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A Survey on Single-Valued Neutrosophic K-Algebras

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Dedicated to the Memory of Professor L. C. HSU on the Occasion of His 100th Birthday

Abstract In this survey, we first present a brief overview of logical algebras. We then discuss concepts of single-valued neutrosophic K-subalgebras, single-valued neutrosophic soft K-algebras and single-valued neutrosophic topological K-algebras. Moreover, we discuss various fundamental concepts which include interior, closure, C_5-connectivity, super connectivity, compactness and Hausdorffness of single-valued neutrosophic topological K-algebras.

Keywords K-algebras; single-valued neutrosophic sets; homomorphism; neutrosophic topological K-algebras

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1. Introduction

The study of algebra of logic also termed as logical algebra is the algebraic study of certain propositional calculi. This study arose as an attempt to solve logical problems by using algebraic methods. George Boole [1] was the first who developed the "algebra of logic" in 19th century. The work of Boole was further developed by Stanley, Peirce, Schröder, Russell and many others. After the advent of the set theory, propositions and logical operations on them became the main subject of algebra of logic. In an attempt to generalize the set difference in set theory, Imai and Iséki introduced the concept of BCK-algebra [2]. Iséki later generalized the concept of BCK-algebra to BCI-algebra [3]. Interestingly, BCK algebras found several applications including in coding theory [4].

A new kind of logical algebra, known as K-algebra, was introduced by Dar and Akram [5]. The K-algebra was built on a group $(G, \cdot, e)$ by adjoining the induced binary operation $\odot$ on G and joined with an abstract K-algebra $(G, \cdot, \odot, e)$, where e is right identity element of G. The group G is particularly of the type in which each non-identity element is not of order 2. This algebraic structure is, in general, non-commutative and non-associative with right identity element e (see [5, 6]). If the given group G is not an elementary abelian 2-group, then the K-algebra is proper. Thus, a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$ is abelian and non-abelian, proper and improper, purely depends on the base group G. Due to structural basis of group G, it is renamed as "$K(G)$-algebra" and characterized by its left and right mappings [7, 8].

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The concepts and results of K-algebras have been broadened to the fuzzy setting frames by applying Zadeh's fuzzy set theory and its generalizations [9–17]. In 1998, Smarandache [18] introduced neutrosophic sets as a generalization of fuzzy sets [19] and intuitionistic fuzzy sets [20]. A neutrosophic set is identified by three functions called truth-membership (T), indeterminacy-membership (I) and falsity-membership (F) whose values are real standard or non-standard subset of unit interval $]^{-}0, 1^{+}[$, where $^{-}0 = 0 - \epsilon$, $1^{+} = 1 + \epsilon$, ϵ is an infinitesimal number. To apply neutrosophic set in real-life problems more conveniently, Smarandache [18] and Wang et al. [21] defined single-valued neutrosophic sets which take the value from the subset of $[0, 1]$. Thus, a single-valued neutrosophic set is a representative case of neutrosophic set, and can be used expediently to deal with real-world problems, especially in decision support.

A wide range of research has been presiding over by many researchers to model the uncertainties present in environmental sciences, medical sciences, social sciences etc. A number of theories such as the theory of probability, interval mathematics, fuzzy and intuitionistic fuzzy set theories etc. have been used to overcome these uncertainties. Due to the insufficiency of the mathematical tool, these theories have not worked so long. As a consequence, the concept of the soft set was presented by Molodtsov in [22] for modeling vagueness and uncertainty. Soft set theory considers various parameters subject to the problem to get a reliable solution without setting membership function. Molodtsov has put this theory in application to different spheres like game theory, smoothness of functions, operational research etc. In addition, Maji in [23] initiated the concept of neutrosophic soft set just as a coalition of the neutrosophic set and soft set to deal with indeterminate data in a more unified manner. The composition of these two mathematical techniques provided a new mathematical model namely, neutrosophic soft set.

Algebraic structures have a vital place with vast applications in various areas of life. Algebraic structures provide a mathematical modeling of related study. Neutrosophic set theory has also been applied to many algebraic structures. Agboola and Davazz introduced the concept of neutrosophic BCI/BCK-algebras and discuss elementary properties in [24]. Jun et al. [25–27] defined interval neutrosophic sets on BCK/BCI-algebra and proposed neutrosophic positive implicative N-ideals and study their extension property. Currently, work on neutrosophic set theory and its instances is in advancement. Soft sets have also been applied to K-algebras. Alshari et al. [17] applied soft set to K-algebras. Akram et al. [10, 11, 16] introduced the notion of fuzzy soft K-algebras, intuitionistic fuzzy soft K-algebras, bipolar fuzzy soft K-algebras. Bakhat and Das [28] introduced the notion of $(\in, \in \vee q)$-fuzzy subgroups and Yuan et al. [29] introduced generalized fuzzy subgroups.

A number of set theories and their topological structures have been introduced by many researchers in order to deal with uncertainities. Chang [30] was the first who introduced the notion of fuzzy topology. Later, Lowan [31], Pu and Liu [32], Wong [33], Chattopadhyay and Samanta [34], Lupianez [35] and Hanafy [36] introduced other concepts related to fuzzy topology. Coker [37, 38] introduced the notion of intuitionistic fuzzy topology as a generalization of fuzzy topology. Salama and Alblowi [39] defined the topological structure of neutrosophic set theory. These set theories with their topological structures have also been applied to K-algebras. Akram and Dar [14, 15] introduced the concept of fuzzy topological K-algebras and intuitionistic fuzzy topological K-algebras. In this survey, we apply neutrosophic set theory to K-algebras. We organize the rest of the survey article as follows:

In Section 2, we give overview of the concept of K-algebras. In Section 3, we present the notion of single-valued neutrosophic K-algebras. In Section 4, we study the concept of single-valued neutrosophic soft K-algebras. In Section 5, we describe the topological structure of single-valued neutrosophic sets to K-algebras. The same section deals with certain concepts of interior, closure, C_5-connected, super connected, compact and Hausdorff of single-valued neutrosophic topological K-algebras with numerical examples.

2. K-algebras

A non-associative and non-commutative algebra, K-algebra, was firstly instigated by Dar and Akram in 2005 (see [5]). A K-algebra is such a structure in which the base group G is explicitly of the form that each of its non-identity element is not of order two. It is such a structure which is constructed on a group $(G, \cdot, e)$ by adjoining an induced binary operation $\odot$ and connected to an abstract K-algebra $(G, \cdot, \odot, e)$, where e is right identity element of G.

Definition 2.1 ([5]) *Let $(G, \cdot, e)$ be a group such that each non-identity element is not of order 2. Let a binary operation $\odot$ be introduced on the group G and defined by $\odot(s, t) = s \odot t = st^{-1}$ for all s, $t \in G$. If e is the identity of the group G, then:*

(1) e takes the shape of right $\odot$-identity and not that of left $\odot$-identity.

(2) Each non-identity element $(s \neq e)$ is $\odot$-involutionary because $s \odot s = ss^{-1} = e$.

(3) G is $\odot$-nonassociative because $(s \odot t) \odot u = s \odot (u \odot t^{-1}) \neq s \odot (t \odot u)$ for all s, y, $u \in G$.

(4) G is $\odot$-noncommutative since $s \odot t \neq t \odot s$ for all s, $t \in G$.

(5) If G is an elementary Abelian 2-group, then $s \odot t = s \cdot t$.

Definition 2.2 ([5]) *A K-algebra is a structure $(G, \cdot, \odot, e)$ on a group G, where $\odot : G \times G \to G$ defined by $\odot(s, t) = s \odot t = s.t^{-1}$, if it satisfies the following axioms:*

(i) $((s \odot t) \odot (s \odot u)) = (s \odot (u^{-1} \odot t^{-1}) \odot s)$,

(ii) $(s \odot (s \odot t)) = ((s \odot t^{-1}) \odot s)$,

(iii) $s \odot s = e$,

(iv) $s \odot e = s$,

(v) $e \odot s = s^{-1}$ for all s, t, $u \in G$.

Example 2.3 Consider $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$

is the cyclic group of order 9 and Caley's table for $\odot$ is given as:

$\odot$	e	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8
e	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x
x	x	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2
x^2	x^2	x	e	x^8	x^7	x^6	x^5	x^4	x^3
x^3	x^3	x^2	x	e	x^8	x^7	x^6	x^5	x^4
x^4	x^4	x^3	x^2	x	e	x^8	x^7	x^6	x^5
x^5	x^5	x^4	x^3	x^2	x	e	x^8	x^7	x^6
x^6	x^6	x^5	x^4	x^3	x^2	x	e	x^8	x^7
x^7	x^7	x^6	x^5	x^4	x^3	x^2	x	e	x^8
x^8	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x	e

Example 2.4 Consider additive groups $(\mathbb{Z}, +, 0)$, $(\mathbb{Q}, +, 0)$, $(\mathbb{R}, +, 0)$. Then $(\mathbb{Z}, +, \odot, 0)$, $(\mathbb{Q}, +, \odot, 0)$, $(\mathbb{R}, +, \odot, 0)$. Form K-algebras with an induced binary operation $\odot$, defined by $\odot(s, t) = s \odot t = s - t$ for all $s, t \in G$.

Definition 2.5 ([5]) *Let $\mathcal{K}$ be a K-algebra and let $\mathcal{H}$ be a nonempty subset of $\mathcal{K}$. Then $\mathcal{H}$ is called a subalgebra of $\mathcal{K}$ if $u \odot v \in \mathcal{H}$ for all $u, v \in \mathcal{H}$.*

Definition 2.6 ([5]) *Let $\mathcal{K}_1$ and $\mathcal{K}_2$ be two K-algebras. A mapping $f : \mathcal{K}_1 \to \mathcal{K}_2$ is called a homomorphism if it satisfies the following condition:*

- $f(u \odot v) = f(u) \odot f(v)$ *for all $u, v \in \mathcal{K}$.*

Definition 2.7 ([5]) *Let $\mathcal{K}$ be a K-algebra. Then $\mathcal{K}$ is called an Abelian K-algebra if and only if $u \odot (e \odot v) = v \odot (e \odot u)$ for all $u, v \in G$.*

Definition 2.8 ([5]) *Let $\mathcal{K}$ be a K-algebra and u be a fixed element of $\mathcal{K}$. Then a mapping $\rho_u : \mathcal{K} \to \mathcal{K}$, defined by $\rho_u(v) = v \odot u$ for all $v \in \mathcal{K}$, is called right map of $\mathcal{K}$.*

Definition 2.9 ([5]) *Let $\mathcal{K}$ be a K-algebra and u be a fixed element of $\mathcal{K}$. Then a mapping $\rho_u : \mathcal{K} \to \mathcal{K}$, defined by $\rho_u(v) = u \odot v$ for all $v \in \mathcal{K}$, is called left map of $\mathcal{K}$.*

Definition 2.10 ([5]) *Let $\mathcal{K}$ be a K-algebra and f be a mapping. Then $f : \mathcal{K} \to \mathcal{K}$ is called an automorphism of K-algebras if it satisfies the following conditions:*

(i) f is an endomorphism of $\mathcal{K}$.

(ii) f is a bijective mapping.

3. Single-valued neutrosophic K-algebras

Akram et al. [40] presented the concept of single-valued neutrosophic K-algebras. This section is based on [40].

Definition 3.1 *A single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in a K-algebra $\mathcal{K}$ is called a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if it satisfies the following conditions:*

(a) $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\}$,
(b) $\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\}$,
(c) $\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}$, *for all* $s, t \in G$.

Note that $\mathcal{T}_{\mathcal{A}}(e) \geq \mathcal{T}_{\mathcal{A}}(s)$, $\mathcal{I}_{\mathcal{A}}(e) \geq \mathcal{I}_{\mathcal{A}}(s)$, $\mathcal{F}_{\mathcal{A}}(e) \leq \mathcal{F}_{\mathcal{A}}(s)$, *for all* $s \in G$.

Example 3.2 Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra, where group G is given as $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ which is the cyclic group of order 9 and Caley's table for $\odot$ is given as:

$\odot$	e	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8
e	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x
x	x	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2
x^2	x^2	x	e	x^8	x^7	x^6	x^5	x^4	x^3
x^3	x^3	x^2	x	e	x^8	x^7	x^6	x^5	x^4
x^4	x^4	x^3	x^2	x	e	x^8	x^7	x^6	x^5
x^5	x^5	x^4	x^3	x^2	x	e	x^8	x^7	x^6
x^6	x^6	x^5	x^4	x^3	x^2	x	e	x^8	x^7
x^7	x^7	x^6	x^5	x^4	x^3	x^2	x	e	x^8
x^8	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x	e

We define a single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in K-algebra as follows:

$\mathcal{T}_{\mathcal{A}}(e) = 0.8, \mathcal{I}_{\mathcal{A}}(e) = 0.7, \mathcal{F}_{\mathcal{A}}(e) = 0.4$, and $\mathcal{T}_{\mathcal{A}}(s) = 0.2, \mathcal{I}_{\mathcal{A}}(s) = 0.3, \mathcal{F}_{\mathcal{A}}(s) = 0.6$, for all $s \neq e \in G$. Clearly, $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$.

Example 3.3 Consider a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$ on dihedral group $D4$ given as $G = \{e, a, b, c, x, y, u, v\}$, where $c = ab, x = a^2, y = a^3$, $u = a^2b, v = a^3b$ and Caley's table for $\odot$ is given as:

$\odot$	e	a	b	c	x	y	u	v
e	e	y	b	c	x	a	u	v
a	a	e	c	u	y	x	v	b
b	b	c	e	y	u	v	x	a
c	c	u	a	e	v	b	y	x
x	x	a	u	v	e	y	b	c
y	y	x	v	b	a	e	c	u
u	u	v	x	a	b	c	e	y
v	v	b	y	x	c	u	a	e

We define a single-valued neutrosophic set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ in K-algebra as follows:

$\mathcal{T}_{\mathcal{A}}(e) = 0.9, \mathcal{I}_{\mathcal{A}}(e) = 0.3, \mathcal{F}_{\mathcal{A}}(e) = 0.3$, and $\mathcal{T}_{\mathcal{A}}(s) = 0.6, \mathcal{I}_{\mathcal{A}}(s) = 0.2, \mathcal{F}_{\mathcal{A}}(s) = 0.4$, for all $s \neq e \in G$. By routine calculations, it is easy to verify that $\mathcal{A}$ is a single-valued neutrosophic K-subalgebra ok $\mathcal{K}$.

Proposition 3.4 *If* $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ *is a single-valued neutrosophic* K*-subalgebra of* $\mathcal{K}$*, then for all* $s, t \in G$*, we have*

(1) $(\mathcal{T}_{\mathcal{A}}(s\odot t)=\mathcal{T}_{\mathcal{A}}(t)\Rightarrow \mathcal{T}_{\mathcal{A}}(s)=\mathcal{T}_{\mathcal{A}}(e))$, $(\mathcal{T}_{\mathcal{A}}(s)=\mathcal{T}_{\mathcal{A}}(e)\Rightarrow \mathcal{T}_{\mathcal{A}}(s\odot t)\geq \mathcal{T}_{\mathcal{A}}(t))$.

(2) $\mathcal{I}_{\mathcal{A}}(s\odot t)=\mathcal{I}_{\mathcal{A}}(t)\Rightarrow \mathcal{I}_{\mathcal{A}}(s)=\mathcal{I}_{\mathcal{A}}(e))$, $(\mathcal{I}_{\mathcal{A}}(s)=\mathcal{I}_{\mathcal{A}}(e)\Rightarrow \mathcal{I}_{\mathcal{A}}(s\odot t)\geq \mathcal{I}_{\mathcal{A}}(t))$.

(3) $(\mathcal{F}_{\mathcal{A}}(s\odot t)=\mathcal{F}_{\mathcal{A}}(t)\Rightarrow \mathcal{F}_{\mathcal{A}}(s)=\mathcal{F}_{\mathcal{A}}(e))$, $(\mathcal{F}_{\mathcal{A}}(s)=\mathcal{F}_{\mathcal{A}}(e)\Rightarrow \mathcal{F}_{\mathcal{A}}(s\odot t)\leq \mathcal{F}_{\mathcal{A}}(t))$.

