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Central Reflexive Rings with an Involution

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Abstract We study the central reflexive properties of rings with an involution. The concept of central $*$-reflexive rings is introduced and investigated. It is shown that central $*$-reflexive rings are a generalization of reflexive rings, central reflexive rings and $*$-reflexive rings. Some characterizations of this class of rings are given. The related ring extensions including trivial extension, Dorroh extension and polynomial extensions are also studied.

Keywords $*$-reflexive rings; central $*$-reflexive rings; central $*$-semicommutative ring; Dorroh extension

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1. Introduction

Throughout this paper, R denotes an associative ring with identity. We denote by $C(R)$ the centre of R and the ring of integers is denoted by $\mathbb{Z}$. Reversible rings were defined by Cohn [1] in 1999. He showed that the Kothe conjecture is ture for the reversible rings. According to [2], a ring R is reflexive if $aRb = 0$ implies $bRa = 0$ for all $a, b \in R$. It is clear that every reversible ring is reflexive. An involution $*$ of a ring R is an anti-isomorphism such that $(a+b)^* = a^* + b^*$, $(ab)^* = b^*a^*$ and $(a^*)^* = a$ for all $a, b \in R$. A ring R with the involution $*$ is called a $*$-ring. A ring R is said to be a $*$-reflexive ring if for all $a, b \in R$, $aRb = 0$ implies $bRa^* = 0$. According to [3], a ring R is central reflexive if for any $a, b \in R$, whenever $aRb = 0$, then $bRa \subseteq C(R)$.

This is a further study of reflexive rings [2–6] and $*$-reflexive rings [3, 5]. We introduce and study the concept of central $*$-reflexive rings, which is a generalization of reflexive rings, central reflexive rings and $*$-reflexive rings. A ring R is said to be a central $*$-reflexive ring if for all $a, b \in R$, $aRb = 0$ implies $bRa^* \subseteq C(R)$. The connection among central $*$-reversible rings, central $*$-reflexive rings and central $*$-semicommutative rings are studied. Furthermore, we obtain the related ring extensions including trivial extension, Dorroh extension and polynomial extensions.

2. Central $*$-reflexive rings

In this section, we define and study central $*$-reflexive rings. Some characterizations of this class of rings are given, including four equivalent conditions and two sufficient conditions. Then

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we discuss the relations among central $*$-reversible rings, central $*$-reflexive rings and central $*$-semicommutative rings.

Definition 2.1 *An involution $*$ of a ring R is said to be central reflexive, if for all $a,b \in R$, $aRb = 0$ implies $bRa^* \subseteq C(R)$. A ring R with a central reflexive involution is said to be a central $*$-reflexive ring.*

Proposition 2.2 *For a ring R, the following statements are equivalent:*

(1) R is a central $*$-reflexive;

(2) $ARB = 0$ implies $BRA^* \subseteq C(R)$ for any nonempty subsets A, B of R;

(3) $IJ = 0$ implies $JI^* \subseteq C(R)$ for all right(left) ideals I, J of R;

(4) $IJ = 0$ implies $JI^* \subseteq C(R)$ for all ideals I, J of R.

Proof (1)$\Rightarrow$(2). Let A, B be nonempty subsets of R such that $ARB = 0$. Then for any $a \in A$, $b \in B$, $aRb = 0$. Since R is central $*$-reflexive, $bRa^* \subseteq C(R)$. Thus $BRA^* = \Sigma_{a\in A, b\in B} bRa^* \subseteq C(R)$.

(2)$\Rightarrow$(3). Let I, J be two right ideals of R such that $IJ = 0$. Since $IR = I$, $IRJ = 0$. By (2), $JI^* = JRI^* \subseteq C(R)$. A similar proof can be given for all left ideals of R.

(3)$\Rightarrow$(4) is straightforward.

(4)$\Rightarrow$(1). Let $aRb = 0$ for $a, b \in R$. Then we have $RaRRbR = 0$, and so $bRa^* \subseteq RbRRa^*R \subseteq C(R)$ by (4). Thus R is central $*$-reflexive. $\square$

Proposition 2.3 *Let R and S be rings and $\tau : R \to S$ be an isomorphism. Then*

(1) $*$ is an involution on R if and only if $\tau(*) := \tau \circ * \circ \tau^{-1}$ is an involution on S.

