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# Fekete-Szegö Functional Problems for Certain Subclasses of Bi-Univalent Functions Involving the Hohlov Operator

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**Abstract** In the paper the new subclasses $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$ and $\mathcal{M}_{\sum}^{a,b,c}(\lambda;\phi)$ of the function class $\sum$ of bi-univalent functions involving the Hohlov operator are introduced and investigated. Then, the corresponding Fekete-Szegö functional inequalities as well as the bound estimates of the coefficients $a_2$ and $a_3$ are obtained. Furthermore, several consequences and connections to some of the earlier known results also are given.

**Keywords** Fekete-Szegö problem; analytic function; bi-univalent function; Gaussian hypergeometric function; Hohlov operator

**MR(2010) Subject Classification** 30C45; 30C50; 30C55

## 1. Introduction

For the set $\mathbb{C}$ of complex numbers, let $\mathcal{A}$ be the class of normalized analytic function $f(z)$ by

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n \tag{1.1}$$

in the open unit disk $\Delta=\{z\in\mathbb{C}:|z|<1\}$.

Let $\mathcal{S}$ denote the subclass of $\mathcal{A}$ consisting of all functions which are univalent in $\Delta$. Due to the Koebe one quarter theorem [1], the inverse $f^{-1}$ of $f\in\mathcal{S}$ satisfies

$$f^{-1}(f(z))=z,\quad z\in\Delta$$

and

$$f(f^{-1}(w))=w,\quad w\in\Delta_\rho,$$

where $\rho\in[\frac{1}{4},1]$ denotes the radius of the image $f(\Delta)$ and $\Delta_\rho=\{z\in\mathbb{C}:|z|<\rho\}$. It is well known that

$$f^{-1}(w)=w-a_2w^2+(2a_2^2-a_3)w^3-(5a_2^3-5a_2a_3+a_4)w^4+\cdots. \tag{1.2}$$

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If the function $f \in \mathcal{A}$ and its inverse $f^{-1}$ are univalent in $\Delta$, then it is bi-univalent. Denote by $\Sigma$ the class of all bi-univalent functions $f \in \mathcal{A}$ in $\Delta$.

Given two analytic functions $f$ and $g$, if there exists an analytic $w$ with $w(0) = 0$ and $|w(z)| < 1$ for $z \in \Delta$ so that $f(z) = g(w(z))$, then $f$ is subordinate to $g$, i.e., $f \prec g$.

For given $f, g \in \mathcal{A}$, define the Hadamard product or convolution $f * g$ by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \quad (z \in \Delta), \tag{1.3}$$

where $f(z)$ is given by Eq. (1.1) and $g(z) = z + \sum_{k=2} b_k z^k$, and the Gaussian hypergeometric function ${}_2F_1(a, b, c; z)$ by

$${}_2F_1(a, b, c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} = 1 + \sum_{n=2}^{\infty} \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}} \frac{z^{n-1}}{(n-1)!}, \quad z \in \Delta \tag{1.4}$$

for the complex parameters $a, b$ and $c$ with $c \neq 0, -1, -2, -3, \ldots$, where $(\ell)_n$ denotes the Pochhammer symbol or shifted factorial by

$$(\ell)_n = \frac{\Gamma(\ell + n)}{\Gamma(\ell)} = \begin{cases} 1, & \text{if } n = 0, \ell \in \mathbb{C} \setminus \{0\} \\ \ell(\ell+1)(\ell+2)\cdots(\ell+n-1), & \text{if } n \in \mathbb{N} = \{1, 2, 3, \ldots\}. \end{cases}$$

Hohlov [2,3] ever considered the convolution operator $\mathcal{I}_c^{a,b}$ later named by himself as follows:

$$\mathcal{I}_c^{a,b} f(z) = z {}_2F_1(a, b, c; z) * f(z) = z + \sum_{n=2}^{\infty} p_n a_n z^n, \quad z \in \Delta, \tag{1.5}$$

where

$$p_n = \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}. \tag{1.6}$$

Here, we also note that there exist some reduced versions of Hohlov operator $\mathcal{I}_c^{a,b}$ for suitable parameters $a, b$ and $c$, for example, Carlson-Shaffer operator $\mathcal{L}(a, c) = \mathcal{I}_c^{a,1}$ (see [4]), Ruscheweyh derivative operator $\mathcal{D}^\delta = \mathcal{I}_1^{1+\delta,1}$ $(-1 < \delta)$ (see [5]), Owa-Srivastava fractional differential operator $\Omega_z^\lambda = \mathcal{I}_{2-\lambda}^{2,1}$ $(0 \leq \lambda < 1)$ (see [6,7]), Choi-Saigo-Srivastava operator $\mathcal{I}_{\lambda,\mu} = \mathcal{I}_{\lambda+1}^{\mu,1}$ $(-1 < \lambda, 0 \leq \mu)$ (see [8]), Noor integral operator $\mathcal{I}_n = \mathcal{I}_{n+1}^{2,1}$ (see [9]).