Proof (1) Assume that $\mathcal{T}_{\mathcal{A}}(s\odot t)=\mathcal{T}_{\mathcal{A}}(t)$, for all $s,t\in G$. Taking $t=e$ and using (iii) of Definition 2.1, we have $\mathcal{T}_{\mathcal{A}}(s)=\mathcal{T}_{\mathcal{A}}(s\odot e)=\mathcal{T}_{\mathcal{A}}(e)$. Let $s,t\in G$ be such that $\mathcal{T}_{\mathcal{A}}(s)=\mathcal{T}_{\mathcal{A}}(e)$. Then $\mathcal{T}_{\mathcal{A}}(s\odot t)\geq \min\{\mathcal{T}_{\mathcal{A}}(s),\mathcal{T}_{\mathcal{A}}(t)\}=\min\{\mathcal{T}_{\mathcal{A}}(e),\mathcal{T}_{\mathcal{A}}(t)\}=\mathcal{T}_{\mathcal{A}}(t)$.

(2) Again assume that $\mathcal{I}_{\mathcal{A}}(s\odot t)=\mathcal{I}_{\mathcal{A}}(t)$, for all $s,t\in G$. Taking $t=e$ and by (iii) of Definition 2.1, we have $\mathcal{I}_{\mathcal{A}}(s)=\mathcal{I}_{\mathcal{A}}(s\odot e)=\mathcal{I}_{\mathcal{A}}(e)$. Also let $s,t\in G$ be such that $\mathcal{I}_{\mathcal{A}}(s)=\mathcal{I}_{\mathcal{A}}(e)$. Then $\mathcal{I}_{\mathcal{A}}(s\odot t)\geq \min\{\mathcal{I}_{\mathcal{A}}(s),I_(t)\}=\min\{\mathcal{I}_{\mathcal{A}}(e),\mathcal{I}_{\mathcal{A}}(t)\}=\mathcal{I}_{\mathcal{A}}(t)$.

(3) Consider that $\mathcal{F}_{\mathcal{A}}(s\odot t)=\mathcal{F}_{\mathcal{A}}(t)$, for all $s,t\in G$. Taking $t=e$, we have $\mathcal{F}_{\mathcal{A}}(s)=\mathcal{F}_{\mathcal{A}}(s\odot e)=\mathcal{F}_{\mathcal{A}}(e)$. Let $s,t\in G$ be such that $\mathcal{F}_{\mathcal{A}}(s)=F_(e)$. Then $\mathcal{F}_{\mathcal{A}}(s\odot t)\leq \max\{\mathcal{F}_{\mathcal{A}}(s),\mathcal{F}_{\mathcal{A}}(t)\}=\max\{\mathcal{F}_{\mathcal{A}}(e),\mathcal{F}_{\mathcal{A}}(t)\}=\mathcal{F}_{\mathcal{A}}(t)$. This completes the proof. □

Definition 3.5 *Let $\mathcal{A}=(\mathcal{T}_{\mathcal{A}},\mathcal{I}_{\mathcal{A}},\mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in a K-algebra $\mathcal{K}$ and let $(\alpha,\beta,\gamma)\in[0,1]\times[0,1]\times[0,1]$ with $\alpha+\beta+\gamma\leq 3$. Then level subsets of $\mathcal{A}$ defined as:*

$$\mathcal{A}_{(\alpha,\beta,\gamma)}=\{s\in G|\mathcal{T}_{\mathcal{A}}(s)\geq\alpha,\mathcal{I}_{\mathcal{A}}(s)\geq\beta,\mathcal{F}_{\mathcal{A}}(s)\leq\gamma\},$$
$$\mathcal{A}_{(\alpha,\beta,\gamma)}=\{s\in G|\mathcal{T}_{\mathcal{A}}(s)\geq\alpha\}\cap\{s\in G|\mathcal{I}_{\mathcal{A}}(s)\geq\beta\}\cap\{s\in G|\mathcal{F}_{\mathcal{A}}(s)\leq\gamma\},$$
$$\mathcal{A}_{(\alpha,\beta,\gamma)}=\cup(\mathcal{T}_{\mathcal{A}},\alpha)\cap\cup'(\mathcal{I}_{\mathcal{A}},\beta)\cap L(\mathcal{F}_{\mathcal{A}},\gamma)$$

are called (α,β,γ) -level subsets of single-valued neutrosophic set $\mathcal{A}$. The set of all $(\alpha,\beta,\gamma)\in$ $\mathrm{Im}(\mathcal{T}_{\mathcal{A}})\times\mathrm{Im}(\mathcal{I}_{\mathcal{A}})\times\mathrm{Im}(\mathcal{I}_{\mathcal{A}})$ is known as image of $\mathcal{A}=(\mathcal{T}_{\mathcal{A}},\mathcal{I}_{\mathcal{A}},\mathcal{F}_{\mathcal{A}})$. The set $\mathcal{A}_{(\alpha,\beta,\gamma)}=\{s\in G|\mathcal{T}_{\mathcal{A}}(s)>\alpha,\mathcal{I}_{\mathcal{A}}(s)>\beta,\mathcal{F}_{\mathcal{A}}(s)<\gamma\}$ is known as strong (α,β,γ)-level subset of $\mathcal{A}$.

Proposition 3.6 *If $\mathcal{A}=(\mathcal{T}_{\mathcal{A}},\mathcal{I}_{\mathcal{A}},\mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$, then the level subsets $\cup(\mathcal{T}_{\mathcal{A}},\alpha)=\{s\in G\mid \mathcal{T}_{\mathcal{A}}(s)\geq\alpha\}$, $\cup'(\mathcal{I}_{\mathcal{A}},\beta)=\{s\in G|\mathcal{I}_{\mathcal{A}}(s)\geq\beta\}$ and $L(\mathcal{F}_{\mathcal{A}},\gamma)=\{s\in G|\mathcal{F}_{\mathcal{A}}(s)\leq\gamma\}$ are k-subalgebras of $\mathcal{K}$, for every $(\alpha,\beta,\gamma)\in\mathrm{Im}(\mathcal{T}_{\mathcal{A}})\times\mathrm{Im}(\mathcal{I}_{\mathcal{A}})\times\mathrm{Im}(\mathcal{F}_{\mathcal{A}})\subseteq[0,1]$, where $\mathrm{Im}(\mathcal{T}_{\mathcal{A}})$, $\mathrm{Im}(\mathcal{I}_{\mathcal{A}})$ and $\mathrm{Im}(\mathcal{F}_{\mathcal{A}})$ are sets of values of $T_(\mathcal{A})$, $\mathcal{I}_(\mathcal{A})$ and $F_(\mathcal{A})$, respectively.*

Proof Assume that $\mathcal{A}=(\mathcal{T}_{\mathcal{A}},\mathcal{I}_{\mathcal{A}},\mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ and let $(\alpha,\beta,\gamma)\in\mathrm{Im}(\mathcal{T}_{\mathcal{A}})\times\mathrm{Im}(\mathcal{I}_{\mathcal{A}})\times\mathrm{Im}(\mathcal{F}_{\mathcal{A}})$ be such that $\cup(\mathcal{T}_{\mathcal{A}},\alpha)\neq\emptyset,\cup'(\mathcal{I}_{\mathcal{A}},\beta)\neq\emptyset$ and $L(\mathcal{F}_{\mathcal{A}},\gamma)\neq\emptyset$. Now to prove that $\cup,\cup'$ and L are level K-subalgebras. Let $s,t\in\cup(\mathcal{T}_{\mathcal{A}},\alpha)$, $\mathcal{T}_{\mathcal{A}}(s)\geq\alpha$ and $\mathcal{T}_{\mathcal{A}}(t)\geq\alpha$. It implies that $\mathcal{T}_{\mathcal{A}}(s\odot t)\geq\min\{\mathcal{T}_{\mathcal{A}}(s),\mathcal{T}_{\mathcal{A}}(t)\}\geq\alpha$. It implies that $s\odot t\in\cup(\mathcal{T}_{\mathcal{A}},\alpha)$. Hence $\cup(\mathcal{T}_{\mathcal{A}},\alpha)$ is a level K-subalgebra of $\mathcal{K}$. Similar result can be proved for $\cup'(\mathcal{I}_{\mathcal{A}},\beta)$ and $L(\mathcal{F}_{\mathcal{A}},\gamma)$. □

Theorem 3.7 *Let $\mathcal{A}=(\mathcal{T}_{\mathcal{A}},\mathcal{I}_{\mathcal{A}},\mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in K-algebra $\mathcal{K}$. Then $\mathcal{A}=(\mathcal{T}_{\mathcal{A}},\mathcal{I}_{\mathcal{A}},\mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if and only if $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-sublagebra of $\mathcal{K}$, for every $(\alpha,\beta,\gamma)\in\mathrm{Im}(\mathcal{T}_{\mathcal{A}})\times\mathrm{Im}(\mathcal{I}_{\mathcal{A}})\times\mathrm{Im}(\mathcal{F}_{\mathcal{A}})$ with $\alpha+\beta+\gamma\leq 3$.*

Proof Let $\mathcal{A}=(\mathcal{T}_{\mathcal{A}},\mathcal{I}_{\mathcal{A}},\mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic set in a K-algebra $\mathcal{K}$. Assume that $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic K-subalgebra of $\mathcal{K}$, i.e., the following conditions hold:

(1) $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\}$,

(2) $\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\}$,

(3) $\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}$, for all $s, t \in G$.

$\mathcal{T}_{\mathcal{A}}(e) \geq \mathcal{T}_{\mathcal{A}}(s)$, $\mathcal{I}_{\mathcal{A}}(e) \geq \mathcal{I}_{\mathcal{A}}(s)$, $\mathcal{F}_{\mathcal{A}}(e) \leq \mathcal{F}_{\mathcal{A}}(s)$, for all $s \in G$.

Let $(\alpha, \beta, \gamma) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}})$ with $\alpha + \beta + \gamma \leq 3$ be such that $\mathcal{A}_{(\alpha,\beta,\gamma)} \neq \emptyset$. Let $s, t \in \mathcal{A}_{(\alpha,\beta,\gamma)}$ be such that

$$\begin{aligned}
&\mathcal{T}_{\mathcal{A}}(s) \geq \alpha, \mathcal{T}_{\mathcal{A}}(t) \geq \alpha',\\
&\mathcal{I}_{\mathcal{A}}(s) \geq \beta, \mathcal{I}_{\mathcal{A}}(t) \geq \beta',\\
&\mathcal{F}_{\mathcal{A}}(s) \leq \gamma, \mathcal{F}_{\mathcal{A}}(t) \leq \gamma'.
\end{aligned}$$

Without loss of generality we can assume that $\alpha \leq \alpha'$, $\beta \leq \beta'$ and $\gamma \geq \gamma'$, then

$$\begin{aligned}
&\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \alpha = \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},\\
&\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \beta = \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},\\
&\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \gamma = \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}.
\end{aligned}$$

It implies that $s \odot t \in \mathcal{A}_{(\alpha,\beta,\gamma)}$. So, $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$. Conversely, we suppose that $\mathcal{A}_{(\alpha,\beta,\gamma)}$ is a K-subalgebra of $\mathcal{K}$. If the condition of the Definition 3.1 is not true, then there exist $u, v \in G$ such that

$$\begin{aligned}
&\mathcal{T}_{\mathcal{A}}(u \odot v) < \min\{\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)\},\\
&\mathcal{I}_{\mathcal{A}}(u \odot v) < \min\{\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)\},\\
&\mathcal{F}_{\mathcal{A}}(u \odot v) > \max\{\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)\}.
\end{aligned}$$

We take

$$\begin{aligned}
\alpha_1 &= \frac{1}{2}(\mathcal{T}_{\mathcal{A}}(u \odot v) + \min\{\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)\}),\\
\beta_1 &= \frac{1}{2}(\mathcal{I}_{\mathcal{A}}(u \odot v) + \min\{\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)\}),\\
\gamma_1 &= \frac{1}{2}(\mathcal{F}_{\mathcal{A}}(u \odot v) + \min\{\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)\}).
\end{aligned}$$

We have $\mathcal{T}_{\mathcal{A}}(u \odot v) < \alpha_1 < \min\{\mathcal{T}_{\mathcal{A}}(u), \mathcal{T}_{\mathcal{A}}(v)\}$, $\mathcal{I}_{\mathcal{A}}(u \odot v) < \beta_1 < \min\{\mathcal{I}_{\mathcal{A}}(u), \mathcal{I}_{\mathcal{A}}(v)\}$ and $\mathcal{F}_{\mathcal{A}}(u \odot v) > \gamma_1 > \max\{\mathcal{F}_{\mathcal{A}}(u), \mathcal{F}_{\mathcal{A}}(v)\}$. It implies that $u, v \in \mathcal{A}_{(\alpha,\beta,\gamma)}$ and $u \odot v \notin \mathcal{A}_{(\alpha,\beta,\gamma)}$, a contradiction. Therefore, the condition of Definition 3.1 is true. Hence $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic k-subalgebra of $\mathcal{K}$. $\square$

Theorem 3.8 *Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ be a single-valued neutrosophic k-subalgebra and $(\alpha_1, \beta_1, \gamma_1)$, $(\alpha_2, \beta_2, \gamma_2) \in \mathrm{Im}(\mathcal{T}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) \times \mathrm{Im}(\mathcal{F}_{\mathcal{A}})$ with $\alpha_j + \beta_j + \gamma_j \leq 3$ for $j = 1, 2$. Then $\mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} = \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$ if $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$.*

Proof If $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$, then clearly $\mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} = \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$. Assume that $\mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} =$

$\mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$. Since $(\alpha_1,\beta_1,\gamma_1) \in \mathrm{Im}(\mathcal{T_A}) \times \mathrm{Im}(\mathcal{I_A}) \times \mathrm{Im}(\mathcal{F_A})$, then there exists $s \in G$ such that $\mathcal{T_A}(s) = \alpha_1, \mathcal{I_A}(s) = \beta_1$ and $\mathcal{F_A}(s) = \gamma_1$. It follows that $s \in \mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)} = \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)}$.

Therefore, $\alpha_1 = \mathcal{T_A}(s) \geq \alpha_2, \beta_1 = \mathcal{I_A}(s) \geq \beta_2$ and $\gamma_1 = \mathcal{F_A}(s) \leq \gamma_2$. Also $(\alpha_2,\beta_2,\gamma_2) \in \mathrm{Im}(\mathcal{T_A}) \times \mathrm{Im}(\mathcal{I_A}) \times \mathrm{Im}(\mathcal{F_A})$, there exists $t \in G$ such that $\mathcal{T_A}(t) = \alpha_2, \mathcal{I_A}(t) = \beta_2$ and $\mathcal{F_A}(t) = \gamma_2$. It follows that $t \in \mathcal{A}_{(\alpha_2,\beta_2,\gamma_2)} = \mathcal{A}_{(\alpha_1,\beta_1,\gamma_1)}$. Therefore, $\alpha_2 = \mathcal{T_A}(t) \geq \alpha_1$, $\beta_2 = \mathcal{I_A}(t) \geq \beta_1$ and $\gamma_2 = \mathcal{F_A}(t) \leq \gamma_1$. Hence $(\alpha_1,\beta_1,\gamma_1) = (\alpha_2,\beta_2,\gamma_2)$. □

Theorem 3.9 *Let H be a K-subalgebra of K-algebra $\mathcal{K}$. Then there exists a single-valued neutrosophic K-subalgebra $\mathcal{A} = (\mathcal{T_A}, \mathcal{I_A}, \mathcal{F_A})$ of K-algebra $\mathcal{K}$ such that $\mathcal{A} = (\mathcal{T_A}, \mathcal{I_A}, \mathcal{F_A}) = H$, for some $\alpha, \beta \in (0,1], \gamma \in [0,1)$.*

Proof Let $\mathcal{A} = (\mathcal{T_A}, \mathcal{I_A}, \mathcal{F_A})$ be a single-valued neutrosophic set in K-algebra $\mathcal{K}$ given as:

$$\mathcal{T_A}(s) = \begin{cases} \alpha \in (0,1], & \text{if } s \in H, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{I_A}(s) = \begin{cases} \beta \in (0,1], & \text{if } s \in H, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{F_A}(s) = \begin{cases} \gamma \in [0,1), & \text{if } s \in H, \\ 0, & \text{otherwise.} \end{cases}$$

Let $s, t \in G$. If $s, t \in H$, then $s \odot t \in H$, and

$$\begin{aligned} &\mathcal{T_A}(s \odot t) \geq \min\{\mathcal{T_A}(s), \mathcal{T_A}(t)\}, \\ &\mathcal{I_A}(s \odot t) \geq \min\{\mathcal{I_A}(s), \mathcal{I_A}(t)\}, \\ &\mathcal{F_A}(s \odot t) \leq \max\{\mathcal{F_A}(s), \mathcal{F_A}(t)\}. \end{aligned}$$

But if $s \notin H$ or $t \notin H$, then $\mathcal{T_A}(s) = 0$ or $\mathcal{T_A}(t), \mathcal{I_A}(s) = 0$ or $\mathcal{I_A}(t)$ and $\mathcal{F_A}(s) = 0$ or $\mathcal{F_A}(t)$. It follows that $\mathcal{T_A}(s \odot t) \geq \min\{\mathcal{T_A}(s), \mathcal{T_A}(t)\}$, $\mathcal{I_A}(s \odot t) \geq \min\{\mathcal{I_A}(s), \mathcal{I_A}(t)\}$, $\mathcal{F_A}(s \odot t) \leq \max\{\mathcal{F_A}(s), \mathcal{F_A}(t)\}$. Hence $\mathcal{A} = (\mathcal{T_A}, \mathcal{I_A}, \mathcal{F_A})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Consequently $\mathcal{A}_{(\alpha,\beta,\gamma)} = H$.