(2) R is $*$-reflexive if and only if S is $\tau(*)$-reflexive.

(3) R is central $*$-reflexive if and only if S is central $\tau(*)$-reflexive.

Proof (1) First note that $\tau \circ * \circ \tau^{-1} := \tau(*)$ is an anti-automorphism on S of order two and similarly $\tau^{-1} \circ \tau(*) \circ \tau = *$ is also an anti-automorphism on R of order two. It is easy to check that if $*$ is an involution on R, then $\tau(*)$ is an involution on S. Similarly, if $\tau(*)$ is an involution on S, then $*$ is an involution on R.

(2) Assume that R is $*$-reflexive. Then for any $a, b \in S$ satisfying $aSb = 0$, we have $\tau^{-1}(aSb) = \tau^{-1}(a)R\tau^{-1}(b) = 0$. Thus $\tau^{-1}(a)R(\tau^{-1}(b))^* = 0$. It follows that

$$0 = \tau[\tau^{-1}(a)R(\tau^{-1}(b))^*] = aS\tau[(\tau^{-1}(b))^*] = aSb^*.$$

Hence, S is $\tau(*)$-reflexive. The inverse is straightforward.

The proofs of (2) and (3) are analogous. $\square$

An involution $*$ of R is said to be proper (resp., semiproper) if $aa^* = 0$ (resp., $aRa^* = 0$) implies $a = 0$ for all $a \in R$.

Proposition 2.4 *Let R be a $*$-ring. If the involution $*$ is semiproper, then R is central $*$-reflexive.*

Proof Let R be a ring with a semiproper involution $*$. Then for any $a, b, r \in R$ satisfying $aRb = 0$, we have $0 = (ar^*b^*Rb)ra^* = (bra^*)^*Rbra^*$. It follows that $bra^* = 0$ since $*$ is semiproper. Thus $bRa^* = 0$ and $bRa^* \subseteq C(R)$. Therefore, R is central $*$-reflexive. □

For a $*$-ring R, if an ideal I is closed under $*$ (i.e., $I^* = I$), then I is a $*$-ring (possibly without identity). It is clear that $\overline{*} : R/I \to R/I$ defined by $(a + I)^{\overline{*}} = a^* + I$ is an involution of R/I.

Proposition 2.5 *Let R be a $*$-ring and I be an ideal which is closed under $*$. If R/I is $\overline{*}$-reflexive and $*$ is a semiproper involution of I, then R is central $*$-reflexive.*

Proof For any $a, b \in R$ satisfying $aRb = 0$, we have $bRa^* \subseteq I$ since R/I is $\overline{*}$-reflexive. And $aRb = 0$ also implies $(bra^*)^*R(bra^*) = (ar^*b^*Rb)ra^* = 0$ for any $r \in R$. Since $bra^* \in I$ and $*$ is a semiproper involution of I, we have $bra^* = 0$. Thus $bRa^* \subseteq C(R)$, as needed. □

A ring R is called semicommutative if for all $a, b \in R$, $ab = 0$ implies $aRb = 0$. It is clear that a ring R is reversible if and only if R is semicommutative and reflexive.

Definition 2.6 *A $*$-ring R is said to be central $*$-reversible if $ab = 0$ implies $ba^* \in C(R)$ for all $a, b \in R$.*

Definition 2.7 *A $*$-ring R is said to be central $*$-semicommutative if $ab = 0$ implies $bRa^* \subseteq C(R)$ for all $a, b \in R$.*

Proposition 2.8 *Every $*$-reversible is central $*$-reflexive.*

Proof If R is a $*$-reversible ring, then R is symmetric by the proof of [2, Proposition 6]. For any $a, b \in R$ satisfying $aRb = 0$, we have $ab = 0$, and thus $ba^* = 0$. Then we have $bRa^* = 0$ since every symmetric ring is semicommutative. This shows that R is a $*$-reflexive ring. Thus R is central $*$-reflexive. □

