_1 = (S + R'W^{-1}R)^{-1}(S - A)(S + R'W^{-1}R)^{-1}, \quad D_2 = C - B. \tag{20}$$

In the following theorems, we will give the necessary and sufficient conditions for the new estimator to be superior to the OME and the s–K estimator in the MSEM sense.

Theorem 3.1 *When $s > 1$, then the stochastic restricted s–K estimator $\hat{\beta}_s^*(K)$ is better than the OME estimator $\hat{\beta}_{\rm OME}$ in the MSEM sense if and only if $b_1' D_1^{-1} b_1 < \sigma^2$.*

Proof It follows from $Q'SQ = \Lambda$ that

$$F_s(K) = Q\Lambda(s\Lambda + K)^{-1}Q',$$

which yields

$$A = F_s(K)SF_s(K)' = Q\Lambda^3(s\Lambda + K)^{-2}Q'.$$

Consequently,

$$S - A = Q\Lambda Q' - Q\Lambda^3(s\Lambda + K)^{-2}Q' = Q\text{diag}(\tau_1, \tau_2, \ldots, \tau_p)Q',$$

where $\tau_i = \lambda_i - \frac{\lambda_i^3}{(s\lambda_i + k_i)^2}$, $i = 1, 2, \ldots, p$. Note that $s > 1$, we get $\tau_i > 0$, $i = 1, 2, \ldots, p$, which implies that $S - A > 0$. Thus, we have $D_1 > 0$. Then by Lemma 2.2 and expression (18), we obtain that $\Delta_1 > 0$ if and only if $b_1' D_1^{-1} b_1 < \sigma^2$. $\Box$

Theorem 3.2 *When $\lambda_{\max}(BC^{-1}) < 1$, then the necessary and sufficient condition for the new estimator $\hat{\beta}_s^*(K)$ to be superior to the s–K estimator $\hat{\beta}_s(K)$ in the MSEM sense is*

$$b_1'(\sigma^2 D_2 + b_2 b_2')^{-1} b_1 < 1.$$

Proof It is straightforward that $B > 0$ and $C > 0$. Thus when $\lambda_{\max}(BC^{-1}) < 1$, we can get $D_2 > 0$ by applying Lemma 2.3. Consequently, it follows from Lemma 2.2 and expression (19) that $\Delta_2 > 0$ if and only if $b_1'(\sigma^2 D_2 + b_2 b_2')^{-1} b_1 < 1$. $\Box$

4. Numerical example

In this section, we apply the proposed estimator to the well known data set on Total National Research and Development Expenditures as a per cent of Gross National Product by Country, which was discussed in Gruber [18]. The data has then been widely analyzed in literature by Akdeniz and Erol [19], Li and Yang [13], and Chang and Yang [20]. We assemble the data as follows:

$$X = \begin{pmatrix} 1.9 & 2.2 & 1.9 & 3.7 \\ 1.8 & 2.2 & 2.0 & 3.8 \\ 1.8 & 2.4 & 2.1 & 3.6 \\ 1.8 & 2.4 & 2.2 & 3.8 \\ 2.0 & 2.5 & 2.3 & 3.8 \\ 2.1 & 2.6 & 2.4 & 3.7 \\ 2.1 & 2.6 & 2.6 & 3.8 \\ 2.2 & 2.6 & 2.6 & 4.0 \\ 2.3 & 2.8 & 2.8 & 3.7 \\ 2.3 & 2.7 & 2.8 & 3.8 \end{pmatrix}, \quad Y = \begin{pmatrix} 2.3 \\ 2.2 \\ 2.2 \\ 2.3 \\ 2.4 \\ 2.5 \\ 2.6 \\ 2.6 \\ 2.7 \\ 2.7 \end{pmatrix}.$$

From the data, we can obtain the following results:

(i) The eigenvalues of $X'X$: 302.96, 0.7283, 0.0446, 0.0345.

(i) The LSE of β: $(0.6455, 0.0896, 0.1436, 0.1526)'$.