The above theorem shows that any K-subalgebra of $\mathcal{K}$ can be perceived as a level K-subalgebra of some single-valued neutrosophic K-subalgebras of $\mathcal{K}$. □

Theorem 3.10 *Let $\mathcal{K}$ be a K-algebra. Given a chain of K-subalgebras: $\mathcal{A}_0 \subset \mathcal{A}_1 \subset \mathcal{A}_2 \subset \cdots \subset A_n = G$. Then there exists a single-valued neutrosophic K-subalgebra whose level K-subalgebras are exactly the K-subalgebras in this chain.*

Proof Let $\{\alpha_k | k = 0, 1, \ldots, n\}$, $\{\beta_k | k = 0, 1, \ldots, n\}$ be finite decreasing sequences and $\{\gamma_k | k = 0, 1, \ldots, n\}$ be finite increasing sequence in $[0,1]$ such that $\alpha_i + \beta_i + \gamma_i \leq 3$, for $i = 0, 1, 2, \ldots, n$. Let $\mathcal{A} = (\mathcal{T_A}, \mathcal{I_A}, \mathcal{F_A})$ be a single-valued neutrosophic set in $\mathcal{K}$ defined by $\mathcal{T_A}(\mathcal{A}_0) = \alpha_0, \mathcal{I_A}(\mathcal{A}_0) = \beta_0$, $\mathcal{F_A}(\mathcal{A}_0) = \gamma_0$, $\mathcal{T_A}(\mathcal{A}_k \setminus \mathcal{A}_{k-1}) = \alpha_k$, $\mathcal{I_A}(\mathcal{A}_k \setminus \mathcal{A}_{k-1}) = \beta_k$ and $\mathcal{F_A}(\mathcal{A}_k \setminus \mathcal{A}_{k-1}) = \gamma_k$, for $0 < k \leq n$. We claim that $\mathcal{A} = (\mathcal{T_A}, \mathcal{I_A}, \mathcal{F_A})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Let $s, t \in G$. If $s, t \in \mathcal{A}_k \setminus \mathcal{A}_{k-1}$, then it implies that $\mathcal{T_A}(s) = \alpha_k = \mathcal{T_A}(t), \mathcal{I_A}(s) = \beta_k = \mathcal{I_A}(t)$

and $\mathcal{F}_{\mathcal{A}}(s) = \gamma_k = \mathcal{F}_{\mathcal{A}}(t)$. Since each $\mathcal{A}_k$ is a K-subalgebra, it follows that $s \odot t \in \mathcal{A}_k$. So that either $s \odot t \in \mathcal{A}_k \setminus \mathcal{A}_{k-1}$ or $s \odot t \in \mathcal{A}_{k-1}$. In any case, we conclude that

$$\begin{aligned}
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \alpha_k = \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},\\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \beta_k = \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},\\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \gamma_k = \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}.
\end{aligned}$$

For $i > j$, if $s \in \mathcal{A}_i \setminus \mathcal{A}_{i-1}$ and $t \in \mathcal{A}_j \setminus \mathcal{A}_{j-1}$, then $\mathcal{T}_{\mathcal{A}}(s) = \alpha_i$, $\mathcal{T}_{\mathcal{A}}(t) = \alpha_j$, $\mathcal{I}_{\mathcal{A}}(s) = \beta_i$, $\mathcal{I}_{\mathcal{A}}(t) = \beta_j$ and $\mathcal{F}_{\mathcal{A}}(s) = \gamma_i$, $\mathcal{F}_{\mathcal{A}}(t) = \gamma_j$ and $s \odot t \in \mathcal{A}_i$ because $\mathcal{A}_i$ is a K-subalgebra and $\mathcal{A}_j \subset \mathcal{A}_i$. It follows that

$$\begin{aligned}
\mathcal{T}_{\mathcal{A}}(s \odot t) &\geq \alpha_i = \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},\\
\mathcal{I}_{\mathcal{A}}(s \odot t) &\geq \beta_i = \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},\\
\mathcal{F}_{\mathcal{A}}(s \odot t) &\leq \gamma_i = \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\}.
\end{aligned}$$

Thus, $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ and all its non empty level subsets are level K-subalgebras of $\mathcal{K}$. Since $\mathrm{Im}(\mathcal{T}_{\mathcal{A}}) = \{\alpha_0, \alpha_1, \ldots, \alpha_{\mathrm{n}}\}, \mathrm{Im}(\mathcal{I}_{\mathcal{A}}) = \{\beta_0, \beta_1, \ldots, \beta_{\mathrm{n}}\}$, $\mathrm{Im}(\mathcal{F}_{\mathcal{A}}) = \{\gamma_0, \gamma_1, \ldots, \gamma_{\mathrm{n}}\}$, the level K-subalgebras of $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ are given by the chain of K-subalgebras:

$$\begin{aligned}
\cup\,(\mathcal{T}_{\mathcal{A}}, \alpha_0) \subset \cup(\mathcal{T}_{\mathcal{A}}, \alpha_1) \subset \cdots \subset \cup(\mathcal{T}_{\mathcal{A}}, \alpha_n) = G,\\
\cup'\,(\mathcal{I}_{\mathcal{A}}, \beta_0) \subset \cup'(\mathcal{I}_{\mathcal{A}}, \beta_1) \subset \cdots \subset \cup'(\mathcal{I}_{\mathcal{A}}, \beta_n) = G,\\
\mathrm{L}(\mathcal{F}_{\mathcal{A}}, \gamma_0) \subset \mathrm{L}(\mathcal{F}_{\mathcal{A}}, \gamma_1) \subset \cdots \subset \mathrm{L}(\mathcal{F}_{\mathcal{A}}, \gamma_n) = G,
\end{aligned}$$

respectively. Indeed,

$$\begin{aligned}
\cup\,(\mathcal{T}_{\mathcal{A}}, \alpha_0) &= \{s \in G | \mathcal{T}_{\mathcal{A}}(s) \geq \alpha_0\} = \mathcal{A}_0,\\
\cup'\,(\mathcal{I}_{\mathcal{A}}, \beta_0) &= \{s \in G | \mathcal{I}_{\mathcal{A}}(s) \geq \beta_0\} = \mathcal{A}_0,\\
\mathrm{L}(\mathcal{F}_{\mathcal{A}}, \gamma_0) &= \{s \in G | \mathcal{F}_{\mathcal{A}}(s) \leq \gamma_0\} = \mathcal{A}_0.
\end{aligned}$$

Now we prove that $\cup(\mathcal{T}_{\mathcal{A}}, \alpha_k) = \mathcal{A}_k, \cup'(\mathcal{I}_{\mathcal{A}}, \beta_k) = \mathcal{A}_k$ and $L(\mathcal{F}_{\mathcal{A}}, \gamma_k) = \mathcal{A}_k$, for $0 < k \leq n$. Clearly, $\mathcal{A}_k \subseteq \cup(\mathcal{T}_{\mathcal{A}}, \alpha_k)$, $\mathcal{A}_k \subseteq \cup'(\mathcal{I}_{\mathcal{A}}, \beta_k)$ and $\mathcal{A}_k \subseteq L(\mathcal{F}_{\mathcal{A}}, \gamma_k)$. If $s \in \cup(\mathcal{T}_{\mathcal{A}}, \alpha_k)$, then $\mathcal{T}_{\mathcal{A}}(s) \geq \alpha_k$ and so $s \notin \mathcal{A}_i$, for $i > k$.

Hence $\mathcal{T}_{\mathcal{A}}(s) \in \{\alpha_0, \alpha_1, \ldots, \alpha_k\}$ which implies that $s \in \mathcal{A}_i$, for some $i \leq k$ since $\mathcal{A}_i \subseteq \mathcal{A}_k$. It follows that $s \in \mathcal{A}_k$.

Consequently, $\cup(\mathcal{T}_{\mathcal{A}}, \alpha_k) = \mathcal{A}_k$ for some $0 < k \leq n$. Similar case can be proved for $\cup'(\mathcal{I}_{\mathcal{A}}, \beta_k) = \mathcal{A}_k$. Now if $t \in L(\mathcal{F}_{\mathcal{A}}, \gamma_k)$, then $\mathcal{F}_{\mathcal{A}}(s) \leq \gamma_k$ and so $t \notin \mathcal{A}_i$, for some $j \leq k$. Thus, $\mathcal{F}_{\mathcal{A}}(s) \in \{\gamma_0, \gamma_1, \ldots, \gamma_k\}$ which implies that $s \in \mathcal{A}_j$, for some $j \leq k$. Since $\mathcal{A}_j \subseteq \mathcal{A}_k$, It follows that $t \in \mathcal{A}_k$.

Thus, $L(\mathcal{F}_{\mathcal{A}}, \gamma_k) = \mathcal{A}_k$, for some $0 < k \leq n$. The proof is completed. □

4. Single-valued neutrosophic soft K-algebras

Akram et al. [41, 42] discussed single-valued neutrosophic soft K-algebras. This section is due to [41, 42].

Definition 4.1 *Let $(\zeta, \mathbb{M})$ be a single-valued neutrosophic soft set (SNSS) over $\mathcal{K}$. The pair $(\zeta, \mathbb{M})$ is called a single-valued neutrosophic soft K-subalgebra of $\mathcal{K}$ if the following conditions are satisfied:*

(i) $\mathcal{T}_{\zeta_\theta}(s \odot t) \geq \min\{\mathcal{T}_{\zeta_\theta}(s), \mathcal{T}_{\zeta_\theta}(t)\}$,

(ii) $\mathcal{I}_{\zeta_\theta}(s \odot t) \geq \min\{\mathcal{I}_{\zeta_\theta}(s), \mathcal{I}_{\zeta_\theta}(t)\}$,

(iii) $\mathcal{F}_{\zeta_\theta}(s \odot t) \leq \max\{\mathcal{F}_{\zeta_\theta}(s), \mathcal{F}_{\zeta_\theta}(t)\}$ *for all* $s, t \in G$.

A single-valued neutrosophic soft K-algebra also satisfies the following properties:

$$\mathcal{T}_{\zeta_\theta}(e) \geq \mathcal{T}_{\zeta_\theta}(s), \quad \mathcal{I}_{\zeta_\theta}(e) \geq \mathcal{I}_{\zeta_\theta}(s),$$
$$\mathcal{F}_{\zeta_\theta}(e) \leq \mathcal{F}_{\zeta_\theta}(s) \text{ for all } s \neq e \in G.$$

Example 4.2 Consider a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where G is the cyclic group of order 9 given as $G = \{e, w, w^2, w^3, w^4, w^5, w^6, w^7, w^8\}$. Consider the following Cayley's table:

$\odot$	e	w	w^2	w^3	w^4	w^5	w^6	w^7	w^8
e	e	w^8	w^7	w^6	w^5	w^4	w^3	w^2	w
w	w	e	w^8	w^7	w^6	w^5	w^4	w^3	w^2
w^2	w^2	w	e	w^8	w^7	w^6	w^5	w^4	w^3
w^3	w^3	w^2	w	e	w^8	w^7	w^6	w^5	w^4
w^4	w^4	w^3	w^2	w	e	w^8	w^7	w^6	w^5
w^5	w^5	w^4	w^3	w^2	w	e	w^8	w^7	w^6
w^6	w^6	w^5	w^4	w^3	w^2	w	e	w^8	w^7
w^7	w^7	w^6	w^5	w^4	w^3	w^2	w	e	w^8
w^8	w^8	w^7	w^6	w^5	w^4	w^3	w^2	w	e

Consider a set of parameters $\mathbb{M} = \{l_1, l_2, l_3,\}$ and a set-valued function $\zeta : \mathbb{M} \to P(G)$, where the membership, indeterminacy-membership and non-membership values of the elements of G at parameters l_1, l_2, l_3 are given as:

(i) $\mathcal{T}_{\zeta_{l_1}}(e) = 0.9$, $\mathcal{I}_{\zeta_{l_1}}(e) = 0.3$, $\mathcal{F}_{\zeta_{l_1}}(e) = 0.3$, $\mathcal{T}_{\zeta_{l_1}}(s) = 0.6$, $\mathcal{I}_{\zeta_{l_1}}(s) = 0.2$, $\mathcal{F}_{\zeta_{l_1}}(s) = 0.4$,

(ii) $\mathcal{T}_{\zeta_{l_2}}(e) = 0.8$, $\mathcal{I}_{\zeta_{l_2}}(e) = 0.7$, $\mathcal{F}_{\zeta_{l_2}}(e) = 0.4$, $\mathcal{T}_{\zeta_{l_2}}(s) = 0.7$, $\mathcal{I}_{\zeta_{l_2}}(s) = 0.6$, $\mathcal{F}_{\zeta_{l_2}}(s) = 0.5$,

(iii) $\mathcal{T}_{\zeta_{l_3}}(e) = 0.9$, $\mathcal{I}_{\zeta_{l_3}}(e) = 0.6$, $\mathcal{F}_{\zeta_{l_3}}(e) = 0.6$, $\mathcal{T}_{\zeta_{l_3}}(s) = 0.8$, $\mathcal{I}_{\zeta_{l_3}}(s) = 0.5$, $\mathcal{F}_{\zeta_{l_3}}(s) = 0.7$

for all $s \neq e \in G$. The function ζ is defined as:

$$\begin{aligned}
\zeta(l_1) =& \{(e, 0.9, 0.3, 0.3), (w, 0.6, 0.2, 0.4), (w^2, 0.6, 0.2, 0.4), (w^3, 0.6, 0.2, 0.4),\\
&(w^4, 0.6, 0.2, 0.4), (w^5, 0.6, 0.2, 0.4), (w^6, 0.6, 0.2, 0.4),\\
&(w^7, 0.6, 0.2, 0.4), (w^8, 0.6, 0.2, 0.4)\},\\
\zeta(l_2) =& \{(e, 0.8, 0.7, 0.4), (w, 0.7, 0.6, 0.5), (w^2, 0.7, 0.6, 0.5), (w^3, 0.7, 0.6, 0.5),\\
&(w^4, 0.7, 0.6, 0.5), (w^5, 0.7, 0.6, 0.5), (w^6, 0.7, 0.6, 0.5),\\
&(w^7, 0.7, 0.6, 0.5), (w^8, 0.7, 0.6, 0.5)\},\\
\zeta(l_3) =& \{(e, 0.9, 0.6, 0.6), (w, 0.8, 0.5, 0.7), (w^2, 0.8, 0.5, 0.7), (w^3, 0.8, 0.5, 0.7),
\end{aligned}$$

$$(w^4, 0.8, 0.5, 0.7), (w^5, 0.8, 0.5, 0.7), (w^6, 0.8, 0.5, 0.7),$$
$$(w^7, 0.8, 0.5, 0.7), (w^8, 0.8, 0.5, 0.7)\}.$$