Recall that a $*$-ring R is said to be $*$-semicommutative if $ab = 0$ implies $bRa^* = 0$ for all $a, b \in R$. More generally, we give the following

Proposition 2.9 *Every $*$-semicommutative ring is a central $*$-reflexive ring.*

Proof Let R be a $*$-semicommutative ring and let $a, b \in R$ such that $aRb = 0$. Then we have $ab = 0$, and thus $bRa^* = 0$. This implies that $bRa^* \subseteq C(R)$ and thus R is a central $*$-reflexive ring. □

Corollary 2.10 *Every $*$-semicommutative ring is a central $*$-semicommutative ring.*

Proposition 2.11 *Let R be a $*$-ring. Then*

(1) If R is central $*$-reversible, then it is central $*$-reflexive.

(2) If R is central $*$-reflexive and semicommutative, then it is central $*$-reversible.

Proof (1) Let R be a central $*$-reversible ring. For any $a, b \in R$ such that $aRb = 0$, we have $ab = 0$. Then $ba^* \in C(R)$. Therefore, $abr = 0$ for all $r \in R$. By the central $*$-reversible property of R, $bra^* \in C(R)$. Hence, $bRa^* \subseteq C(R)$. Therefore, R is central $*$-reflexive.

(2) Let R be a central $*$-reflexive and semicommutative ring and let $a, b \in R$ such that $ab = 0$. Then we have $aRb = 0$ by the semicommutative property of R. And thus $bRa^* \subseteq C(R)$. Then $ba^* \in C(R)$ since $1 \in R$. □

Corollary 2.12 *Every central $*$-semicommutative ring is a central $*$-reflexive ring.*

Corollary 2.13 *Let R be a semicommutative ring. If R is a central $*$-reflexive ring, then R is central $*$-semicommutative.*

The next example shows that a central $*$-reflexive ring need not be central $*$-semicommutative.

Example 2.14 Let $R = (M_n(\mathbb{C}), M_n(\mathbb{C}))$ and $\overline{*}$: $R \to R$ defined by $(A, B)^{\overline{*}} = (A^*, B^*)$ for any $A, B \in M_n(\mathbb{C})$, where $*$ is the conjugate transpose of matrices. $M_n(\mathbb{C})$ is central $*$-reflexive, then so is R. Now we show that R is not central $*$-semicommutative. Let

$$a = \left(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right) \in R.$$

Then we have $a^2 = 0$. However,

$$aa^{\overline{*}} = \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right) \notin C(R).$$

It follows that $aRa^{\overline{*}} \nsubseteq C(R)$, proving R is not central $*$-semicommutative.

3. Extensions of central $*$-reflexive rings

An element $a \in R$ is called self-adjoint if $a = a^*$. In particular, an idempotent e of a $*$-ring R is called a projection if $e^* = e = e^2$.

Proposition 3.1 *For a $*$-ring R, the following are equivalent:*

(1) R is central $*$-reflexive;

(2) eRe is central $*$-reflexive for any projection $e \in R$;

(3) eR is central $*$-reflexive for any central idempotent $e \in R$.

Proof (1)⇒(2). Assume that R is a central $*$-reflexive ring and there is $e^2 = e = e^* \in R$. For any $eae, ebe \in eRe$ satisfying $0 = eae(eRe)ebe = eaeRebe$, we have

$$ebeR(eae)^* = ebeR(eaee)^* = ebeeRe^*(eae)^* = (ebe)(eRe)(eae)^* \subseteq C(R).$$

Thus eRe is central $*$-reflexive.

(2)⇒(1). This is obvious if we let the projection $e = 1$.

(1)⇔(3) is straightforward by the above proof since every central idempotent is a projection. □

Recall that for a ring R and an (R,R)-bimodule M, the trivial extension of R by M is the ring $T(R, M) = R \oplus M$ with the usual addition and the following multiplication:

$$(r_1, m_1)(r_2, m_2) = (r_1 r_2, r_1 m_2 + m_1 r_2).$$

This is isomorphic to the ring of all matrices of the form $\left(\begin{smallmatrix} r & m \\ 0 & r \end{smallmatrix}\right)$, where $r \in R$, $m \in M$ and the usual matrix operations are used.