(ii) The LSE of σ^2: $\hat{\sigma}^2 = 0.0015$.

(iv) The condition number of $X'X$: 5537.7.

Following Li and Yang [13], we consider the following stochastic linear restriction:

$$r = R\beta + e, \quad R = (1, -2, -2, -2), \quad e \sim N(0, \hat{\sigma}^2).$$

Now let us compare the MSE of the proposed estimator with that of the OME and the s–K estimator. The estimated MSE values are obtained by replacing all the unknown parameters with the corresponding LSE. Here, we select $s = 1.01$ and 1.1. For the sake of convenience, we choose $K = kI$, where $k = 0,\ 0.001,\ 0.005,\ 0.009,\ 0.01,\ 0.02$ and 0.1. All the results are computed by R2.8.0, which are listed in Tables 1–2.

	k=0	k=0.001	k=0.005	k=0.009	k=0.01	k=0.02	k=0.1
$\hat{\beta}_{\mathrm{OME}}$	0.0445	0.0445	0.0445	0.0445	0.0445	0.0445	0.0445
$\hat{\beta}_s(K)$	0.0777	0.0741	0.0643	0.0595	0.0589	0.0597	0.1249
$\hat{\beta}_s^*(K)$	0.0437	0.0415	0.0342	0.0288	0.0282	0.0194	0.0058

Table 1 Estimated MSE values of the OME, the s–K estimator and the new estimator with $s = 1.01$.

	k=0	k=0.001	k=0.005	k=0.009	k=0.01	k=0.02	k=0.1
$\hat{\beta}_{\mathrm{OME}}$	0.0445	0.0445	0.0445	0.0445	0.0445	0.0445	0.0445
$\hat{\beta}_s(K)$	0.0693	0.0673	0.0621	0.0603	0.0602	0.0642	0.1293
$\hat{\beta}_s^*(K)$	0.0412	0.0395	0.0338	0.0295	0.0286	0.0218	0.0102

Table 2 Estimated MSE values of the OME, the s–K estimator and the new estimator with $s = 1.1$.

From Tables 1–2, we can observe the following: when s is slightly bigger than 1 and k is relatively small, then the estimated MSE values of the new estimator are indeed smaller than those of the OME and the s–K estimator, which agrees with the theoretical findings in Theorems 3.1–3.2.

5. Monte Carlo simulation study

In this section, we carry out a Monte Carlo simulation study to further illustrate the performance of the new estimator. In the simulation, the explanatory variables and the dependent variables are generated respectively by

$$x_{ij} = (1-\gamma^2)^{1/2}\omega_{ij} + \gamma\omega_{i,p}, \quad i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, p-1,$$

$$y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \varepsilon_i, \ \varepsilon_i \sim N(0,1), \quad i = 1, 2, \ldots, n,$$

where $p = 5$, $n = 100$, and $\omega_{i,j}, i = 1, \ldots, n, \ j = 1, \ldots, p$ are generated from independent standard normal distributions, ε_i, $i = 1, \ldots, n$ are independent standard normal pseudo-random numbers, and γ is specified so that the correlation between any two explanatory variables is given by γ^2. Following Liu [21], four sets of correlation are considered here, which are $\gamma = 0.9$, $\gamma = 0.99$, $\gamma = 0.999$ and $\gamma = 0.9999$. The resulting condition numbers of $X'X$ are 13.3, 125.9, 1251.4, 12494.4. We choose the normalized eigenvector corresponding to the largest eigenvalue of $X'X$ as the parameter vector β suggested by Chang and Yang [3]. For simplicity, we choose $K = kI$, where k =0.001, 0.01 and 0.1. In addition, the following stochastic linear constraint to the model is considered:

$$r = R\beta + e, \quad R = (1, -2, -2, -2), \quad e \sim N(0, 1).$$

For each choice of γ, the experiment is replicated 10000 times by generating new error terms while X and β are fixed. Then, the estimated MSE for any estimator $\widetilde{\beta}$ of β is calculated as follows:

$$\mathrm{MSE}(\widetilde{\beta}) = \frac{1}{N}\sum_{m=1}^{N}(\widetilde{\beta}^{(m)} - \beta)'(\widetilde{\beta}^{(m)} - \beta),$$

where $\widetilde{\beta}^{(m)}$ is the estimator of β in the mth replication of the experiment, and $N = 10000$. The simulation results are summarized in Tables 3–6.