Consider a set $\mathbb{N} = \{l_1, l_2\}$ of parameters and a set-valued function $\eta : \mathbb{N} \rightarrow P(G)$, where the membership, indeterminacy-membership and non- membership values of the elements of G at parameters l_1, l_2 are defined as:

(i) $\mathcal{T}_{\eta_{l_1}}(e) = 0.9$, $\mathcal{I}_{\eta_{l_1}}(e) = 0.8$, $\mathcal{F}_{\eta_{l_1}}(e) = 0.2$, $\mathcal{T}_{\eta_{l_1}}(s) = 0.5$, $\mathcal{I}_{\eta_{l_1}}(s) = 0.2$, $\mathcal{F}_{\eta_{l_1}}(s) = 0.5$,

(ii) $\mathcal{T}_{\eta_{l_2}}(e) = 0.3$, $\mathcal{I}_{\eta_{l_2}}(e) = 0.5$, $\mathcal{F}_{\eta_{l_2}}(e) = 0.6$, $\mathcal{T}_{\eta_{l_2}}(s) = 0.1$, $\mathcal{I}_{\eta_{l_2}}(s) = 0.4$, $\mathcal{F}_{\eta_{l_2}}(s) = 0.8$

for all $s \neq e \in G$. The function η is defined as:

$$\begin{aligned}\eta(l_1) =& \{(e, 0.9, 0.8, 0.2), (w, 0.5, 0.2, 0.5), (w^2, 0.5, 0.2, 0.5), (w^3, 0.5, 0.2, 0.5),\\ &(w^4, 0.5, 0.2, 0.5), (w^5, 0.5, 0.2, 0.5), (w^6, 0.5, 0.2, 0.5),\\ &(w^7, 0.5, 0.2, 0.5), (w^8, 0.5, 0.2, 0.5)\},\\ \eta(l_2) =& \{(e, 0.3, 0.5, 0.6), (w, 0.1, 0.4, 0.8), (w^2, 0.1, 0.4, 0.8), (w^3, 0.1, 0.4, 0.8),\\ &(w^4, 0.1, 0.4, 0.8), (w^5, 0.1, 0.4, 0.8), (w^6, 0.1, 0.4, 0.8),\\ &(w^7, 0.1, 0.4, 0.8), (w^8, 0.1, 0.4, 0.8)\}.\end{aligned}$$

Evidently, the set $(\zeta, \mathbb{M})$ and the set $(\eta, \mathbb{N})$ are SNSSs. Since $\zeta(\theta)$, $\eta(\theta)$ are single-valued neutrosophic K-subalgebrasfor all $\theta \in \mathbb{M}$ and $\theta \in \mathbb{N}$. It concludes that the pairs $(\zeta, \mathbb{M})$, $(\eta, \mathbb{N})$ are single-valued neutrosophic soft K-subalgebras.

Example 4.3 Consider K-algebra on dihedral group $D4$ given as $G = \{e, a, b, c, w, x, y, z\}$, where $c = ab, w = a^2, x = a^3, y = a^2b, z = a^3b$ and Caley's table for $\odot$ is given as:

$\odot$	e	a	b	c	w	x	y	z
e	e	x	b	c	w	a	y	z
a	a	e	c	y	x	w	z	b
b	b	c	e	x	y	z	w	a
c	c	y	a	e	z	b	x	w
w	w	a	y	z	e	x	b	c
x	x	w	z	b	a	e	c	y
y	y	z	w	a	b	c	e	x
z	z	b	x	w	c	y	a	e

Consider a set of parameters $\mathbb{M} = \{l_1, l_2, l_3, \}$ and a set-valued function $\zeta : \mathbb{M} \rightarrow P(G)$, where the membership, indeterminacy-membership and non-membership values of the elements of G at parameters l_1, l_2, l_3 are given as:

(i) $\mathcal{T}_{\zeta_{l_1}}(e) = 0.7$, $\mathcal{I}_{\zeta_{l_1}}(e) = 0.7$, $\mathcal{F}_{\zeta_{l_1}}(e) = 0.3$, $\mathcal{T}_{\zeta_{l_1}}(s) = 0.5$, $\mathcal{I}_{\zeta_{l_1}}(s) = 0.2$, $\mathcal{F}_{\zeta_{l_1}}(s) = 0.7$,

(ii) $\mathcal{T}_{\zeta_{l_2}}(e) = 0.9$, $\mathcal{I}_{\zeta_{l_2}}(e) = 0.8$, $\mathcal{F}_{\zeta_{l_2}}(e) = 0.4$, $\mathcal{T}_{\zeta_{l_2}}(s) = 0.2$, $\mathcal{I}_{\zeta_{l_2}}(s) = 0.2$, $\mathcal{F}_{\zeta_{l_2}}(s) = 0.9$,

(iii) $\mathcal{T}_{\zeta_{l_3}}(e) = 0.5$, $\mathcal{I}_{\zeta_{l_3}}(e) = 0.5$, $\mathcal{F}_{\zeta_{l_3}}(e) = 0.3$, $\mathcal{T}_{\zeta_{l_3}}(s) = 0.1$, $\mathcal{I}_{\zeta_{l_3}}(s) = 0.3$, $\mathcal{F}_{\zeta_{l_3}}(s) = 0.8$

for all $s \neq e \in G$. The function ζ is defined as:

$$\zeta(l_1) =\{(e, 0.7, 0.7, 0.3), (a, 0.5, 0.2, 0.7), (b, 0.5, 0.2, 0.7), (c, 0.5, 0.2, 0.7),$$

$$
\begin{aligned}
&(w,0.5,0.2,0.7),(x,0.5,0.2,0.7),(y,0.5,0.2,0.7),(z,0.5,0.2,0.7)\},\\
\zeta(l_2)=&\{(e,0.9,0.8,0.4),(a,0.2,0.2,0.9),(b,0.2,0.2,0.9),(c,0.2,0.2,0.9),\\
&(w,0.2,0.2,0.9),(x,0.2,0.2,0.9),(y,0.2,0.2,0.9),(z,0.2,0.2,0.9)\},\\
\zeta(l_3)=&\{(e,0.5,0.5,0.3),(a,0.1,0.3,0.8),(b,0.1,0.3,0.8),(c,0.1,0.3,0.8),\\
&(w,0.1,0.3,0.8),(x,0.1,0.3,0.8),(y,0.1,0.3,0.8),(z,0.1,0.3,0.8)\}.
\end{aligned}
$$

Consider a set $\mathbb{N}=\{l_1,l_2\}$ of parameters and a set-valued function $\eta:\mathbb{N}\to P(G)$, where the truth, indeterminacy and falsity membership values of the elements of G at parameters l_1,l_2 are defined as:

(i) $\mathcal{T}_{\eta_{l_1}}(e)=0.8$, $\mathcal{I}_{\eta_{l_1}}(e)=0.8$, $\mathcal{F}_{\eta_{l_1}}(e)=0.2$, $\mathcal{T}_{\eta_{l_1}}(s)=0.6$, $\mathcal{I}_{\eta_{l_1}}(s)=0.3$, $\mathcal{F}_{\eta_{l_1}}(s)=0.7$,

(ii) $\mathcal{T}_{\eta_{l_2}}(e)=0.6$, $\mathcal{I}_{\eta_{l_2}}(e)=0.4$, $\mathcal{F}_{\eta_{l_2}}(e)=0.3$, $\mathcal{T}_{\eta_{l_2}}(s)=0.5$, $\mathcal{I}_{\eta_{l_2}}(s)=0.4$, $\mathcal{F}_{\eta_{l_2}}(s)=0.9$

for all $s\neq e\in G$. The function η is defined as:

$$
\begin{aligned}
\eta(l_1)=&\{(e,0.8,0.8,0.2),(a,0.6,0.3,0.7),(b,0.6,0.3,0.7),(c,0.6,0.3,0.7),\\
&(w,0.6,0.3,0.7),(x,0.6,0.3,0.7),(y,0.6,0.3,0.7),(z,0.6,0.3,0.7)\},\\
\eta(l_2)=&\{(e,0.6,0.4,0.3),(a,0.5,0.4,0.9),(b,0.5,0.4,0.9),(c,0.5,0.4,0.9),\\
&(w,0.5,0.4,0.9),(x,0.5,0.4,0.9),(y,0.5,0.4,0.9),(z,0.5,0.4,0.9)\}.
\end{aligned}
$$

Obviously, the set $(\zeta,\mathbb{M})$ and $(\eta,\mathbb{N})$ are SNSSs. Since for $\theta\in\mathbb{M}$ and $\theta\in\mathbb{N}$, the sets $\zeta(\theta)$, $\eta(\theta)$ are single-valued neutrosophic K-subalgebras. This concludes that the pair $(\zeta,\mathbb{M})$ and $(\eta,\mathbb{N})$ are single-valued neutrosophic soft K-subalgebras.

Proposition 4.4 *Let $(\zeta,\mathbb{M})$ and $(\eta,\mathbb{N})$ be two single-valued neutrosophic soft K-subalgebras. Then the extended intersection of $(\zeta,\mathbb{M})$ and $(\eta,\mathbb{N})$ is a single-valued neutrosophic soft K-subalgebra.*

Proof For any $\theta\in\mathbb{Q}$, following three cases arise.

First Case. If $\theta\in\mathbb{M}-\mathbb{N}$, then $\vartheta(\theta)=\zeta(\theta)$ and $\zeta(\theta)$ being single-valued neutrosophic K-subalgebra implies that $\vartheta(\theta)$ is also a single-valued neutrosophic K-subalgebra since $(\zeta,\mathbb{M})$ is a SNS K-subalgebra.

Second Case. If $\theta\in\mathbb{N}-\mathbb{M}$, then $\vartheta(\theta)=\eta(\theta)$ and $\eta(\theta)$ being single-valued neutrosophic K-subalgebra implies that $\vartheta(\theta)$ is a single-valued neutrosophic K-subalgebra since $(\eta,\mathbb{N})$ is a SNS K-subalgebra.

Third Case. Now if $\theta\in\mathbb{M}\cap\mathbb{N}$, then $\vartheta(\theta)=\zeta(\theta)\cap\eta(\theta)$ which is again a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. Thus, in any case, $\vartheta(\theta)$ is a single-valued neutrosophic K-subalgebra. Consequently, $(\zeta,\mathbb{M})\cap_{ex}(\eta,\mathbb{N})$ is a over K-algebra K-subalgebra over $\mathcal{K}$. □

Proposition 4.5 *If $(\zeta,\mathbb{M})$ and $(\eta,\mathbb{N})$ are two SNS K-subalgebras over $\mathcal{K}$, then $(\zeta,\mathbb{M})\wedge(\eta,\mathbb{N})$ is an SNS K-subalgebra.*

Proof Let $(l,m)\in\mathbb{Q}$, $\zeta(l)$, $\zeta(m)$ be single-valued neutrosophic K-subalgebras of $\mathcal{K}$, where $\mathbb{Q}=\mathbb{M}\times\mathbb{N}$, which implies that $\vartheta(l,m)=\zeta(l)\cap\eta(m)$ is also a single-valued neutrosophic K-

subalgebra over $\mathcal{K}$. Hence $(\zeta, \mathbb{M}) \wedge (\eta, \mathbb{N})$ is an SNS K-subalgebra of $\mathcal{K}$. $\square$

Proposition 4.6 *If $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ are two SNS K-subalgebras and $\zeta(l) \subseteq \eta(l)$ for all $l \in \mathbb{M}$, then $(\zeta, \mathbb{M})$ is an SNS K-subalgebra of $(\eta, \mathbb{N})$.*

Proof Since $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ are SNS K-subalgebras and $\zeta(l)$, $\eta(l)$ be two single-valued neutrosophic K-subalgebras also $\zeta(l) \subseteq \eta(l)$. Therefore, $(\zeta, \mathbb{M})$ is an SNS K-subalgebra of $(\eta, \mathbb{N})$. $\square$

Definition 4.7 Let $(\zeta, \mathbb{M})$ be a single-valued neutrosophic soft over Z. Then for each $\alpha, \beta, \gamma \in [0,1]$, the set $(\zeta, \mathbb{M})^{(\alpha,\beta,\gamma)} = (\zeta^{(\alpha,\beta,\gamma)}, \mathbb{M})$ is called an (α, β, γ)- level soft set of $(\zeta, \mathbb{M})$ and defined as:

$$\zeta_\theta^{(\alpha,\beta,\gamma)} = \{\mathcal{T}_{\zeta_\theta} \geq \alpha, \mathcal{I}_{\zeta_\theta} \geq \beta, \mathcal{F}_{\zeta_\theta} \leq \gamma\}, \quad \text{for all } \theta \in \mathbb{M}.$$

Theorem 4.8 *If $(\zeta, \mathbb{M})$ is a single-valued neutrosophic soft set over $\mathcal{K}$, then $(\zeta, \mathbb{M})$ is a single-valued neutrosophic soft K-subalgebra if and only if $(\zeta, \mathbb{M})^{(\alpha,\beta,\gamma)}$ is a soft K-subalgebrafor all $\alpha, \beta, \gamma \in [0, 1]$.*

Proof Consider that $(\zeta, \mathbb{M})$ is an SNS K-subalgebra. Then for all $\alpha, \beta, \gamma \in [0, 1]$, $\theta \in \mathbb{M}$ and $u_1, u_2 \in \zeta_\theta{}^{(\alpha,\beta,\gamma)}$, $\mathcal{T}_{\zeta_\theta}(u_1) \geq \alpha$, $\mathcal{T}_{\zeta_\theta}(u_2) \geq \alpha, \mathcal{I}_{\zeta_\theta}(u_1) \geq \beta, \mathcal{I}_{\zeta_\theta}(u_2) \geq \beta, \mathcal{F}_{\zeta_\theta}(u_1) \leq \gamma, \mathcal{F}_{\zeta_\theta}(u_2) \leq \gamma$.