Note that if R is a $*$-ring and $T(R,R)$ is the trivial extension of R, then $\overline{*}$: $T(R,R) \to T(R,R)$ defined by

$$\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}^{\overline{*}} = \begin{pmatrix} a^* & b^* \\ 0 & a^* \end{pmatrix}$$

is an involution on $T(R,R)$.

Proposition 3.2 *If the trivial extension $T(R,R)$ is central $\overline{*}$-reflexive, then R is central $*$-reflexive.*

Proof Suppose that R is a $*$-ring and $T(R,R)$ is central $\overline{*}$-reflexive. Let $a, b \in R$ such that $aRb = 0$. We have

$$\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} T(R,R) \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} = 0.$$

It follows that

$$\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} T(R,R) \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}^{\overline{*}} = \begin{pmatrix} 0 & bRa^* \\ 0 & 0 \end{pmatrix} \in C(T(R,R))$$

since $T(R,R)$ is central $\overline{*}$-reflexive. Thus, we have $bRa^* \subseteq C(R)$. Hence R is central $*$-reflexive. □

Proposition 3.3 *Let R be a reduced ring. If R is a central $*$-reflexive ring, then $T(R,R)$ is central $\overline{*}$-reflexive ring.*

Proof Let

$$A = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}, \quad C = \begin{pmatrix} c & d \\ 0 & c \end{pmatrix} \in T(R,R)$$

such that $ABC = 0$ for every $B = \left(\begin{smallmatrix} m & n \\ 0 & m \end{smallmatrix}\right) \in T(R,R)$. Then we have

$$amc = 0, \tag{3.1}$$

$$amd + amc + bmc = 0. \tag{3.2}$$

In the following, we freely use the fact that R is reduced and every reduced ring is semicommutative. From Eq. (3.1), we see that $amc = 0$, and so $abmc = amdc = 0$. Multiplying Eq. (3.2) on the right side by c, we have $amdc + (an + bm)c^2 = (an + bm)c^2 = (an + bm)c = 0$. This shows that

$$anc + bmc = 0. \tag{3.3}$$

Next multiplying Eq. (3.3) on the left side by a, we obtain $a^2nc + abmc = a^2nc = 0$ and so $anc = 0$. Hence Eq. (3.2) becomes:

$$amd + bmc = 0. \tag{3.4}$$

Multiplying Eq. (3.4) on the left side by a, we obtain $a^2md + abmc = a^2md = 0$. This shows that $amd = 0$, and so $bmc = 0$. Now we obtain $amc = amd = anc = bmc = 0$. Since R is a central $*$-reflexive ring, we have $cma^* \in C(R)$, $dma^* \in C(R)$, $cna^* \in C(R)$, $cmb^* \in C(R)$. Therefore, we get

$$CBA^{\overline{*}} = \begin{pmatrix} cma^* & cmb^* + cna^* + dma^* \\ 0 & cma^* \end{pmatrix} \in C(T(R,R)),$$

which implies that $T(R,R)$ is central $\overline{*}$-reflexive. □

Proposition 3.4 *If $\{R_i : i \in I\}$ is a class of central $*$-reflexive rings, then $\Pi_{i\in I} R_i$ is central $*$-reflexive.*

Proof Let R_i be central $*$-reflexive rings for all $i \in I$. Let $S = \Pi_{i\in I} R_i$ and $(a_i), (b_i) \in S$ such that $(a_i)S(b_i) = 0$. This gives $a_i R_i b_i = 0$ for all $i \in I$. Since R_i is central $*$-reflexive for each $i \in I$, $b_i R_i a_i^* \subseteq C(R_i)$ for all $i \in I$. Thus $(b_i)S(a_i^*) \subseteq C(R)$. This implies that S is central $*$-reflexive. □

Recall that an element u of a ring R is right regular if $ur = 0$ implies $r = 0$ for $r \in R$. Left regular elements can be similarly defined. An element is regular if it is both left and right regular. For a $*$-ring R, let $\triangle$ be a multiplicative monoid in R consisting of central regular elements. Then

$$\triangle^{-1}R = \{u^{-1}a \mid u \in \triangle, a \in R\}$$

is a ring. If $\triangle$ is closed under $*$, then $\overline{*}$: $\triangle^{-1}R \to \triangle^{-1}R$ defined by $(u^{-1}a)^{\overline{*}} = (u^*)^{-1}a^*$ is an involution of $\triangle^{-1}R$.