In 1967, Lewin [10] introduced the analytic and bi-univalent function and proved that $|a_2| < 1.51$. Moreover, Brannan and Clunie [11] conjectured that $|a_2| \leq \sqrt{2}$, and Netanyahu [12] obtained that $\max_{f \in \sum} |a_2| = \frac{4}{3}$. Later, Styer and Wright [13] showed that there exists function $f(z)$ so that $|a_2| > \frac{4}{3}$. However, so far the upper bound estimate $|a_2| < 1.485$ of coefficient for functions in $\sum$ by Tan [14] is best. Unfortunately, as for the coefficient estimate problem for every Taylor-Maclaurin coefficient $|a_n| (n \in \mathbb{N} \setminus \{1, 2\})$ it is probably still an open problem.

For the work of Brannan and Taha [15] and Srivastava et al. [16], a great deal of subclasses of analytic and bi-univalent functions class $\sum$ were introduced and investigated, and the non-sharp estimates of first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ were given; refer to Deniz [17], Frasin and Aouf [18], Hayami and Owa [19], Li and Wang [20], Ma and Minda [21], Magesh and Yamini [22], Patil and Naik [23,24], Srivastava et al. [25,26], Tang et al. [27,28] and Xu et al. [29,30] for more detailed information. Recently, Srivastava et al. [31,32] gave some new subclasses of the function class $\sum$ of analytic and bi-univalent functions to unify the work of Deniz [17], Frasin [33], Srivastava et al. [34], Srivastava et al. [35], Keerthi and Raja [36] and Xu et al. [29], etc.

Since Fekete-Szegö [37] considered the determination of the sharp upper bounds for the subclass of $\mathcal{S}$, Fekete-Szegö functional problem was studied in many classes of functions; refer to Orhan and Răducanu [38] for class of starlike functions, Abdel-Gawad [39] for class of quasi-convex functions, Magesh and Balaji [40] for class of convex and starlike functions, Koepf [41] for class of close-to-convex functions, Tang et al. [28] for classes of $m$-mold symmetric bi-univalent functions, Panigrahi and Raina [42] for class of quasi-subordination functions.

Besides, Murugusundaramoorthy et al. [35,43,44] and Patil and Naik [45] ever introduced and investigated several new subclasses of the function class $\sum$ of analytic and bi-univalent functions associated with the Hohlov operator. Stimulated by the statements above, in the paper we will introduce and investigate the new subclasses of the function class $\sum$ of analytic and bi-univalent functions involving the Hohlov operator, and consider the corresponding bound estimates of the coefficients $a_2$ and $a_3$ and Fekete-Szegö functional inequalities. Moreover, several consequences and connections to some of the earlier known results also will be given.

Now we will introduce the following general subclasses of bi-univalent functions.

**Definition 1.1** *A function $f(z) \in \sum$ given by (1.1), belongs to the class $\mathcal{N}_{\sum}^{a,b,c}(\mu, \lambda; \phi)$ if the following subordinations are satisfied:*

$$(1-\lambda)(\frac{\mathcal{I}_c^{a,b} f(z)}{z})^{\mu} + \lambda(\mathcal{I}_c^{a,b} f)'(z)(\frac{\mathcal{I}_c^{a,b} f(z)}{z})^{\mu-1} \prec \phi(z) \tag{1.7}$$

*and*

$$(1-\lambda)(\frac{\mathcal{I}_c^{a,b} g(w)}{w})^{\mu} + \lambda(\mathcal{I}_c^{a,b} g)'(w)(\frac{\mathcal{I}_c^{a,b} g(w)}{w})^{\mu-1} \prec \phi(w) \tag{1.8}$$

*for $z, w \in \Delta$, where $\mu, \lambda \in [0, \infty)$ satisfy $\mu^2 + \lambda^2 > 0$ and the function $g$ is the inverse of $f$ given by (1.2).*

**Definition 1.2** *A function $f(z) \in \sum$ given by (1.1), belongs to the class $\mathcal{M}_{\sum}^{a,b,c}(\lambda; \phi)$ if the following subordinations are satisfied:*

$$(1-\lambda)\frac{z(\mathcal{I}_c^{a,b} f)'(z)}{\mathcal{I}_c^{a,b} f(z)} + \lambda(1 + \frac{z(\mathcal{I}_c^{a,b} f)''(z)}{(\mathcal{I}_c^{a,b} f)'(z)}) \prec \phi(z) \tag{1.9}$$

*and*

$$(1-\lambda)\frac{w(\mathcal{I}_c^{a,b} g)'(w)}{\mathcal{I}_c^{a,b} g(w)} + \lambda(1 + \frac{w(\mathcal{I}_c^{a,b} g)''(w)}{(\mathcal{I}_c^{a,b} g)'(w)}) \prec \phi(w) \tag{1.10}$$