It follows that $\mathcal{T}_{\zeta_\theta}(u_1 \odot u_2) \geq \min(\mathcal{T}_{\zeta_\theta}(u_1), \mathcal{T}_{\zeta_\theta}(u_2)) \geq \alpha, \mathcal{I}_{\zeta_\theta}(u_1 \odot (u_2)) \geq \min(\mathcal{I}_{\zeta_\theta}(u_1), \mathcal{I}_{\zeta_\theta}(u_2)) \geq \beta$, $\mathcal{F}_{\zeta_\theta}(u_1 \odot (u_2)) \leq \max(\mathcal{F}_{\zeta\theta}(u_1), \mathcal{F}_{\zeta_\theta}(u_2)) \leq \gamma$, which implies that $u_1 \odot u_2 \in \zeta_\theta{}^{(\alpha,\beta,\gamma)}$. Hence $\zeta_\theta{}^{(\alpha,\beta,\gamma)}$ is a soft K-subalgebrafor all $\alpha, \beta, \gamma \in [0, 1]$. Converse part is obvious. $\square$

Definition 4.9 *Let φ and ρ be two functions, where $\varphi : S_1 \to S_2$ and $\rho : \mathbb{M} \to \mathbb{N}$ and $\mathbb{M}$ and $\mathbb{N}$ are subsets of universe of parameters $\mathbb{R}$ from S_1 and S_2, respectively. The pair (φ, ρ) is said to be a single-valued neutrosophic soft function from S_1 to S_2.*

Definition 4.10 *Let the pair (φ, ρ) be a single-valued neutrosophic soft function from $\mathcal{K}_1$ into $\mathcal{K}_2$, then the pair (φ, ρ) is called a single-valued neutrosophic soft homomorphism if φ is a homomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$ and said to be a single-valued neutrosophic soft bijective homomorphism if φ is an isomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$ and ρ is an injective map from $\mathbb{M}$ to $\mathbb{N}$.*

Definition 4.11 *Let $(\zeta, \mathbb{M})$ and $(\eta, \mathbb{N})$ be two single-valued neutrosophic soft sets over G_1 and G_2, respectively and let (φ, ρ) be SNS function from G_1 into G_2. Then under the single-valued neutrosophic soft function (φ, ρ), image of $(\zeta, \mathbb{M})$ is a single-valued neutrosophic soft set on $\mathcal{K}_2$, denoted by $(\varphi, \rho)(\zeta, \mathbb{M})$ and defined as for all $l \in \rho(\mathbb{M})$ and $v \in G_2$, $(\varphi, \rho)(\zeta, \mathbb{M}) = (\varphi(\zeta), \rho(\mathbb{M}))$, where*

$$\mathcal{T}_{\varphi(\zeta)_l}(v) = \begin{cases} \bigvee_{\varphi(u)=v} \bigvee_{\rho(a)=l} \zeta_a(u), & \text{if } u \in \rho^{-1}(v), \\ 1, & \text{otherwise}, \end{cases}$$

$$\mathcal{I}_{\varphi(\zeta)_l}(v) = \begin{cases} \bigvee_{\varphi(u)=v} \bigvee_{\rho(a)=l} \zeta_a(u), & \text{if } u \in \rho^{-1}(v), \\ 1, & \text{otherwise}, \end{cases}$$

$$\mathcal{F}_{\varphi(\zeta)_l}(v) = \begin{cases} \bigwedge_{\varphi(u)=v} \bigwedge_{\rho(a)=l} \zeta_a(u), & \text{if } u \in \rho^{-1}(v), \\ 0, & \text{otherwise}. \end{cases}$$

Under the single-valued neutrosophic soft function (φ, ρ), *preimage of* $(\eta, \mathbb{N})$ *is denoted as* $(\varphi, \rho)^{-1}(\eta, \mathbb{N})$ *and defined as for all* $a \in \rho^{-1}(\mathbb{N})$ *and for all* $u \in G_1$, $(\varphi, \rho)^{-1}(\eta, \mathbb{N}) = (\varphi^{-1}(\eta), \rho^{-1}(\mathbb{N}))$, *where*

$$\mathcal{T}_{\varphi^{-1}(\eta)_a}(u) = \mathcal{T}_{\eta_{\rho(a)}}(\varphi(u)),$$
$$\mathcal{I}_{\varphi^{-1}(\eta)_a}(u) = \mathcal{I}_{\eta_{\rho(a)}}(\varphi(u)),$$
$$\mathcal{F}_{\varphi^{-1}(\eta)_a}(u) = \mathcal{F}_{\eta_{\rho(a)}}(\varphi(u)).$$

Theorem 4.12 *Let* (φ, ρ) *be a single-valued neutrosophic soft homomorphism from* $\mathcal{K}_1$ *to* $\mathcal{K}_2$ *and* $(\eta, \mathbb{N})$ *be a single-valued neutrosophic soft* K*-subalgebra on* $\mathcal{K}_2$. *Then* $(\varphi, \rho)^{-1}(\eta, \mathbb{N})$ *is an SNS* K*-subalgebra on* $\mathcal{K}_1$.

Proof Assume that $u_1, u_2 \in \mathcal{K}_1$, then we have

$$\varphi^{-1}(\mathcal{T}_{\eta\theta})(u_1 \odot u_2) = \mathcal{T}_{\eta_{\rho(\theta)}}(\varphi(u_1 \odot u_2)) = \mathcal{T}_{\eta_{\rho(\theta)}}(\varphi(u_1) \odot \varphi(u_1)),$$
$$\varphi^{-1}(\mathcal{T}_{\eta\theta})(u_1 \odot u_2) \geq \min\{\mathcal{T}_{\eta_{\rho(\theta)}}(\varphi(u_1)), \mathcal{T}_{\eta_{\rho(\theta)}}(\varphi(u_2))\},$$
$$\varphi^{-1}(\mathcal{T}_{\eta\theta})(u_1 \odot u_2) \geq \min\{\varphi^{-1}(\mathcal{T}_{\eta\theta})(u_1), \varphi^{-1}(\mathcal{T}_{\eta\theta})(u_2)\},$$
$$\varphi^{-1}(\mathcal{I}_{\eta\theta})(u_1 \odot u_2) = \mathcal{I}_{\eta_{\rho(\theta)}}(\varphi(u_1 \odot u_2)) = \mathcal{I}_{\eta_{\rho(\theta)}}(\varphi(u_1) \odot \varphi(u_1)),$$
$$\varphi^{-1}(\mathcal{I}_{\eta\theta})(u_1 \odot u_2) \geq \min\{\mathcal{I}_{\eta_{\rho(\theta)}}(\varphi(u_1)), \mathcal{I}_{\eta_{\rho(\theta)}}(\varphi(u_2))\},$$
$$\varphi^{-1}(\mathcal{I}_{\eta\theta})(u_1 \odot u_2) \geq \min\{\varphi^{-1}(\mathcal{I}_{\eta\theta})(u_1), \varphi^{-1}(\mathcal{I}_{\eta\theta})(u_2)\},$$
$$\varphi^{-1}(\mathcal{F}_{\eta\theta})(u_1 \odot u_2) = \mathcal{F}_{\eta_{\rho(\theta)}}(\varphi(u_1 \odot u_2)) = \mathcal{F}_{\eta_{\rho(\theta)}}(\varphi(u_1) \odot \varphi(u_1)),$$
$$\varphi^{-1}(\mathcal{F}_{\eta\theta})(u_1 \odot u_2) \leq \max\{\mathcal{F}_{\eta_{\rho(\theta)}}(\varphi(u_1)), \mathcal{F}_{\eta_{\rho(\theta)}}(\varphi(u_2))\},$$
$$\varphi^{-1}(\mathcal{F}_{\eta\theta})(u_1 \odot u_2) \leq \max\{\varphi^{-1}(\mathcal{F}_{\eta\theta})(u_1), \varphi^{-1}(\mathcal{F}_{\eta\theta})(u_2)\}.$$

Therefore, $(\varphi, \rho)^{-1}(\eta, \mathbb{N})$ is SNS K-subalgebra over $\mathcal{K}_1$. □

Remark 4.13 Let $(\zeta, \mathbb{M})$ be a single-valued neutrosophic soft K-subalgebra and let (φ, ρ) be a single-valued neutrosophic soft homomorphism from $\mathcal{K}_1$ into $\mathcal{K}_2$. Then $(\varphi, \rho)(\zeta, \mathbb{M})$ may not be a single-valued neutrosophic soft K-subalgebra over $\mathcal{K}_2$.

5. Neutrosophic topological K-algebras

Akram et al. [43] studied certain notions of neutrosophic topological K-algebras. This section is due to [43].

Definition 5.1 *Let* Z *be a nonempty set. A collection* χ *of single-valued neutrosophic sets in* Z *is called a single-valued neutrosophic topology on* Z *if the following conditions hold:*

(a) $\emptyset_{SN}, 1_{SN} \in \chi$;

(b) *If* $\mathcal{A}, \mathcal{B} \in \chi$, *then* $\mathcal{A} \bigcap \mathcal{B} \in \chi$;

(c) *If* $\mathcal{A}_i \in \chi$, $\forall i \in I$, *then* $\bigcup_{i \in I} \mathcal{A}_i \in \chi$.

The pair (Z, χ) *is called a single-valued neutrosophic topological space. Each member of* χ *is said to be* χ*-open or single-valued neutrosophic open set (SNOS) and compliment of each* *open single-valued neutrosophic set is a single-valued neutrosophic closed set (SNCS). A discrete topology is a topology which contains all single-valued neutrosophic subsets of Z and indiscrete if its elements are only $\emptyset_{SN}$, 1_{SN}.*

Definition 5.2 *Let $\mathcal{A} = (\mathcal{T}_\mathcal{A}, \mathcal{I}_\mathcal{A}, \mathcal{F}_\mathcal{A})$ be a single-valued neutrosophic set in $\mathcal{K}$. Then $\mathcal{A}$ is called a single-valued neutrosophic K-subalgebra of $\mathcal{K}$ if following conditions hold for $\mathcal{A}$:*

(i) $\mathcal{T}_\mathcal{A}(e) \geq \mathcal{T}_\mathcal{A}(s)$, $\mathcal{I}_\mathcal{A}(e) \geq \mathcal{I}_\mathcal{A}(s)$, $\mathcal{F}_\mathcal{A}(e) \leq \mathcal{F}_\mathcal{A}(s)$.

(ii) $\mathcal{T}_\mathcal{A}(s \odot t) \geq \min\{\mathcal{T}_\mathcal{A}(s), \mathcal{T}_\mathcal{A}(t)\}$, $\mathcal{I}_\mathcal{A}(s \odot t) \geq \min\{\mathcal{I}_\mathcal{A}(s), \mathcal{I}_\mathcal{A}(t)\}$, $\mathcal{F}_\mathcal{A}(s \odot t) \leq \max\{\mathcal{F}_\mathcal{A}(s), \mathcal{F}_\mathcal{A}(t)\}$ $\forall\ s, t \in \mathcal{K}$.

Example 5.3 Consider a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where group $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Caley's table for $\odot$ is given as:

$\odot$	e	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8
e	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x
x	x	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2
x^2	x^2	x	e	x^8	x^7	x^6	x^5	x^4	x^3
x^3	x^3	x^2	x	e	x^8	x^7	x^6	x^5	x^4
x^4	x^4	x^3	x^2	x	e	x^8	x^7	x^6	x^5
x^5	x^5	x^4	x^3	x^2	x	e	x^8	x^7	x^6
x^6	x^6	x^5	x^4	x^3	x^2	x	e	x^8	x^7
x^7	x^7	x^6	x^5	x^4	x^3	x^2	x	e	x^8
x^8	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x	e

We define a single-valued neutrosophic set $\mathcal{A}, \mathcal{B}$ in $\mathcal{K}$ such that:

$$\mathcal{A} = \{(e, 0.4, 0.5, 0.8), (s, 0.3, 0.4, 0.7)\},$$
$$\mathcal{B} = \{(e, 0.3, 0.4, 0.8), (s, 0.2, 0.3, 0.7)\}$$

$\forall\ s \neq e \in G$. According to Definition 5.1, the family $\{\emptyset_{SN}, 1_{SN}, \mathcal{A}, \mathcal{B}\}$ of single-valued neutrosophic sets of K-algebra is a single-valued neutrosophic topology on $\mathcal{K}$. We define a single-valued neutrosophic set $\mathcal{A} = \{\mathcal{T}_\mathcal{A}, \mathcal{I}_\mathcal{A}, \mathcal{F}_\mathcal{A}\}$ in $\mathcal{K}$ such that $\mathcal{T}_\mathcal{A}(e) = 0.7, \mathcal{I}_\mathcal{A}(e) = 0.5, \mathcal{F}_\mathcal{A}(e) = 0.2$, $\mathcal{T}_\mathcal{A}(s) = 0.2, \mathcal{I}_\mathcal{A}(s) = 0.4, \mathcal{F}_\mathcal{A}(s) = 0.6$. Clearly, $\mathcal{A} = (\mathcal{T}_\mathcal{A}, \mathcal{I}_\mathcal{A}, \mathcal{F}_\mathcal{A})$ is an SN K-subalgebra of $\mathcal{K}$.

Definition 5.4 *Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra and let $\chi_\mathcal{K}$ be a topology on $\mathcal{K}$. Let $\mathcal{A}$ be a single-valued neutrosophic set in $\mathcal{K}$ and let $\chi_\mathcal{K}$ be a topology on $\mathcal{K}$. Then an induced single-valued neutrosophic topology on $\mathcal{A}$ is a collection or family of single-valued neutrosophic subsets of $\mathcal{A}$ which are the intersection with $\mathcal{A}$ and single-valued neutrosophic open sets in $\mathcal{K}$ defined as: $\chi_\mathcal{A} = \{\mathcal{A} \cap F : F \in \chi_\mathcal{K}\}$. Then $\chi_\mathcal{A}$ is called single-valued neutrosophic induced topology on $\mathcal{A}$ or relative topology and the pair $(\mathcal{A}, \chi_\mathcal{A})$ is called an induced topological space or single-valued neutrosophic subspace of $(\mathcal{K}, \chi_\mathcal{K})$.*

Definition 5.5 *Let $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$ be two single-valued neutrosophic topologies and let $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$. Then f is called single-valued neutrosophic continuous if following* *conditions hold:*

(i) *For each single-valued neutrosophic set* $\mathcal{A} \in \chi_2$, $f^{-1}(\mathcal{A}) \in \chi_1$.

(ii) *For each SN* K*-subalgebra* $\mathcal{A} \in \chi_2$, $f^{-1}(\mathcal{A})$ *is an SN* K*-subalgebra* $\in \chi_1$.

Definition 5.6 *Let* $(\mathcal{K}_1, \chi_1)$ *and* $(\mathcal{K}_2, \chi_2)$ *be two single-valued neutrosophic topologies and let* $(\mathcal{A}, \chi_{\mathcal{A}})$ *and* $(\mathcal{B}, \chi_{\mathcal{B}})$ *be two single-valued neutrosophic subspaces over* $(\mathcal{K}_1, \chi_1)$ *and* $(\mathcal{K}_2, \chi_2)$*. Let* f *be a mapping from* $(\mathcal{K}_1, \chi_1)$ *into* $(\mathcal{K}_2, \chi_2)$*. Then* f *is a mapping from* $(\mathcal{A}, \chi_{\mathcal{A}})$ *to* $(\mathcal{B}, \chi_{\mathcal{B}})$ *if* $f(\mathcal{A}) \subset \mathcal{B}$.

Definition 5.7 *Let* f *be a mapping from* $(\mathcal{A}, \chi_{\mathcal{A}})$ *to* $(\mathcal{B}, \chi_{\mathcal{B}})$*. Then* f *is relatively single-valued neutrosophic continuous if for every SNOS* $Y_{\mathcal{B}}$ *in* $\chi_{\mathcal{B}}$, $f^{-1}(Y_{\mathcal{B}}) \cap \mathcal{A} \in \chi_{\mathcal{A}}$.

Definition 5.8 *Let* f *be a mapping from* $(\mathcal{A}, \chi_{\mathcal{A}})$ *to* $(\mathcal{B}, \chi_{\mathcal{B}})$*. Then* f *is relatively single-valued neutrosophic open if for every SNOS* $X_{\mathcal{A}}$ *in* $\chi_{\mathcal{A}}$*, the image* $f(X_{\mathcal{A}}) \in \chi_{\mathcal{B}}$.

Proposition 5.9 *Let* $(\mathcal{A}, \chi_{\mathcal{A}})$ *and* $(\mathcal{B}, \chi_{\mathcal{B}})$ *be single-valued neutrosophic subspaces of* $(\mathcal{K}_1, \chi_1)$ *and* $(\mathcal{K}_2, \chi_2)$*, where* $\mathcal{K}_1$ *and* $\mathcal{K}_2$ *are* K*-algebras. If* f *is a single-valued neutrosophic continuous function from* $\mathcal{K}_1$ *to* $\mathcal{K}_2$ *and* $f(\mathcal{A}) \subset \mathcal{B}$*. Then* f *is relatively single-valued neutrosophic continuous function from* $\mathcal{A}$ *into* $\mathcal{B}$.

Definition 5.10 *Let* $(\mathcal{K}_1, \chi_1)$ *and* $(\mathcal{K}_2, \chi_2)$ *be two single-valued neutrosophic topologies. A mapping* $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ *is called a single-valued neutrosophic homomorphism if following conditions hold:*

(i) f *is a one-one and onto function.*

(ii) f *is a single-valued neutrosophic continuous function from* $\mathcal{K}_1$ *to* $\mathcal{K}_2$.

(iii) f^{-1} *is a single-valued neutrosophic continuous function from* $\mathcal{K}_2$ *to* $\mathcal{K}_1$.

Theorem 5.11 *Let* $(\mathcal{K}_1, \chi_1)$ *be a single-valued neutrosophic topology and* $(\mathcal{K}_2, \chi_2)$ *be an indiscrete single-valued neutrosophic topology on* K*-algebras* $\mathcal{K}_1$ *and* $\mathcal{K}_2$*, respectively. Then each function* f *defined as:* $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ *is a single-valued neutrosophic continuous function from* $\mathcal{K}_1$ *to* $\mathcal{K}_2$*. If* $(\mathcal{K}_1, \chi_1)$ *and* $(\mathcal{K}_2, \chi_2)$ *are two discrete single-valued neutrosophic topology* $\mathcal{K}_1$ *and* $\mathcal{K}_2$*, respectively, then each homomorphism* $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ *is a single values neutrosophic continuous function from* $\mathcal{K}_1$ *to* $\mathcal{K}_2$.