	s=1.1			s=1.15			s=1.2		
	k=0.001	k=0.01	k=0.1	k=0.001	k=0.01	k=0.1	k=0.001	k=0.01	k=0.1
$\hat{\beta}_{\mathrm{OME}}$	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666
$\hat{\beta}_s(K)$	0.1624	0.1601	0.1599	0.1585	0.1574	0.1547	0.1589	0.1583	0.1566
$\hat{\beta}_s^*(K)$	0.1487	0.1464	0.1460	0.1483	0.1462	0.1434	0.1496	0.1494	0.1489

Table 3 Estimated MSE values of the three estimators with $\gamma = 0.9$

	s=1.1			s=1.15			s=1.2		
	k=0.001	k=0.01	k=0.1	k=0.001	k=0.01	k=0.1	k=0.001	k=0.01	k=0.1
$\hat{\beta}_{\mathrm{OME}}$	1.2524	1.2524	1.2524	1.2524	1.2524	1.2524	1.2524	1.2524	1.2524
$\hat{\beta}_s(K)$	1.4589	1.4663	1.3245	1.3550	1.3315	1.2459	1.2400	1.2426	1.1445
$\hat{\beta}^*_s(K)$	1.0637	1.0586	0.9752	1.0005	0.9894	0.9359	0.9435	0.9382	0.8756

Table 4 Estimated MSE values of the three estimators with $\gamma = 0.99$

	s=1.1			s=1.15			s=1.2		
	k=0.001	k=0.01	k=0.1	k=0.001	k=0.01	k=0.1	k=0.001	k=0.01	k=0.1
$\hat{\beta}_{\mathrm{OME}}$	11.489	11.489	11.489	11.489	11.489	11.489	11.489	11.489	11.489
$\hat{\beta}_s(K)$	14.421	13.074	6.0899	13.193	12.083	5.7394	12.179	11.223	5.4544
$\hat{\beta}^*_s(K)$	9.4382	8.5644	4.1622	8.6449	7.8918	3.9398	8.0563	7.4471	3.7631

Table 5 Estimated MSE values of the three estimators with $\gamma = 0.999$

	s=1.1			s=1.15			s=1.2		
	k=0.001	k=0.01	k=0.1	k=0.001	k=0.01	k=0.1	k=0.001	k=0.01	k=0.1
$\hat{\beta}_{\mathrm{OME}}$	113.08	113.08	113.08	113.08	113.08	113.08	113.08	113.08	113.08
$\hat{\beta}_s(K)$	129.21	60.364	3.6971	119.66	57.867	3.6405	111.47	54.373	3.6184
$\hat{\beta}^*_s(K)$	82.352	40.475	2.6868	77.493	38.409	2.7029	71.842	35.497	2.6739

Table 6 Estimated MSE values of the three estimators with $\gamma = 0.9999$

From the simulation results shown in Tables 3–6, we can see that with the increase of the levels of multicollinearity, the estimated MSE values of the OME, the s–K estimator and the proposed estimator increase in general. Moreover, under the cases considered, the new estimator has smaller MSE values than those of the OME and the s–K estimator. In particular, the more severe the multicollinearity is, the more pronounced the superiority of the new estimator is. Therefore, the proposed estimator is meaningful in practice.

6. Conclusion

In this paper, we introduce a stochastic restricted s–K estimator for the vector of parameters in a linear regression model when additional stochastic linear restrictions are assumed to hold. Necessary and sufficient conditions are derived for the proposed estimator to be superior to the s–K estimator and the OME in the sense of the MSEM criterion. Finally, we illustrate our theoretical results by a numerical example and a Monte Carlo simulation.

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