Proposition 3.5 *For a $*$-ring R, R is central $*$-reflexive if and only if $\triangle^{-1}R$ is central $\overline{*}$-reflexive.*

Proof Let R be a central $*$-reflexive ring. Let $u^{-1}a, v^{-1}b \in \triangle^{-1}R$ with $u, v \in \triangle$ and $a, b \in R$ such that $(u^{-1}a) \triangle^{-1} R(v^{-1}b) = 0$. Then we have $aRb = 0$. By assumption, $bRa^* \subseteq C(R)$. Therefore, we have

$$(v^{-1}b) \triangle^{-1} R(u^{-1}a)^{\overline{*}} = (v^{-1}b) \triangle^{-1} R(u^*)^{-1}a^* \subseteq C(\triangle^{-1}R),$$

and hence $\triangle^{-1}R$ is central $\overline{*}$-reflexive. Conversely, assume that $\triangle^{-1}R$ is central $\overline{*}$-reflexive. Let $a, b \in R$ such that $aRb = 0$. This implies that $a(\triangle^{-1}R)b = 0$. Since $\triangle^{-1}R$ is central $\overline{*}$-reflexive, $b(\triangle^{-1}R)a^{\overline{*}} = b(\triangle^{-1}R)a^* \subseteq C(\triangle^{-1}R)$. Therefore, we get $bRa^* \subseteq C(R)$. This shows that R is central $*$-reflexive. □

The ring of Laurent polynomials in x, over a ring R, consists of all formal sums $\sum_{i=k}^{n} r_i x^i$ with usual addition and multiplication, where $r_i \in R$ and $k, n \in \mathbb{Z}$. This ring is denoted by $R[x; x^{-1}]$ [7]. Moreover, if R is a ring with involution $*$, then $\overline{*} : R[x; x^{-1}] \to R[x; x^{-1}]$ defined by $(\sum_{i=k}^{n} a_i x^i)^{\overline{*}} = \sum_{i=k}^{n} a_i^* x^i$ extends $*$ and also is an involution of $R[x; x^{-1}]$. Let $\triangle = \{1, x, x^2, \ldots\}$. Then clearly $\triangle$ is a multiplicative monoid in $R[x]$ consisting of central regular elements, and $\triangle$ is closed under $\overline{*}$ (in fact, $x^{\overline{*}} = x$). Then we have the following

Corollary 3.6 *$R[x]$ is central $\overline{*}$-reflexive if and only if $\triangle^{-1}R[x]$ is central $\overline{*}$-reflexive.*

Corollary 3.7 *For a ring R, $R[x]$ is central $\overline{*}$-reflexive if and only if $R[x; x^{-1}]$ is central $\overline{*}$-*

reflexive.

For an algebra R over a commutative ring S, the Dorroh extension of R by S is the Abelian group $D = R \oplus S$ with multiplication given by

$$(r_1, s_1)(r_2, s_2) = (r_1 r_2 + s_1 r_2 + s_2 r_1, s_1 s_2),$$

where $r_i \in R$ and $s_i \in R$. If R is an algebra with involution $*$, then $*$ can induce an involution $\overline{*}$:$D \to D$ defined by $(r, s)^{\overline{*}} = (r^*, s)$.