Proof Let f be a mapping defined as $f : \mathcal{K}_1 \to \mathcal{K}_2$. Let χ_1 be single-valued neutrosophic topology on $\mathcal{K}_1$ and χ_2 be single-valued neutrosophic topology on $\mathcal{K}_2$, where $\chi_2 = \{\emptyset_{SN}, 1_{SN}\}$. We show that $f^{-1}(\mathcal{A})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$, i.e., for each $\mathcal{A} \in \chi_2$, $f^{-1}(\mathcal{A}) \in \chi_1$. Since $\chi_2 = \{\emptyset_{SN}, 1_{SN}\}$, then for any $u \in \chi_1$, consider $\emptyset_{SN} \in \chi_2$ such that $f^{-1}(\emptyset_{SN})(u) = \emptyset_{SN}(f(u)) = \emptyset_{SN}(u)$.

Therefore, $(f^{-1}(\emptyset_{SN})) = \emptyset_{SN} \in \chi_1$. Likewise, $(f^{-1}(1_{SN})) = 1_{SN} \in \chi_1$. Hence f is an SN continuous function from $\mathcal{K}_1$ to $\mathcal{K}_2$.

Now, for the second part of the theorem, where both χ_1 and χ_2 are single-valued neutrosophic topology on $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively and $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ is a homomorphism. Therefore, for all $\mathcal{A} \in \chi_2$ and $f^{-1}\mathcal{A} \in \chi_1$, where f is not a usual inverse homomorphism. To prove that $f^{-1}(\mathcal{A})$ is a single-valued neutrosophic K-subalgebra in of $\mathcal{K}_1$. Let for $u, v \in \mathcal{K}_1$,

$$\begin{aligned}
f^{-1}(\mathcal{T}_\mathcal{A})(u \odot v) =& \mathcal{T}_\mathcal{A}(f(u \odot v)) = \mathcal{T}_\mathcal{A}(f(u) \odot f(v)) \\
\geq& \min\{\mathcal{T}_\mathcal{A}(f(u)) \odot \mathcal{T}_(f(v))\} \\
=& \min\{f^{-1}(\mathcal{T}_\mathcal{A})(u), f^{-1}(\mathcal{T}_\mathcal{A})(v)\}, \\
f^{-1}(\mathcal{I}_\mathcal{A})(u \odot v) =& \mathcal{I}_\mathcal{A}(f(u \odot v)) = \mathcal{I}_\mathcal{A}(f(u) \odot f(v)) \\
\geq& \min\{\mathcal{I}_\mathcal{A}(f(u)) \odot \mathcal{I}_(f(v))\} \\
=& \min\{f^{-1}(\mathcal{I}_\mathcal{A})(u), f^{-1}(\mathcal{I}_\mathcal{A})(v)\}, \\
f^{-1}(\mathcal{F}_\mathcal{A})(u \odot v) =& \mathcal{F}_\mathcal{A}(f(u \odot v)) = \mathcal{F}_\mathcal{A}(f(u) \odot f(v)) \\
\leq& \max\{\mathcal{F}_\mathcal{A}(f(u)) \odot \mathcal{F}_(f(v))\} \\
=& \max\{f^{-1}(\mathcal{F}_\mathcal{A})(u), f^{-1}(\mathcal{F}_\mathcal{A})(v)\}.
\end{aligned}$$

Hence f is a single-valued neutrosophic continuous function from $\mathcal{K}_1$ to $\mathcal{K}_2$. □

Proposition 5.12 *Let χ_1 and χ_2 be two single-valued neutrosophic topologies on $\mathcal{K}$. Then each homomorphism $f : (\mathcal{K}, \chi_1) \to (\mathcal{K}, \chi_2)$ is a single-valued neutrosophic continuous function.*

Proof Let $(\mathcal{K}, \chi_1)$ and $(\mathcal{K}, \chi_2)$ be two single-valued neutrosophic topologies, where $\mathcal{K}$ is a K-algebra. To prove the above result, it suffices to show that the result is false for a particular topology. Let $\mathcal{A} = (\mathcal{T}_\mathcal{A}, \mathcal{I}_\mathcal{A}, \mathcal{F}_\mathcal{A})$ and $\mathcal{B} = (\mathcal{T}_\mathcal{B}, \mathcal{I}_\mathcal{B}, \mathcal{F}_\mathcal{B})$ be two single-valued neutrosophic sets in $\mathcal{K}$. Take $\chi_1 = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}\}$ and $\chi_2 = \{\emptyset_{SN}, 1_{SN}, \mathcal{B}\}$. If $f : (\mathcal{K}, \chi_1) \to (\mathcal{K}, \chi_2)$, defined by $f(u) = e \odot u$, for all $u \in \mathcal{K}$. Then f is a homomorphism. Now, for $u \in \mathcal{A}$, $v \in \chi_2$, $(f^{-1}(\mathcal{B}))(u) = \mathcal{B}(f(u)) = \mathcal{B}(e \odot u) = \mathcal{B}(u)$, $\forall\, u \in \mathcal{K}$, i.e., $f^{-1}(\mathcal{B}) = \mathcal{B}$. Therefore, $(f^{-1}(\mathcal{B})) \notin \chi_1$. Hence f is not a single-valued neutrosophic continuous mapping. □

Definition 5.13 *Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra and χ be a single-valued neutrosophic topology on $\mathcal{K}$. Let $\mathcal{A}$ be a single-valued neutrosophic K-algebra (K-subalgebra) of $\mathcal{K}$ and $\chi_\mathcal{A}$ be a single-valued neutrosophic topology on $\mathcal{A}$. Then $\mathcal{A}$ is said to be a single-valued neutrosophic topological K-algebra (K-subalgebra) on $\mathcal{K}$ if the self mapping $\rho_a : (\mathcal{A}, \chi_\mathcal{A}) \to (\mathcal{A}, \chi_\mathcal{A})$ defined as $\rho_a(u) = u \odot a$, $\forall a \in \mathcal{K}$, is a relatively single-valued neutrosophic continuous mapping.*

Theorem 5.14 *Let χ_1 and χ_2 be two single-valued neutrosophic topology on $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively and $f : \mathcal{K}_1 \to \mathcal{K}_2$ be a homomorphism such that $f^{-1}(\chi_2) = \chi_1$. If $\mathcal{A} = \{\mathcal{T}_\mathcal{A}, \mathcal{I}_\mathcal{A}, \mathcal{F}_\mathcal{A}\}$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}_2$, then $f^{-1}(\mathcal{A})$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}_1$.*

Proof Let $\mathcal{A} = \{\mathcal{T}_\mathcal{A}, \mathcal{I}_\mathcal{A}, \mathcal{F}_\mathcal{A}\}$ be a single-valued neutrosophic topological K-algebra of $\mathcal{K}_2$. We prove that $f^{-1}(\mathcal{A})$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}_1$. Let for any $u, v \in \mathcal{K}_1$,

$$\mathcal{T}_{f^{-1}(\mathcal{A})}(u \odot v) = \mathcal{T}_\mathcal{A}(f(u \odot v)) \geq \min\{\mathcal{T}_\mathcal{A}(f(u)), \mathcal{T}_\mathcal{A}(f(v))\}$$

$$
\begin{aligned}
&= \min\{\mathcal{T}_{f^{-1}(\mathcal{A})}(u), \mathcal{T}_{f^{-1}(\mathcal{A})}(v)\},\\
\mathcal{I}_{f^{-1}(\mathcal{A})}(u \odot v) &= \mathcal{I}_{\mathcal{A}}(f(u \odot v)) \geq \min\{\mathcal{I}_{\mathcal{A}}(f(u)), \mathcal{I}_{\mathcal{A}}(f(v))\}\\
&= \min\{\mathcal{I}_{f^{-1}(\mathcal{A})}(u), \mathcal{I}_{f^{-1}(\mathcal{A})}(v)\},\\
\mathcal{F}_{f^{-1}(\mathcal{A})}(u \odot v) &= \mathcal{F}_{\mathcal{A}}(f(u \odot v)) \leq \max\{\mathcal{F}_{\mathcal{A}}(f(u)), \mathcal{F}_{\mathcal{A}}(f(v))\}\\
&= \max\{\mathcal{F}_{f^{-1}(\mathcal{A})}(u), \mathcal{F}_{f^{-1}(\mathcal{A})}(v)\}.
\end{aligned}
$$

Hence $f^{-1}(\mathcal{A})$ is a single-valued neutrosophic K-algebra of $\mathcal{K}_1$.

Now, we prove that $f^{-1}(\mathcal{A})$ is single-valued neutrosophic topological K-algebra of $\mathcal{K}_1$. Since f is a single-valued neutrosophic continuous function, by Proposition 5.12, f is also a relatively single-valued neutrosophic continuous function which maps $(f^{-1}(\mathcal{A}), \chi_{f^{-1}(\mathcal{A})})$ to $(\mathcal{A}, \chi_{\mathcal{A}})$.

Let $a \in \mathcal{K}_1$ and Y be a single-valued neutrosophic set in $\chi_{\mathcal{A}}$, and let X be a single-valued neutrosophic set in $\chi_{f^{-1}(\mathcal{A})}$ such that

$$f^{-1}(Y) = X. \tag{5.1}$$

We are to prove that $\rho_a : (f^{-1}(\mathcal{A}), \chi_{f^{-1}(\mathcal{A})}) \to (f^{-1}(\mathcal{A}), \chi_{f^{-1}(\mathcal{A})})$ is relatively single-valued neutrosophic continuous mapping, then for any $a \in \mathcal{K}_1$, we have

$$
\begin{aligned}
\mathcal{T}_{\rho_a^{-1}(X)}(u) &= \mathcal{T}_{(X)}(\rho_a(u)) = \mathcal{T}_{(X)}(u \odot a)\\
&= \mathcal{T}_{f^{-1}(Y)}(u \odot a) = \mathcal{T}_{(Y)}(f(u \odot a))\\
&= \mathcal{T}_{(Y)}(f(u) \odot f(a)) = \mathcal{T}_{(Y)}(\rho_{f(a)}(f(u)))\\
&= \mathcal{T}_{\rho^{-1}f(a)Y}(f(u)) = \mathcal{T}_{f^{-1}}(\rho_{f(a)}^{-1}(Y)(u)),\\
\mathcal{I}_{\rho_a^{-1}(X)}(u) &= \mathcal{I}_{(X)}(\rho_a(u)) = \mathcal{I}_{(X)}(u \odot a)\\
&= \mathcal{I}_{f^{-1}(Y)}(u \odot a) = \mathcal{I}_{(Y)}(f(u \odot a))\\
&= \mathcal{I}_{(Y)}(f(u) \odot f(a)) = \mathcal{I}_{(Y)}(\rho_{f(a)}(f(u)))\\
&= \mathcal{I}_{\rho^{-1}f(a)Y}(f(u)) = \mathcal{I}_{f^{-1}}(\rho_{f(a)}^{-1}(Y)(u)),\\
\mathcal{F}_{\rho_a^{-1}(X)}(u) &= \mathcal{F}_{(X)}(\rho_a(u)) = \mathcal{F}_{(X)}(u \odot a)\\
&= \mathcal{F}_{f^{-1}(Y)}(u \odot a) = \mathcal{F}_{(Y)}(f(u \odot a))\\
&= \mathcal{F}_{(Y)}(f(u) \odot f(a)) = \mathcal{F}_{(Y)}(\rho_{f(a)}(f(u)))\\
&= \mathcal{F}_{\rho^{-1}f(a)Y}(f(u)) = \mathcal{F}_{f^{-1}}(\rho_{f(a)}^{-1}(Y)(u)).
\end{aligned}
$$

It concludes that $\rho_a^{-1}(X) = f^{-1}(\rho_{f(a)}^{-1}(Y))$. Thus, $\rho_a^{-1}(X) \cap f^{-1}(\mathcal{A}) = f^{-1}(\rho_{f(a)}^{-1}(Y)) \cap f^{-1}(\mathcal{A})$ is a single-valued neutrosophic set in $f^{-1}(\mathcal{A})$ and a single-valued neutrosophic set in $\chi_{f^{-1}(\mathcal{A})}$. Hence $f^{-1}(\mathcal{A})$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}$. The proof is completed. $\Box$

Theorem 5.15 *Let $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$ be two single-valued neutrosophic topologyies on $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively and let f be a bijective homomorphism of $\mathcal{K}_1$ into $\mathcal{K}_2$ such that $f(\chi_1) = \chi_2$. If $\mathcal{A}$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}_1$, then $f(\mathcal{A})$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}_2$.*

Proof Suppose that $\mathcal{A} = \{\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}}\}$ is an SN topological K-algebra of $\mathcal{K}_1$. We are to prove that $f(\mathcal{A})$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}_2$. Let for $u, v \in \mathcal{K}_2$,

$$f(\mathcal{A}) = (f_{\sup}(\mathcal{T}_{\mathcal{A}})(v), f_{\sup}(\mathcal{I}_{\mathcal{A}})(v), f_{\inf}(\mathcal{F}_{\mathcal{A}})(v)).$$

Let $a_o \in f^{-1}(u)$, $b_o \in f^{-1}(v)$ such that

$$\begin{aligned}
&\sup_{x\in f^{-1}(u)} \mathcal{T}_{\mathcal{A}}(x) = \mathcal{T}_{\mathcal{A}}(a_o), \quad \sup_{x\in f^{-1}(v)} \mathcal{T}_{\mathcal{A}}(x) = \mathcal{T}_{\mathcal{A}}(b_o),\\
&\sup_{x\in f^{-1}(u)} \mathcal{I}_{\mathcal{A}}(x) = \mathcal{I}_{\mathcal{A}}(a_o), \quad \sup_{x\in f^{-1}(v)} \mathcal{I}_{\mathcal{A}}(x) = \mathcal{I}_{\mathcal{A}}(b_o),\\
&\inf_{x\in f^{-1}(u)} \mathcal{F}_{\mathcal{A}}(x) = \mathcal{F}_{\mathcal{A}}(a_o), \quad \inf_{x\in f^{-1}(v)} \mathcal{F}_{\mathcal{A}}(x) = \mathcal{F}_{\mathcal{A}}(b_o).
\end{aligned}$$

Now

$$\begin{aligned}
\mathcal{T}_{f(\mathcal{A})}(u\odot v) &= \sup_{x\in f^{-1}(u\odot v)} \mathcal{T}_{\mathcal{A}}(x) \geq \ \mathcal{T}_{\mathcal{A}}(a_o, b_o) \geq \min\{\mathcal{T}_{\mathcal{A}}(a_o), \mathcal{T}_{\mathcal{A}}(b_o)\}\\
&= \min\{\sup_{x\in f^{-1}(u)} \mathcal{T}_{\mathcal{A}}(x), \sup_{x\in f^{-1}(v)} \mathcal{T}_{\mathcal{A}}(x)\}\\
&= \min\{\mathcal{T}_{f(\mathcal{A})}(u), \mathcal{T}_{f(\mathcal{A})}(v)\},\\
\mathcal{I}_{f(\mathcal{A})}(u\odot v) &= \sup_{x\in f^{-1}(u\odot v)} \mathcal{I}_{\mathcal{A}}(x) \geq \mathcal{I}_{\mathcal{A}}(a_o, b_o) \geq \min\{\mathcal{I}_{\mathcal{A}}(a_o), \mathcal{I}_{\mathcal{A}}(b_o)\}\\
&= \min\{\sup_{x\in f^{-1}(u)} \mathcal{I}_{\mathcal{A}}(x), \sup_{x\in f^{-1}(v)} \mathcal{I}_{\mathcal{A}}(x)\}\\
&= \min\{\mathcal{I}_{f(\mathcal{A})}(u), \mathcal{I}_{f(\mathcal{A})}(v)\},\\
\mathcal{F}_{f(\mathcal{A})}(u\odot v) &= \inf_{x\in f^{-1}(u\odot v)} \mathcal{F}_{\mathcal{A}}(x) \leq \ \mathcal{F}_{\mathcal{A}}(a_o, b_o) \leq \ \max\{\mathcal{F}_{\mathcal{A}}(a_o), \mathcal{F}_{\mathcal{A}}(b_o)\}\\
&= \max\{\inf_{x\in f^{-1}(u)} \mathcal{F}_{\mathcal{A}}(x), \inf_{x\in f^{-1}(v)} \mathcal{F}_{\mathcal{A}}(x)\}\\
&= \max\{\mathcal{F}_{f(\mathcal{A})}(u), \mathcal{F}_{f(\mathcal{A})}(v)\}.
\end{aligned}$$