*for $z, w \in \Delta$, where $0 \le \lambda \le 1$ and the function $g$ is the inverse of $f$ given by (1.2).*

**Remark 1.3** Let

$$\phi(z) = (\frac{1+z}{1-z})^{\alpha} \text{ for } 0 < \alpha \le 1 \tag{1.11}$$

or

$$\phi(z)=\frac{1+(1-2\beta)z}{1-z} \quad \text{for } 0\leq\beta<1 \tag{1.12}$$

in Definitions 1.1 and 1.2. The class $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$ (resp., $\mathcal{M}_{\sum}^{a,b,c}(\lambda;\phi)$) reduces to $\widetilde{\mathcal{N}}_{\sum}^{a,b,c}(\mu,\lambda;\alpha)$ (resp., $\widetilde{\mathcal{M}}_{\sum}^{a,b,c}(\lambda;\alpha)$) or $\widetilde{\mathcal{N}}_{\sum}^{a,b,c}(\mu,\lambda;\beta)$ (resp., $\widetilde{\mathcal{M}}_{\sum}^{a,b,c}(\lambda;\beta)$). Further, if $a=c$ and $b=1$, then the classes $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$ and $\mathcal{M}_{\sum}^{a,b,c}(\lambda;\phi)$ are just $\mathcal{N}_{\sum}^{a,1,a}(\mu,\lambda;\phi)=\mathcal{N}_{\sum}(\mu,\lambda;\phi)$ and $\mathcal{M}_{\sum}^{a,1,a}(\lambda;\phi)=\mathcal{M}_{\sum}(\lambda;\phi)$, respectively; refer to Tang et al. [27] and Ali et al. [46].

**Lemma 1.4** ([1,47]) *Let $\mathcal{P}$ be the class of all analytic functions $h(z)$ of the following form*

$$h(z)=1+\sum_{n=1}^{\infty}c_nz^n,\quad z\in\Delta$$

*satisfying $\Re h(z)>0$ and $h(0)=1$. Then the sharp estimates $|c_n|\leq 2$ $(n\in\mathbb{N})$. Particularly, the equality holds for all $n$ for the next function*

$$h(z)=\frac{1+z}{1-z}=1+\sum_{n=1}^{\infty}2z^n.$$

## 2. Coefficient estimates for the class $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$

Define the functions $s$ and $t$ in $\mathcal{P}$ by

$$s(z)=\frac{1+u(z)}{1-u(z)}=1+\sum_{n=1}^{\infty}c_nz^n,\quad t(w)=\frac{1+v(w)}{1-v(w)}=1+\sum_{n=1}^{\infty}d_nw^n,\quad z,w\in\Delta. \tag{2.1}$$

Therefore, from (2.1) we infer that

$$u(z)=\frac{s(z)-1}{s(z)+1}=\frac{c_1}{2}z+\frac{1}{2}(c_2-\frac{c_1^2}{2})z^2+\cdots,\quad z\in\Delta \tag{2.2}$$

and

$$v(w)=\frac{t(w)-1}{t(w)+1}=\frac{d_1}{2}w+\frac{1}{2}(d_2-\frac{d_1^2}{2})w^2+\cdots,\quad w\in\Delta. \tag{2.3}$$

Let $\phi\in\mathcal{P}$ with $\phi'(0)>0$ satisfying $\phi(\Delta)$ being symmetric with respect to the real axis. Assume that the series expansion form of $\phi$ is denoted by

$$\phi(z)=1+\sum_{n=1}^{\infty}E_nz^n,\quad E_1>0,\ z\in\Delta. \tag{2.4}$$

By (2.2–2.4), it follows that

$$\phi(u(z))=1+\frac{1}{2}E_1c_1z+[\frac{1}{2}E_1(c_2-\frac{c_1^2}{2})+\frac{1}{4}E_2c_1^2]z^2+\cdots,\quad z\in\Delta \tag{2.5}$$

and

$$\phi(v(w))=1+\frac{1}{2}E_1d_1w+[\frac{1}{2}E_1(d_2-\frac{d_1^2}{2})+\frac{1}{4}E_2d_1^2]w^2+\cdots,\quad w\in\Delta. \tag{2.6}$$

Now we consider the coefficient estimates for the class $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$ and establish the next theorem.