Proposition 3.8 *Let R be an algebra over a commutative ring S. Then R is central $*$-reflexive if and only if the Dorroh extension D of R by S is central $\overline{*}$-reflexive.*

Proof Since every $s \in S$ can be written as $s = s \cdot 1_R$, we have $R = \{r + s : (r, s) \in D\}$. Let R be central $*$-reflexive and $(r_1, s_1)D(r_2, s_2) = 0$. Then $(r_1, s_1)(r, s)(r_2, s_2) = 0$ for any $(r, s) \in D$. This implies

$$r_1 r r_2 + s_1 r r_2 + s r_1 r_2 + s_2 r_1 r + s_1 s r_2 + s_1 s_2 r + s s_2 r_1 = 0 \text{ and } s_1 s s_2 = 0.$$

So $(r_1, s_1)(r, s)(r_2, s_2) = 0$ is equivalent to $(r_1 + s_1)(r + s)(r_2 + s_2) = 0$ with $s_1 s s_2 = 0$. This gives $(r_1 + s_1)R(r_2 + s_2) = 0$ with $s_2 S s_1 = 0$. Since R is central $*$-reflexive and S is commutative, we have $(r_2, s_2)R(r_1, s_1)^* = (r_2, s_2)R(r_1^*, s_1) \subseteq C(R)$ with $s_2 S s_1 = 0$. This gives $(r_2, s_2)(r, s)(r_1^*, s_1) = (r_2, s_2)(r, s)(r_1, s_1)^{\overline{*}} \in C(D)$ and so, $(r_2, s_2)D(r_1, s_1)^{\overline{*}} \subseteq C(D)$. Hence D is central $\overline{*}$-reflexive.

Conversely, suppose D is central $\overline{*}$-reflexive. Let $a, b \in R$ such that $aRb = 0$. Then $(a, 0)D(b, 0) = (aRb + Sab, 0) = 0$. By assumption, we have $(b, 0)D(a, 0)^{\overline{*}} = (b, 0)D(a^*, 0) \subseteq C(D)$. It follows that $(b, 0)(R, 0)(a^*, 0) = (bRa^*, 0) \in C(D)$. Therefore, $bRa^* \subseteq C(R)$, proving that R is central $*$-reflexive. $\square$

A ring R is an Armendariz ring if whenever the product of two polynomials in $R[x]$ is zero, each product of their coefficients is zero. Quasi-Armendariz rings are a generalization of Armendariz rings defined in [8]. A ring R is quasi-Armendariz if whenever

$$f(x) = \sum_{i=0}^{n} a_i x^i, \quad g(x) = \sum_{j=0}^{m} b_j x^j$$

satisfy $f(x)R[x]g(x) = 0$, then $a_i R b_j = 0$ for all i, j.

Proposition 3.9 *Let R be a quasi-Armendariz ring such that it is also central $*$-reflexive. Then $R[x]$ is central $\overline{*}$-reflexive.*

Proof Let $f(x) = \sum_{i=0}^{n} a_i x^i, g(x) = \sum_{j=0}^{m} b_j x^j \in R[x]$ such that $f(x)R[x]g(x) = 0$. Since R is quasi-Armendariz, $a_i R b_j = 0$ for all i, j. Since R is central $*$-reflexive, $b_j R a_i^* \subseteq C(R)$. Thus $g(x)R[x]f(x)^{\overline{*}} \subseteq C(R[x])$ and hence $R[x]$ is central $\overline{*}$-reflexive. $\square$

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References

[1] P. M. COHN. *Reversible rings*. Bull. London Math. Soc., 1999, **31**: 641–648.

[2] G. MASON. *Reflexive ideals*. Comm. Algebra, 1981, **9**(17): 1709–1724.

[3] U. S. CHAKRABORTY. *On some classes of reflexive rings*. Asian-Eur. J. Math., 2015, **8**: 1–15.

[4] W. M. FAKIEH, S. K. NAUMAN. *Reversible rings with involutions and some minimalities*. The Scientific World Journal, 2013, **6**: 650702.

[5] Liang ZHAO, Xiaosheng ZHU, Qinqin GU. *Reflexive rings and their extensions*. Math. Slovaca, 2013, **63**(3): 417–430.

[6] T. K. KWAK, Y. LEE. *Reflexive property of rings*. Comm. Algebra, 2012, **40**(4): 1576–1594.

[7] Y. HIRANO. *On annihilator ideals of a polynomial ring over a noncommutative ring*. J. Pure Appl. Algebra, 2002, **168**: 45–52.

[8] N. K. KIM, Y. LEE. *Armendariz rings and reduced Rings*. J. Algebra, 2000, **223**: 477–488.