Hence $f(\mathcal{A})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$. Now, we prove that the self mapping $\rho_b : (f(\mathcal{A}), \chi_f(\mathcal{A})) \to (f(\mathcal{A}), \chi_f(\mathcal{A}))$, defined by $\rho_b(v) = v \odot b$, for all $b \in \mathcal{K}_2$ is a relatively single-valued neutrosophic continuous mapping. Let $Y_{\mathcal{A}}$ be a single-valued neutrosophic set in $\chi_{\mathcal{A}}$. There exists a single-valued neutrosophic set "Y" in χ_1 such that $Y_{\mathcal{A}} = Y \cap \mathcal{A}$. We show that for a single-valued neutrosophic set in $\chi_{f(\mathcal{A})}$,

$$\rho^{-1}{}_b(Y_{f(\mathcal{A})}) \cap f(\mathcal{A}) \in \chi_{f(\mathcal{A})}.$$

Since f is an injective mapping, $f(Y_{\mathcal{A}}) = f(Y \cap \mathcal{A}) = f(Y) \cap f(\mathcal{A})$ is a single-valued neutrosophic set in $\chi_{f(\mathcal{A})}$ which shows that f is relatively single-valued neutrosophic open. Also f is surjective, then for all $b \in \mathcal{K}_2, a = f(b)$, where $a \in \mathcal{K}_1$. Now

$$\begin{aligned}
\mathcal{T}_{f^{-1}(\rho^{-1}{}_b(Y_{f(\mathcal{A})}))}(u) &= \mathcal{T}_{f^{-1}(\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})}))}(u) = \mathcal{T}_{\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})})}(f(u))\\
&= \mathcal{T}_{(Y_{f(\mathcal{A})})}(\rho_{f(a)}(f(u))) = \mathcal{T}_{(Y_{f(\mathcal{A})})}(f(u)\odot f(a))\\
&= \mathcal{T}_{f^{-1}(Y_{f(\mathcal{A})})}(u\odot a) = \mathcal{T}_{f^{-1}(Y_{f(\mathcal{A})})}(\rho_a(u))\\
&= \mathcal{T}_{\rho^{-1}(a)}(f^{-1}(Y_{f(\mathcal{A})}))(u),\\
\mathcal{I}_{f^{-1}(\rho^{-1}{}_b(Y_{f(\mathcal{A})}))}(u) &= \mathcal{I}_{f^{-1}(\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})}))}(u) = \mathcal{I}_{\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})})}(f(u))
\end{aligned}$$

$$\begin{aligned}
&= \mathcal{I}_{(Y_{f(\mathcal{A})})}(\rho_{f(a)}(f(u))) = \mathcal{I}_{(Y_{f(\mathcal{A})})}(f(u) \odot f(a)) \\
&= \mathcal{I}_{f^{-1}(Y_{f(\mathcal{A})})}(u \odot a) = \mathcal{I}_{f^{-1}(Y_{f(\mathcal{A})})}(\rho_a(u)) \\
&= \mathcal{I}_{\rho^{-1}(a)}(f^{-1}(Y_{f(\mathcal{A})}))(u), \\
\mathcal{F}_{f^{-1}(\rho^{-1}{}_b(Y_{f(\mathcal{A})}))}(u) &= \mathcal{F}_{f^{-1}(\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})}))}(u) = \mathcal{F}_{\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})})}(f(u)) \\
&= \mathcal{F}_{(Y_{f(\mathcal{A})})}(\rho_{f(a)}(f(u))) = \mathcal{F}_{(Y_{f(\mathcal{A})})}(f(u) \odot f(a)) \\
&= \mathcal{F}_{f^{-1}(Y_{f(\mathcal{A})})}(u \odot a) = \mathcal{F}_{f^{-1}(Y_{f(\mathcal{A})})}(\rho_a(u)) \\
&= \mathcal{F}_{\rho^{-1}(a)}(f^{-1}(Y_{f(\mathcal{A})}))(u).
\end{aligned}$$

This implies that $f^{-1}(\rho^{-1}_{(b)}((Y_{f(\mathcal{A})}))) = \rho^{-1}_{(a)}(f^{-1}(Y_{(\mathcal{A})}))$. Since $\rho_a : (\mathcal{A}, \chi_\mathcal{A}) \to (\mathcal{A}, \chi_\mathcal{A})$ is relatively single-valued neutrosophic continuous mapping and f is relatively single-valued neutrosophic continues mapping from $(\mathcal{A}, \chi_\mathcal{A})$ into $(f(\mathcal{A}), \chi_{f(\mathcal{A})})$. Thus, $f^{-1}(\rho^{-1}_{(b)}((Y_{f(\mathcal{A})}))) \cap \mathcal{A} = \rho^{-1}_{(a)}(f^{-1}(Y_{(\mathcal{A})})) \cap \mathcal{A}$ is a single-valued neutrosophic set in $\chi_\mathcal{A}$. Hence $f(f^{-1}(\rho_{(b)}((Y_{f(\mathcal{A})}))) \cap \mathcal{A}) = \rho^{-1}_{(b)}(Y_{f(\mathcal{A})}) \cap f(\mathcal{A})$ is a single-valued neutrosophic set in $\chi_\mathcal{A}$, which completes the proof. $\square$

Example 5.16 Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Caley's table for $\odot$ is given in Example 5.3. We define a single-valued neutrosophic set as:

$$\begin{aligned}
\mathcal{A} &= \{(e, 0.4, 0.5, 0.8), (s, 0.3, 0.4, 0.6)\}, \\
\mathcal{B} &= \{(e, 0.3, 0.4, 0.8), (s, 0.2, 0.3, 0.6)\},
\end{aligned}$$

for all $s \neq e \in G$, where $\mathcal{A}, \mathcal{B} \in [0, 1]$. The collection $\chi_\mathcal{K} = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}, \mathcal{B}\}$ of single-valued neutrosophic sets of $\mathcal{K}$ is a single-valued neutrosophic topology on $\mathcal{K}$ and $(\mathcal{K}, \chi_\mathcal{K})$ is a single-valued neutrosophic topology. Let $\mathcal{C}$ be a single-valued neutrosophic set in $\mathcal{K}$, defined as:

$$\mathcal{C} = \{(e, 0.7, 0.5, 0.2), (s, 0.5, 0.4, 0.6)\}, \quad \forall s \neq e \in G.$$

Clearly, $\mathcal{C}$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. By direct calculations relative topology $\chi_\mathcal{C}$ is obtained as $\chi_\mathcal{C} = \{\emptyset_\mathcal{A}, 1_\mathcal{A}, \mathcal{A}\}$. Then the pair $(\mathcal{C}, \chi_\mathcal{C})$ is a single-valued neutrosophic subspace of $(\mathcal{K}, \chi_\mathcal{K})$. We show that $\mathcal{C}$ is a single-valued neutrosophic topological K-subalgebra of $\mathcal{K}$, i.e., the self mapping $\rho_a : (\mathcal{C}, \chi_\mathcal{C}) \to (\mathcal{C}, \chi_\mathcal{C})$ defined by $\rho_a(u) = u \odot a, \forall a \in \mathcal{K}$ is relatively single-valued neutrosophic continuous mapping, i.e., for an SNOS $\mathcal{A}$ in $(\mathcal{C}, \chi_\mathcal{C})$, $\rho_a^{-1}(\mathcal{A}) \cap \mathcal{C} \in \chi_\mathcal{C}$. Since ρ_a is homomorphism, then $\rho_a^{-1}(\mathcal{A}) \cap \mathcal{C} = \mathcal{A} \in \chi_\mathcal{C}$. Therefore, $\rho_a : (\mathcal{C}, \chi_\mathcal{C}) \to (\mathcal{C}, \chi_\mathcal{C})$ is relatively single-valued neutrosophic continuous mapping. Hence $\mathcal{C}$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}$.

Example 5.17 Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Caley's table for $\odot$ is given in Example 5.3. We define a single-valued neutrosophic set as:

$$\begin{aligned}
\mathcal{A} &= \{(e, 0.4, 0.5, 0.8), (s, 0.3, 0.4, 0.6)\}, \\
\mathcal{B} &= \{(e, 0.3, 0.4, 0.8), (s, 0.2, 0.3, 0.6)\}, \\
\mathcal{D} &= \{(e, 0.2, 0.1, 0.3), (s, 0.1, 0.1, 0.5)\},
\end{aligned}$$

for all $s \neq e \in G$, where $\mathcal{A}, \mathcal{B} \in [0,1]$. The collection $\chi_1 = \{\emptyset_{SN}, 1_{SN}, \mathcal{D}\}$ and $\chi_2 = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}, \mathcal{B}\}$ of single-valued neutrosophic sets of $\mathcal{K}$ are single-valued neutrosophic topology on $\mathcal{K}$ and $(\mathcal{K}, \chi_1)$, $(\mathcal{K}, \chi_2)$ are two single-valued neutrosophic topologies. Let $\mathcal{C}$ be a single-valued neutrosophic set in $(\mathcal{K}, \chi_2)$, defined as:

$$\mathcal{C} = \{(e, 0.7, 0.5, 0.2), (s, 0.5, 0.4, 0.6)\}, \quad \forall s \neq e \in G.$$

Now, let $f : (\mathcal{K}, \chi_1) \to (\mathcal{K}, \chi_2)$ be a homomorphism such that $f^{-1}(\chi_2) = \chi_1$ (we have not considered $\mathcal{K}$ to be distinct). Then by Proposition 5.9, f is a single-valued neutrosophic continuous function also f is relatively single-valued neutrosophic continues mapping from $(\mathcal{K}, \chi_1)$ into $(\mathcal{K}, \chi_2)$. Since $\mathcal{C}$ is a single-valued neutrosophic set in $(\mathcal{K}, \chi_2)$ and with relative topology $\chi_{\mathcal{C}} = \{\emptyset_{\mathcal{A}}, 1_{\mathcal{A}}, \mathcal{A}\}$ is also a single-valued neutrosophic topological K-algebra of $(\mathcal{K}, \chi_2)$. We prove that $f^{-1}(\mathcal{C})$ is a single-valued neutrosophic topological K-algebra in $(\mathcal{K}, \chi_1)$. Since f is a continuous function, by Definition 5.5 $f^{-1}(\mathcal{C})$ is a single-valued neutrosophic K-subalgebra in $(\mathcal{K}, \chi_1)$. To prove that $f^{-1}(c)$ is a single-valued neutrosophic topological K-algebra, then for $b \in \mathcal{K}_1$ take

$$\rho_b : (f^{-1}(\mathcal{C}), \chi_{f^{-1}(\mathcal{C})}) \to (f^{-1}(\mathcal{C}), \chi_{f^{-1}(\mathcal{C})}),$$

for $\mathcal{A} \in \chi_{f^{-1}(C)}, \rho_b^{-1}(\mathcal{A}) \cap f^{-1}(\mathcal{C}) \in \chi_{f^{-1}(C)}$ which shows that $f^{-1}(C)$ is a single-valued neutrosophic topological K-algebra in $(\mathcal{K}, \chi_1)$. Similarly we can show that $f(\mathcal{C})$ is a single-valued neutrosophic topological K-algebra in $(\mathcal{K}, \chi_2)$ by considering a bijective homomorphism.

Definition 5.18 *Let χ be a single-valued neutrosophic topology on $\mathcal{K}$ and $(\mathcal{K}, \chi)$ be a single-valued neutrosophic topology. Then $(\mathcal{K}, \chi)$ is called single-valued neutrosophic C_5-disconnected topological space if there exist an SNOS and SNCS $\mathcal{H}$ such that $\mathcal{H} = (\mathcal{T}_{\mathcal{H}}, \mathcal{I}_{\mathcal{H}}, \mathcal{F}_{\mathcal{H}},) \neq 1_{SN}$ and $\mathcal{H} = (\mathcal{T}_{\mathcal{H}}, \mathcal{I}_{\mathcal{H}}, \mathcal{F}_{\mathcal{H}},) \neq \emptyset_{SN}$, otherwise $(\mathcal{K}, \chi)$ is called single-valued neutrosophic C_5-connected.*

Example 5.19 Every indiscrete single-valued neutrosophic topology space on $\mathcal{K}$ is C_5-connected.

Proposition 5.20 *Let $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$ be two single-valued neutrosophic topologies and $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ be a surjective single-valued neutrosophic continuous mapping. If $(\mathcal{K}_1, \chi_1)$ is a single-valued neutrosophic C_5-connected space, then $(\mathcal{K}_2, \chi_2)$ is also a single-valued neutrosophic C_5-connected space.*

Proof Suppose on contrary that $(\mathcal{K}_2, \chi_2)$ is a single-valued neutrosophic C_5-disconnected space. Then by Definition 5.18, there exist both SNOS and SNCS $\mathcal{H}$ such that $\mathcal{H} \neq 1_{SN}$ and $\mathcal{H} \neq \emptyset_{SN}$. Since f is a single-valued neutrosophic continuous and onto function, $f^{-1}(\mathcal{H}) = 1_{SN}$ or $f^{-1}(\mathcal{H}) = \emptyset_{SN}$, where $f^{-1}(\mathcal{H})$ is both SNOS and SNCS. Therefore,

$$\mathcal{H} = f(f^{-1}(\mathcal{H})) = f(1_{SN}) = 1_{SN}, \quad \mathcal{H} = f(f^{-1}(\mathcal{H})) = f(\emptyset_{SN}) = \emptyset_{SN},$$

a contradiction. Hence $(\mathcal{K}_2, \chi_2)$ is a single-valued neutrosophic C_5-connected space. □

Corollary 5.21 *Let χ be a single-valued neutrosophic topology on $\mathcal{K}$. Then $(\mathcal{K}, \chi)$ is called a single-valued neutrosophic C_5-connected space if and only if there does not exist a single-valued neutrosophic continuous map $f : (\mathcal{K}, \chi) \to (\mathcal{F}_T, \chi_T)$ such that $f \neq 1_{SN}$ and $f \neq \emptyset_{SN}$.*

Definition 5.22 *Let $\mathcal{A} = \{\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}}\}$ be a single-valued neutrosophic set in $\mathcal{K}$. Let χ be a single-valued neutrosophic topology on $\mathcal{K}$. The interior and closure of $\mathcal{A}$ in $\mathcal{K}$ is defined as:*

$\mathcal{A}^{\mathrm{Int}}$*: The union of SNOSs contained in $\mathcal{A}$.*

$\mathcal{A}^{\mathrm{Clo}}$*:The intersection of SNCSs for which $\mathcal{A}$ is a subset of these SNCSs.*

Remark 5.23 Being union of SNOS, $\mathcal{A}^{\mathrm{Int}}$ is an SNO and $\mathcal{A}^{\mathrm{Clo}}$ being intersection of SNCS is SNC.

Theorem 5.24 *Let $\mathcal{A}$ be a single-valued neutrosophic set in a single-valued neutrosophic topology $(\mathcal{K}, \chi)$. Then $\mathcal{A}^{\mathrm{Int}}$ is such an open set which is the largest open set of $\mathcal{K}$ contained in $\mathcal{A}$.*

Corollary 5.25 *$\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is an SNOS in $\mathcal{K}$ if and only if $\mathcal{A}^{\mathrm{Int}} = \mathcal{A}$ and $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is an SNCS in $\mathcal{K}$ if and only if $\mathcal{A}^{\mathrm{Clo}} = \mathcal{A}$.*

Proposition 5.26 *Let $\mathcal{A}$ be a single-valued neutrosophic set in $\mathcal{K}$. Then following results hold for $\mathcal{A}$:*

(i) $(1_{SN})^{\mathrm{Int}} = 1_{SN}$.
(ii) $(\emptyset_{SN})^{\mathrm{Clo}} = \emptyset_{SN}$.
(iii) $\overline{(\mathcal{A})}^{\mathrm{Int}} = \overline{(\mathcal{A})^{\mathrm{Clo}}}$.
(iv) $\overline{(\mathcal{A})}^{\mathrm{Clo}} = \overline{(\mathcal{A})^{\mathrm{Int}}}$.