**Theorem 2.1** *If $f(z)$ given by (1.1) belongs to the class $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$, then*

$$|a_2| \le \min\{\frac{E_1}{(\lambda+\mu)|p_2|}, \sqrt{\frac{E_1+|E_2-E_1|}{(2\lambda+\mu)|\frac{1}{2}(\mu-1)p_2^2+p_3|}}, \frac{E_1^{3/2}}{\sqrt{|\Phi|}}\} \tag{2.7}$$

*and*

$$|a_3| \le \frac{E_1}{(2\lambda+\mu)|p_3|} + \min\{\frac{E_1^2}{(\lambda+\mu)^2p_2^2}, \frac{2(E_1+|E_2-E_1|)}{(2\lambda+\mu)|(\mu-1)p_2^2+2p_3|}\}, \tag{2.8}$$

*where*

$$\Phi = \Phi(\lambda,\mu,E_1,E_2,p_2,p_3) = (2\lambda+\mu)[\frac{1}{2}(\mu-1)p_2^2+p_3]E_1^2 + (E_1-E_2)(\lambda+\mu)^2p_2^2. \tag{2.9}$$

**Proof** Assume that $f(z) \in \mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$. Hence, by Definition 1.1 there exist two analytic functions $u(z), v(z) : \Delta \to \Delta$ with $u(0) = 0$ and $v(0) = 0$ so that

$$(1-\lambda)(\frac{\mathcal{I}_c^{a,b}f(z)}{z})^\mu + \lambda(\mathcal{I}_c^{a,b}f)'(z)(\frac{\mathcal{I}_c^{a,b}f(z)}{z})^{\mu-1} = \phi(u(z)) \tag{2.10}$$

and

$$(1-\lambda)(\frac{\mathcal{I}_c^{a,b}g(w)}{w})^\mu + \lambda(\mathcal{I}_c^{a,b}g)'(w)(\frac{\mathcal{I}_c^{a,b}g(w)}{w})^{\mu-1} = \phi(v(w)). \tag{2.11}$$

Expanding the left half parts of (2.10) and (2.11), we have that

$$\begin{aligned}&(1-\lambda)(\frac{\mathcal{I}_c^{a,b}f(z)}{z})^\mu + \lambda(\mathcal{I}_c^{a,b}f)'(z)(\frac{\mathcal{I}_c^{a,b}f(z)}{z})^{\mu-1}\\ &= 1 + (\lambda+\mu)p_2a_2z + (2\lambda+\mu)[\frac{1}{2}(\mu-1)p_2^2a_2^2 + p_3a_3]z^2 + \cdots\end{aligned} \tag{2.12}$$

and

$$\begin{aligned}&(1-\lambda)(\frac{\mathcal{I}_c^{a,b}g(w)}{w})^\mu + \lambda(\mathcal{I}_c^{a,b}g)'(w)(\frac{\mathcal{I}_c^{a,b}g(w)}{w})^{\mu-1}\\ &= 1 - (\lambda+\mu)p_2a_2w + (2\lambda+\mu)\{\frac{1}{2}[(\mu-1)p_2^2+4p_3]a_2^2 - p_3a_3\}w^2 + \cdots.\end{aligned} \tag{2.13}$$

Obviously, from (2.5), (2.6) and (2.10)–(2.13), we obtain that

$$(\lambda+\mu)p_2a_2 = \frac{E_1c_1}{2}, \tag{2.14}$$

$$(2\lambda+\mu)[\frac{1}{2}(\mu-1)p_2^2a_2^2 + p_3a_3] = \frac{1}{2}(c_2 - \frac{c_1^2}{2})E_1 + \frac{1}{4}c_1^2E_2, \tag{2.15}$$

$$-(\lambda+\mu)p_2a_2 = \frac{E_1d_1}{2} \tag{2.16}$$

and

$$(2\lambda+\mu)\{\frac{1}{2}[(\mu-1)p_2^2+4p_3]a_2^2 - p_3a_3\} = \frac{1}{2}(d_2 - \frac{d_1^2}{2})E_1 + \frac{1}{4}d_1^2E_2. \tag{2.17}$$

From (2.14) and (2.16), we know that

$$a_2 = \frac{E_1c_1}{2p_2(\lambda+\mu)} = -\frac{E_1d_1}{2p_2(\lambda+\mu)}, \tag{2.18}$$

which derives

$$c_1 = -d_1 \tag{2.19}$$

and

$$E_1^2(c_1^2 + d_1^2) = 8(\lambda + \mu)^2 p_2^2 a_2^2. \tag{2.20}$$

By (2.15) and (2.17), we have that

$$c_1^2(E_2 - E_1) + E_1(c_2 + d_2) = 2(2\lambda + \mu)[(\mu - 1)p_2^2 + 2p_3]a_2^2. \tag{2.21}$$

Therefore, from (2.19)–(2.21) we obtain

$$a_2^2 = \frac{(c_2 + d_2)E_1^3}{2(2\lambda + \mu)[(\mu - 1)p_2^2 + 2p_3]E_1^2 + 4(E_1 - E_2)(\lambda + \mu)^2 p_2^2}. \tag{2.22}$$

Hence, by Lemma 1.4 we may remark that

$$|a_2| \leq \frac{E_1^{3/2}}{\sqrt{|\Phi|}}.$$

In addition, from (2.20) and (2.21) we get that

$$|a_2| \leq \frac{E_1}{(\lambda + \mu)|p_2|}$$

and

$$|a_2| \leq \sqrt{\frac{E_1 + |E_2 - E_1|}{(2\lambda + \mu)|\frac{1}{2}(\mu - 1)p_2^2 + p_3|}},$$

which yield the desired results on $|a_2|$ in (2.7).

Similarly, (2.15) and (2.17) imply that

$$E_1(c_2 - d_2) = 4(2\lambda + \mu)p_3(a_3 - a_2^2). \tag{2.23}$$

Then, by (2.19), (2.20) and (2.23), it follows that

$$a_3 = \frac{E_1(c_2 - d_2)}{4(2\lambda + \mu)p_3} + \frac{E_1^2(c_1^2 + d_1^2)}{8(\lambda + \mu)^2 p_2^2}.$$

So, we obtain from Lemma 1.4 that

$$|a_3| \leq \frac{E_1}{(2\lambda + \mu)|p_3|} + \frac{E_1^2}{(\lambda + \mu)^2 p_2^2}.$$

On the other hand, by (2.21) and (2.23) we infer that

$$a_3 = \frac{2[c_1^2(E_2 - E_1) + E_1(c_2 + d_2)]p_3 + E_1(c_2 - d_2)[(\mu - 1)p_2^2 + 2p_3]}{4(2\lambda + \mu)p_3[(\mu - 1)p_2^2 + 2p_3]}.$$

Thus, from Lemma 1.4 we see that

$$|a_3| \leq \frac{E_1}{(2\lambda + \mu)|p_3|} + \frac{2(E_1 + |E_2 - E_1|)}{(2\lambda + \mu)|(\mu - 1)p_2^2 + 2p_3|}. \quad \square$$

When $\mu = 1$, $\mathcal{N}_{\Sigma}^{a,b,c}(1, \lambda; \phi) = \mathcal{N}_{\Sigma}^{a,b,c}(\lambda; \phi)$. Hence, by Theorem 2.1 we immediately get the next corollary.

**Corollary 2.2** *If $f(z)$ given by (1.1) belongs to the class $\mathcal{N}_{\sum}^{a,b,c}(\lambda;\phi)$, then*

$$|a_2| \leq \min\{\frac{E_1}{(1+\lambda)|p_2|}, \sqrt{\frac{E_1+|E_2-E_1|}{(1+2\lambda)|p_3|}}, \frac{E_1^{3/2}}{\sqrt{|\Phi|}}\}$$

*and*

$$|a_3| \leq \frac{E_1}{(1+2\lambda)|p_3|} + \min\{\frac{E_1^2}{(1+\lambda)^2 p_2^2}, \frac{(E_1+|E_2-E_1|)}{(1+2\lambda)|p_3|}\},$$

*where*

$$\Phi = \Phi(\lambda, E_1, E_2, p_2, p_3) = (1+2\lambda)E_1^2 p_3 + (E_1-E_2)(1+\lambda)^2 p_2^2.$$

**Remark 2.3** Moreover, under the conditions of the parameters $a = c$ and $b = 1$ and Remark 1.3, if we choose some suitable parameters $\mu$ and $\lambda$ as well as $\phi$, we also provide the following reduced versions for $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$ in Theorem 2.1:

(i) $\mathcal{N}_{\sum}^{a,1,a}(\mu,\lambda;\alpha) = \mathcal{H}_{\sigma}^{\mu}(\lambda,\alpha)$, $\mathcal{N}_{\sum}^{a,1,a}(\mu,\lambda;\beta) = \mathcal{H}_{\sum}^{\mu}(\lambda,\beta)$, refer to Cağler et al. [48];

(ii) $\mathcal{N}_{\sum}^{a,1,a}(1,\lambda;\phi) = \mathcal{H}_{\sigma}(\lambda,\phi)$, $\mathcal{N}_{\sum}^{a,1,a}(\mu,1;\phi) = \mathcal{H}_{\sigma}^{\mu}(\phi)$, refer to Kumar et al. [49];

(iii) $\mathcal{N}_{\sum}^{a,1,a}(1,1;\phi) = \mathcal{H}_{\sigma}(\phi)$, refer to Ali et al. [46];

(iv) $\mathcal{N}_{\sum}^{a,1,a}(1,\lambda;\alpha) = \mathcal{B}_{\sum}(\alpha,\lambda)$, $\mathcal{N}_{\sum}^{a,1,a}(1,\lambda;\beta) = \mathcal{B}_{\sum}(\beta,\lambda)$, refer to Frasin and Aouf [18];

(v) $\mathcal{N}_{\sum}^{a,1,a}(1,1;\alpha) = \mathcal{B}_{\sum}(\alpha)$, $\mathcal{N}_{\sum}^{a,1,a}(1,1;\beta) = \mathcal{B}_{\sum}(\beta)$, refer to Srivastava et al. [16].

Next, we will consider Fekete-Szegö functional problems for the class $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$.