Definition 5.27 *Let $\mathcal{K}$ be a K-algebra and χ be a single-valued neutrosophic topology on $\mathcal{K}$. An SNOS $\mathcal{A}$ in $\mathcal{K}$ is said to be single-valued neutrosophic regular open if $\mathcal{A} = (\mathcal{A}^{\mathrm{Clo}})^{\mathrm{Int}}$.*

Remark 5.28 Every SNOS which is regular is single-valued neutrosophic open and every single-valued neutrosophic closed and open set is a single-valued neutrosophic regular open.

Definition 5.29 *A single-valued neutrosophic super connected K-algebra is such a K-algebra in which there does not exist a single-valued neutrosophic regular open set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ such that $\mathcal{A} \neq \emptyset_{SN}$ and $\mathcal{A} \neq 1_{SN}$. If there exists such a single-valued neutrosophic regular open set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ such that $\mathcal{A} \neq \emptyset_{SN}$ and $\mathcal{A} \neq 1_{SN}$, then K-algebra is said to be a single-valued neutrosophic super disconnected.*

Example 5.30 Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Caley's table for $\odot$ is given in Example 5.3. We define a single-valued neutrosophic set as:

$$\mathcal{A} = \{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\}.$$

Let $\chi_{\mathcal{K}} = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}\}$ be a single-valued neutrosophic topology on $\mathcal{K}$ and let $\mathcal{B} = \{(e, 0.3, 0.3, 0.8), (s, 0.2, 0.2, 0.6)\}$ be a single-valued neutrosophic set in $\mathcal{K}$. Here

SNOSs: $\emptyset_{SN} = \{0, 0, 1\}, 1_{SN} = \{1, 1, 0\}, \mathcal{A} = \{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\}$.

SNCSs: $(\emptyset_{SN})^c = (\{0, 0, 1\})^c = (\{1, 1, 0\}) = 1_{SN}, (1_{SN})^c = (\{1, 1, 0\})^c = (\{0, 0, 1\}) = \emptyset_{SN}$, $(\mathcal{A})^c = (\{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\})^c = (\{(e, 0.8, 0.3, 0.2), (s, 0.6, 0.2, 0.1)\}) = \mathcal{A}'$ (say). Then closure of $\mathcal{B}$ is the intersection of closed sets which contain $\mathcal{B}$. Therefore, $\mathcal{A}' = \mathcal{B}^{\mathrm{Clo}}$. Now, interior of $\mathcal{B}$ is the union of open sets contained in $\mathcal{B}$. Therefore,

$$\emptyset_{SN} \bigcup \mathcal{A} = \mathcal{A}, \quad \mathcal{A} = \mathcal{B}^{\mathrm{Int}}.$$

Note that $(\mathcal{B}^{\mathrm{Clo}})^{\mathrm{Clo}} = \mathcal{B}^{\mathrm{Clo}}$. Now, if we consider a single-valued neutrosophic set $\mathcal{A} = \{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\}$ in a K-algebra $\mathcal{K}$ and if $\chi_{\mathcal{K}} = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}\}$ is a single-valued neutrosophic topology on $\mathcal{K}$, then $(\mathcal{A})^{\mathrm{Clo}} = \mathcal{A}$ and $(\mathcal{A})^{\mathrm{Int}} = \mathcal{A}$. Consequently, $\mathcal{A} = (\mathcal{A}^{\mathrm{Clo}})^{\mathrm{Int}}$, which shows that $\mathcal{A}$ is an SN regular open set in K-algebra $\mathcal{K}$. Since $\mathcal{A}$ is an SN regular open set in $\mathcal{K}$ and $\mathcal{A} \neq \emptyset_{SN}, \mathcal{A} \neq 1_{SN}$, then by Definition 5.29, K-algebra $\mathcal{K}$ is a single-valued neutrosophic supper disconnected K-algebra.

Proposition 5.31 *Let $\mathcal{K}$ be a K-algebra and let $\mathcal{A}$ be an SNOS. Then the following statements are equivalent:*

(i) A K-algebra is single-valued neutrosophic super connected.

(ii) $(\mathcal{A})^{\mathrm{Clo}} = 1_{SN}$, for each SNOS $\mathcal{A} \neq \emptyset_{SN}$.

(iii) $(\mathcal{A})^{\mathrm{Int}} = \emptyset_{SN}$, for each SNCS $\mathcal{A} \neq 1_{SN}$.

(iv) There do not exist SNOSs $\mathcal{A}, \mathcal{F}$ such that $\mathcal{A} \subseteq \overline{\mathcal{F}}$ and $\mathcal{A} \neq \emptyset_{SN} \neq \mathcal{F}$ in K-algebra $\mathcal{K}$.

Definition 5.32 *Let $(\mathcal{K}, \chi)$ be a single-valued neutrosophic topology, where $\mathcal{K}$ is a K-algebra. Let S be a collection of SNOSs in $\mathcal{K}$ denoted by $S = \{(\mathcal{T}_{\mathcal{A}_j}, \mathcal{I}_{\mathcal{A}_j}, \mathcal{F}_{\mathcal{A}_j}) : j \in J\}$. Let $\mathcal{A}$ be an SNOS in $\mathcal{K}$. Then S is called a single-valued neutrosophic open covering of $\mathcal{A}$ if $\mathcal{A} \subseteq \bigcup S$.*

Definition 5.33 *Let $\mathcal{K}$ be a K-algebra and $(\mathcal{K}, \chi)$ be a single-valued neutrosophic topology. Let L be a finite sub-collection of S. If L is also a single-valued neutrosophic open covering of $\mathcal{A}$, then it is called a finite sub-covering of S and $\mathcal{A}$ is called single-valued neutrosophic compact if each single-valued neutrosophic open covering S of $\mathcal{A}$ has a finite sub-cover. Then $(\mathcal{K}, \chi)$ is called compact K-algebra.*

Remark 5.34 If either $\mathcal{K}$ is a finite K-algebra or χ is a finite topology on $\mathcal{K}$, i.e., consists of finite number of single-valued neutrosophic subsets of $\mathcal{K}$, then the single-valued neutrosophic topology $(\mathcal{K}, \chi)$ is a single-valued neutrosophic compact topological space.

Proposition 5.35 *Let $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$ be two single-valued neutrosophic topologies and f be a single-valued neutrosophic continuous mapping from $\mathcal{K}_1$ into $\mathcal{K}_2$. Let $\mathcal{A}$ be a single-valued neutrosophic set in $(\mathcal{K}_1, \chi_1)$. If $\mathcal{A}$ is single-valued neutrosophic compact in $(\mathcal{K}_1, \chi_1)$, then $f(\mathcal{A})$ is single-valued neutrosophic compact in $(\mathcal{K}_2, \chi_2)$.*

Proof Let $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ be a single-valued neutrosophic continuous function. Let $\acute{S} = (f^{-1}(\mathcal{A}_j : j \in J))$ be a single-valued neutrosophic open covering of $\mathcal{A}$ since $\mathcal{A}$ is a single-valued neutrosophic set in $(\mathcal{K}_1, \chi_1)$. Let $\acute{L} = (\mathcal{A}_j : j \in J)$ be a single-valued neutrosophic open covering of $f(\mathcal{A})$. Since $\mathcal{A}$ is compact, there exists a single-valued neutrosophic finite sub-cover $\bigcup_{j=1}^{n} f^{-1}(\mathcal{A}_j)$ such that

$$\mathcal{A} \subseteq \bigcup_{j=1}^{n} f^{-1}(\mathcal{A}_j).$$

We have to prove that there also exists a finite sub-cover of $\acute{L}$ for $f(\mathcal{A})$ such that $f(\mathcal{A}) \subseteq \bigcup_{j=1}^{n}(\mathcal{A}_j)$. Now

$$\mathcal{A} \subseteq \bigcup_{j=1}^{n} f^{-1}(\mathcal{A}_j), \quad f(\mathcal{A}) \subseteq f(\bigcup_{j=1}^{n} f^{-1}(\mathcal{A}_j))$$

$$f(\mathcal{A}) \subseteq \bigcup_{j=1}^{n}(f(f^{-1}(\mathcal{A}_j))), \quad f(\mathcal{A}) \subseteq \bigcup_{j=1}^{n}(\mathcal{A}_j).$$

Hence $f(\mathcal{A})$ is single-valued neutrosophic compact in $(\mathcal{K}_2, \chi_2)$. □

References

[1] G. BOOLE. *An Investigation of the Laws of Thought, on Which are Founded the Mathematical Theories of Logic and Probabilities*. Cambridge University Press, Cambridge, 2009.

[2] Y. IMAI, K. ISEKI. *On axiom systems of propositional calculi (XIV)*. Proc. Japan Acad., 1966, **42**: 19–22.

[3] K. ISEKI. *An algebra related with a propositional calculus*. Proc. Japan Acad., 1966, **42**: 26–29.

[4] A. B. SAEID, C. FLAUT. *Some connections between BCK-algebras and n-ary block codes*. Springer, Soft Computing, Issue 1, 2018.

[5] K. H. DAR, M. AKRAM. *On a K-algebra built on a group*. Southeast Asian Bull. Math., 2005, **29**(1): 41–49.

[6] K. H. DAR, M. AKRAM. *On K-homomorphisms of K-algebras*. Int. Math. Forum, 2007, **2**(45-48): 2283–2293.

[7] K. H. DAR, M. AKRAM. *Characterization of a $K(G)$-algebra by self maps*. Southeast Asian Bull. Math., 2004, **28**(4): 601–610.

[8] K. H. DAR, M. AKRAM. *Characterization of K-algebras by self maps (II)*. An. Univ. Craiova Ser. Mat. Inform., 2010, **37**(1): 96–103.

[9] M. AKRAM, K. H. DAR, K. P. SHUM. *Interval-valued (α, β)-fuzzy K-algebras*. Appl. Soft Computing, 2011, **11**(1): 1213–1222.

[10] M. AKRAM, N. O. ALSHEHRI, K. P. SHUM, et al. *Application of bipolar fuzzy soft sets in K-algebras*. Ital. J. Pure Appl. Math., 2014, **32**: 533–546.

[11] M. AKRAM, B. DAVVAZ, Feng FENG, *Intuitionistic fuzzy soft K-algebras*. Mathematics in Computer Science, 2013, **7**(3): 353–365.

[12] M. AKRAM, K. H. DAR, Y. B. JUN, et al. *Fuzzy structures of $K(G)$-algebra*. Southeast Asian Bull. Math., 2007, **31**(4): 625–637.

[13] M. AKRAM, K. H. DAR. *Generalized Fuzzy K-Algebras*. VDM Verlag, 2010.

[14] M. AKRAM, K. H. DAR. *On fuzzy topological K-algebras*. Int. Math. Forum, 2016, **1**(21-24): 1113–1124.

[15] M. AKRAM, K. H. DAR. *Intuitionistic fuzzy topological K-algebras*. J. Fuzzy Math., 2009, **17**(1): 19–34.

[16] M. AKRAM, N. O. ALSHEHRI, R. S. ALGHAMDI. *Fuzzy soft K-algebras*. Util. Math., 2013, **90**: 307–325.

[17] N. O. ALSHEHRI, M. AKRAM, R. S. ALGHAMDI. *Applications of soft sets in K-algebras*. Adv. Fuzzy Syst. 2013, Art. ID 319542, 8 pp.

[18] F. SMARANDACHE. *Neutrosophic set–a generalization of the intuitionistic fuzzy set*. Int. J. Pure Appl. Math., 2005, **24**(3): 287–297.

[19] L. A. ZADEH. *Fuzzy sets*. Information and Control, 1965, **8**(3): 338–353.

[20] K. ATANASSOV. *Intuitionistic fuzzy sets*. Fuzzy Sets and Systems, 1986, **20**(1): 87–96.

[21] H. WANG, F. SMARANDACHE, Y. Q. ZHANG, et al. *Single valued neutrosophic sets*. Multispace and Multistruct, 2010, **4**: 410–413.

[22] D. MOLODTSOV. *Soft set theory first results*. Comput. Math. Appl., 1999, **37**(4-5): 19–31.

[23] P. K. MAJI. *Neutrosophic soft set*. Ann. Fuzzy Math. Inform., 2013, **5**(1): 157–168.

[24] A. A. A. AGBOOLA, B. DAVVAZ. *Introduction to neutrosophic BCI/BCK-algebras*. Int. J. Math. Math. Sci., 2015, Art. ID 370267, 6 pp.

[25] Y. B. JUNE, S. J. KIM, F. SMARANDACHE. *Interval neutrosophic sets with applications in BCK/BCI-algebra*. Axioms, 2018, **7**(2): 23.

[26] Y. B. JUN, S. Z. SONG, F. SMARANDACHE, et al. *Neutrosophic quadruple BCK/BCI-algebras*. Axioms, 2018, **7**(2): 41.

[27] Y. B. JUN, F. SMARANDACHE, S. Z. SONG, et al. *Neutrosophic positive implicative N-ideals in BCK-algebras*. Axioms, 2018, **7**(1): 3.

[28] S. K. BAKHAT, P. DAS. *$(\in, \in \vee q)$-fuzzy subgroup*. Fuzzy Sets and Systems, 1996, **80**(3): 359–368.

[29] Xuehai YUAN, Cheng ZHANG, Yonghong REN. *Generalized fuzzy groups and many-valued implications*. Fuzzy Sets and Systems, 2003, **138**(1): 205–211.

[30] C. L. CHANG. *Fuzzy topological spaces*. J. Math. Anal. Appl., 1968, **24**(1): 182–190.

[31] R. LOWEN. *Fuzzy topological spaces and fuzzy compactness*. J. Math. Anal. Appl., 1976, **56**(3): 621–633.

[32] Paoming PU, Yingming LIU. *Fuzzy topology. I. Neighborhood structure of a fuzzy point and Moore-Smith convergence*. J. Math. Anal. Appl., 1980, **76**(2): 571–599.

[33] C. K. WONG. *Fuzzy points and local properties of fuzzy topology*. J. Math. Anal. Appl., 1974, **46**: 316–328.

[34] K. C. CHATTOPADHYAY, S. K. SAMANTA. *Fuzzy topology: fuzzy closure operator, fuzzy compactness and fuzzy connectedness*. Fuzzy Sets and Systems, 1993, **54**(2): 207–212.

[35] F. G. LUPIANEZ. *Hausdorffness in intuitionistic fuzzy topological spaces*. J. Fuzzy Math., 2004, **12**(3): 521–525.

[36] I. M. HANAFY. *Completely continuous functions in intuitionistic fuzzy topological spaces*. Czechoslovak Math. J., 2003, **53**(4): 793–803.

[37] D. COKER. *An introduction to intuitionistic fuzzy topological spaces*. Fuzzy Sets and Systems, 1997, **88**(1): 81–89.

[38] D. COKER, M. DEMIRCI. *On intuitionistic fuzzy points*. Notes IFS, 1995, **1**(2): 79–84.

[39] A. A. SALAMA, S. A. ALBLOWI. *Neutrosophic set and neutrosophic topological spaces*. IOSR J. Math., 2012, **3**(4): 31–35.

[40] M. AKRAM, H. GULZAR, K. P. SHUM. *Certain notions of single-valued neutrosophic K-algebras*. Italian J. Pure Appl. Math., 2019, **42**: 271–289.

[41] M. AKRAM, H. GULZAR, F. SMARANDACHE. *Neutrosophic soft topological K-algebras*. Neutrosophic Sets and Systems, 2019, **25**: 104–124.

[42] M. AKRAM, H. GULZAR, F. SMARANDACHE, et al. *Application of neutrosophic soft sets to K-algebras*. Axioms, 2018, **7**(4): 83.

[43] M. AKRAM, H. GULZAR, F. SMARANDACHE, et al. *Certain notions of neutrosophic topological K-algebras*. Mathematics, 2018, **6**(11): 234.

[44] M. ASLAM, A. B. THAHEEM. *A note on p-semisimple BCI-algebras*. Math. Japon., 1991, **36**(1): 39–45.

[45] Yanling LIU, Hailong YANG. *Further research of single valued neutrosophic rough sets*. J. Intelligent and Fuzzy Systems, 2017, **33**(1): 1–12.