**Corollary 2.4** *If $f(z)$ given by (1.1) belongs to the class $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$ and $\rho \in \mathbb{R}$, then*

$$|a_3 - \rho a_2^2| \leq \begin{cases} \frac{E_1}{(2\lambda+\mu)|p_3|}, & \text{if } \ (2\lambda+\mu)|(1-\rho)p_3|E_1^2 \leq |\Phi| \\ \frac{|1-\rho|E_1^3}{|\Phi|}, & \text{if } \ (2\lambda+\mu)|(1-\rho)p_3|E_1^2 \geq |\Phi|, \end{cases} \tag{2.24}$$

*where $\Phi = \Phi(\lambda,\mu,E_1,E_2,p_2,p_3)$ is the same as in Theorem 2.1.*

**Proof** From (2.23), it follows that

$$a_3 - a_2^2 = \frac{E_1(c_2-d_2)}{4(2\lambda+\mu)p_3}.$$

By (2.22) we easily obtain that

$$a_3 - \rho a_2^2 = \frac{E_1\{[(1-\rho)(2\lambda+\mu)p_3E_1^2+\Phi]c_2 + [(1-\rho)(2\lambda+\mu)p_3E_1^2-\Phi]d_2\}}{4(2\lambda+\mu)p_3\Phi}.$$

Hence, from Lemma 1.4 it follows

$$|a_3 - \rho a_2^2| \leq \begin{cases} \frac{E_1}{(2\lambda+\mu)|p_3|}, & \text{if } \ (2\lambda+\mu)|(1-\rho)p_3|E_1^2 \leq |\Phi|; \\ \frac{|1-\rho|E_1^3}{|\Phi|}, & \text{if } \ (2\lambda+\mu)|(1-\rho)p_3|E_1^2 \geq |\Phi|. \end{cases} \quad \square$$

**Corollary 2.5** *If $f(z)$ given by (1.1) belongs to the class $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$, then*

$$|a_3 - a_2^2| \leq \frac{E_1}{(2\lambda+\mu)|p_3|}.$$

**Corollary 2.6** *If $f(z)$ given by (1.1) belongs to the class $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$, then*

$$|a_3| \leq \begin{cases} \frac{E_1}{(2\lambda+\mu)|p_3|}, & \textit{if } \ (2\lambda+\mu)|p_3|E_1^2 \leq |\Phi|; \\ \frac{E_1^3}{|\Phi|}, & \textit{if } \ (2\lambda+\mu)|p_3|E_1^2 \geq |\Phi|, \end{cases}$$

*where* $\Phi = \Phi(\lambda, \mu, E_1, E_2, p_2, p_3)$ *is the same as in Theorem 2.1.*

**Remark 2.7** Without Hohlov operator, we may refer to the subclass $\mathcal{B}_{\sum,m}(\lambda;\phi)$ of $m$-fold symmetric bi-univalent functions (see Tang et al. [28] for $m = 1$) for Fekete-Szegö functional problems about $\mathcal{N}_{\sum}^{a,b,c}(\mu,\lambda;\phi)$.

## 3. Coefficient estimates for the class $\mathcal{M}_{\sum}^{a,b,c}(\lambda;\phi)$

Now we study the coefficient estimates for the class $\mathcal{M}_{\sum}^{a,b,c}(\lambda;\phi)$ and give the next theorem.

**Theorem 3.1** *If $f(z)$ given by (1.1) belongs to the class $\mathcal{M}_{\sum}^{a,b,c}(\lambda;\phi)$, then*

$$|a_2| \le \min\Big\{\frac{E_1}{(1+\lambda)|p_2|}, \sqrt{\frac{E_1+|E_2-E_1|}{|2(1+2\lambda)p_3-(1+3\lambda)p_2^2|}}, \frac{E_1^{3/2}}{\sqrt{|\Theta|}}\Big\} \tag{3.1}$$

*and*

$$|a_3| \le \frac{E_1}{2(1+2\lambda)|p_3|} + \min\Big\{\frac{E_1^2}{(1+\lambda)^2p_2^2}, \frac{E_1+|E_2-E_1|}{|2(1+2\lambda)p_3-(1+3\lambda)p_2^2|}\Big\}, \tag{3.2}$$

*where*

$$\Theta = \Theta(\lambda, E_1, E_2, p_2, p_3) = [2(1+2\lambda)p_3-(1+3\lambda)p_2^2]E_1^2 + (E_1-E_2)(1+\lambda)^2p_2^2. \tag{3.3}$$

**Proof** Assume that $f(z) \in \mathcal{M}_{\sum}^{a,b,c}(\lambda;\phi)$. Then, by Definition 1.2 there exist two analytic functions $u(z), v(z) : \Delta \to \Delta$ with $u(0) = 0$ and $v(0) = 0$ so that

$$(1-\lambda)\frac{z(\mathcal{I}_c^{a,b}f)'(z)}{\mathcal{I}_c^{a,b}f(z)} + \lambda(1+\frac{z(\mathcal{I}_c^{a,b}f)''(z)}{(\mathcal{I}_c^{a,b}f)'(z)}) = \phi(u(z)) \tag{3.4}$$

and

$$(1-\lambda)\frac{w(\mathcal{I}_c^{a,b}g)'(w)}{\mathcal{I}_c^{a,b}g(w)} + \lambda(1+\frac{w(\mathcal{I}_c^{a,b}g)''(w)}{(\mathcal{I}_c^{a,b}g)'(w)}) = \phi(v(w)). \tag{3.5}$$

Expanding the left half parts of (3.4) and (3.5), we obtain that

$$\begin{aligned}&(1-\lambda)\frac{z(\mathcal{I}_c^{a,b}f)'(z)}{\mathcal{I}_c^{a,b}f(z)} + \lambda(1+\frac{z(\mathcal{I}_c^{a,b}f)''(z)}{(\mathcal{I}_c^{a,b}f)'(z)})\\ &= 1+(1+\lambda)p_2a_2z + [2(1+2\lambda)p_3a_3-(1+3\lambda)p_2^2a_2^2]z^2+\cdots\end{aligned} \tag{3.6}$$

and

$$\begin{aligned}&(1-\lambda)\frac{w(\mathcal{I}_c^{a,b}g)'(w)}{\mathcal{I}_c^{a,b}g(w)} + \lambda(1+\frac{w(\mathcal{I}_c^{a,b}g)''(w)}{(\mathcal{I}_c^{a,b}g)'(w)})\\ &= 1-(1+\lambda)p_2a_2w + [2(1+2\lambda)p_3(2a_2^2-a_3)-(1+3\lambda)p_2^2a_2^2]w^2+\cdots.\end{aligned} \tag{3.7}$$

Hence, with (2.5), (2.6) and (3.4)–(3.7), we deduce that

$$(1+\lambda)p_2a_2 = \frac{E_1c_1}{2}, \tag{3.8}$$

$$2(1+2\lambda)p_3a_3-(1+3\lambda)p_2^2a_2^2 = \frac{1}{2}(c_2-\frac{c_1^2}{2})E_1 + \frac{1}{4}c_1^2E_2, \tag{3.9}$$

$$-(1+\lambda)p_2a_2 = \frac{E_1d_1}{2} \tag{3.10}$$

and

$$2(1+2\lambda)p_3(2a_2^2-a_3)-(1+3\lambda)p_2^2a_2^2 = \frac{1}{2}(d_2-\frac{d_1^2}{2})E_1+\frac{1}{4}d_1^2E_2. \tag{3.11}$$

From (3.8) and (3.10), we know that

$$a_2 = \frac{E_1c_1}{2p_2(1+\lambda)} = -\frac{E_1d_1}{2p_2(1+\lambda)}, \tag{3.12}$$

which implies

$$c_1 = -d_1 \tag{3.13}$$

and

$$E_1^2(c_1^2+d_1^2) = 8(1+\lambda)^2p_2^2a_2^2. \tag{3.14}$$

By (3.9),(3.11) and (3.13), we have that

$$c_1^2(E_2-E_1)+E_1(c_2+d_2) = 8(1+2\lambda)p_3a_2^2-4(1+3\lambda)p_2^2a_2^2. \tag{3.15}$$

Therefore, from (3.12)–(3.15) we obtain

$$a_2^2 = \frac{(c_2+d_2)E_1^3}{[8(1+2\lambda)p_3-4(1+3\lambda)p_2^2]E_1^2+4(E_1-E_2)(1+\lambda)^2p_2^2}. \tag{3.16}$$

Hence, by Lemma 1.4 we derive

$$|a_2| \leq \frac{E_1^{3/2}}{\sqrt{|[2(1+2\lambda)p_3-(1+3\lambda)p_2^2]E_1^2+(E_1-E_2)(1+\lambda)^2p_2^2|}}.$$

In addition, from (3.14) and (3.15) we get that

$$|a_2| \leq \frac{E_1}{(1+\lambda)|p_2|}$$

and

$$|a_2| \leq \sqrt{\frac{E_1+|E_2-E_1|}{|2(1+2\lambda)p_3-(1+3\lambda)p_2^2|}},$$

which yield the desired results on $|a_2|$ in (3.1).

Similarly, from (3.9) and (3.11), it follows

$$E_1(c_2-d_2) = 8(1+2\lambda)p_3(a_3-a_2^2). \tag{3.17}$$

Then, by (3.13), (3.14) and (3.17), one gets

$$a_3 = \frac{E_1(c_2-d_2)}{8(1+2\lambda)p_3}+\frac{E_1^2(c_1^2+d_1^2)}{8(1+\lambda)^2p_2^2}.$$

So, we obtain from Lemma 1.4 that

$$|a_3| \leq \frac{E_1}{2(1+2\lambda)|p_3|}+\frac{E_1^2}{(1+\lambda)^2p_2^2}.$$

On the other hand, by (3.15) and (3.17) we infer that

$$a_3 = \frac{E_1(c_2 - d_2)}{8(1+2\lambda)p_3} + \frac{c_1^2(E_2 - E_1) + E_1(c_2 + d_2)}{8(1+2\lambda)^2 p_3 - 4(1+3\lambda)p_2^2}.$$

Thus, from Lemma 1.4 we see that

$$|a_3| \leq \frac{E_1}{2(1+2\lambda)|p_3|} + \frac{E_1 + |E_2 - E_1|}{|2(1+2\lambda)p_3 - (1+3\lambda)p_2^2|}. \quad \square$$

**Remark 3.2** Clearly, under the conditions of the parameters $a = c$ and $b = 1$ and Remark 1.3, if we take some suitable parameter $\lambda$ and $\phi$, we also provide the following reduced versions for $\mathcal{M}_{\sum}^{a,b,c}(\mu, \lambda; \phi)$ in Theorem 3.1:

(i) $\mathcal{M}_{\sum}^{a,1,a}(1; \phi) = \mathcal{M}_{\sum}(\phi)$, refer to Ali et al. [46];

(ii) $\mathcal{M}_{\sum}^{a,1,a}(1; \alpha)$, $\mathcal{M}_{\sum}^{a,1,a}(1; \beta)$, or $\mathcal{M}_{\sum}^{a,1,a}(0; \alpha)$ and $\mathcal{M}_{\sum}^{a,1,a}(0; \beta)$, refer to Brannan and Taha [15].

**Theorem 3.3** *If $f(z)$ given by (1.1) belongs to the class $\mathcal{M}_{\sum}^{a,b,c}(\lambda; \phi)$ and $\rho \in \mathbb{R}$, then*

$$|a_3 - \rho a_2^2| \leq \begin{cases} \frac{E_1}{4(1+2\lambda)|p_3|}, & \textit{if } 2(1+2\lambda)|(1-\rho)p_3|E_1^2 \leq |\Theta|; \\ \frac{|1-\rho|E_1^3}{2|\Theta|}, & \textit{if } 2(1+2\lambda)|(1-\rho)p_3|E_1^2 \leq |\Theta|, \end{cases} \tag{3.18}$$

*where $\Theta = \Theta(\lambda, E_1, E_2, p_2, p_3)$ is the same as in Theorem 3.1.*

**Proof** From (3.17), it follows that

$$a_3 - a_2^2 = \frac{E_1(c_2 - d_2)}{8(1+2\lambda)p_3}.$$

By (3.16) we easily obtain that

$$a_3 - \rho a_2^2 = \frac{E_1\{[2(1-\rho)(1+2\lambda)p_3E_1^2 + \Theta]c_2 + [2(1-\rho)(1+2\lambda)p_3E_1^2 - \Theta]d_2\}}{8(1+2\lambda)p_3\Theta}. \tag{3.19}$$

Then, from Lemma 1.4 we show that

$$|a_3 - \rho a_2^2| \leq \begin{cases} \frac{E_1}{4(1+2\lambda)|p_3|}, & \text{if } 2(1+2\lambda)|(1-\rho)p_3|E_1^2 \leq |\Theta|; \\ \frac{|1-\rho|E_1^3}{2|\Theta|}, & \text{if } 2(1+2\lambda)|(1-\rho)p_3|E_1^2 \geq |\Theta|. \end{cases} \quad \square$$

**Corollary 3.4** *If $f(z)$ given by (1.1) belongs to the class $\mathcal{M}_{\sum}^{a,b,c}(\lambda; \phi)$, then*

$$|a_3 - a_2^2| \leq \frac{E_1}{4(1+2\lambda)|p_3|}.$$

**Corollary 3.5** *If $f(z)$ given by (1.1) belongs to the class $\mathcal{M}_{\sum}^{a,b,c}(\lambda; \phi)$, then*

$$|a_3| \leq \begin{cases} \frac{E_1}{4(1+2\lambda)|p_3|}, & \textit{if } 2(1+2\lambda)|p_3|E_1^2 \leq |\Theta|; \\ \frac{E_1^3}{2|\Theta|}, & \textit{if } 2(1+2\lambda)|p_3|E_1^2 \geq |\Theta| \end{cases}$$

*where $\Theta = \Theta(\lambda, E_1, E_2, p_2, p_3)$ is the same as in Theorem 3.3.*

**Remark 3.6** Similarly, without Hohlov operator we may refer to the subclass $\mathcal{M}_{\sum,m}(\lambda; \phi)$ of $m$-fold symmetric bi-univalent functions (see Tang et al. [28] for $m = 1$) for Fekete-Szegö functional problems about $\mathcal{M}_{\sum}^{a,b,c}(\lambda; \phi)$.

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