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抽象空间混合型微分积分方程两点边值问题的解

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摘要 本文利用混合单调算子理论及Mönch的一个结果，得到了一类微分方程两个点边值问题的解。

关键词 正规锥，拟解对，非紧性测度。

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1 引言

本文在Banach空间内研究了下列混合型微分方程两点边值问题:

\[ -u'' = f(t, u, u, Tu, Su), \]
\[ u(0) = u(1) = \theta. \]

若上式中的四个不等式均为等号，则称 \( \tau_0, w_0 \) 为问题 (1) 的拟解对。

设 \( \tau_0, w_0 \) 为 (1) 的拟解对。
\[ \alpha ( H ) = \max_{t \in I} \alpha ( H ( t ) ) , \alpha ( H ) = \alpha ( H ( t ) ) . \]

2. Kuratowski Banach Hausdorff \( J = \{ a, b \} \) . \( x_\in \mathbb{E} \). \( x \in \mathbb{N} \). \( x = \beta ( \{ x_n ( t ) | n \in \mathbb{N} \} ) \). \( x \in \mathbb{N} \), \( K \in \mathbb{E} \). \( s \geq 0 \). \( x \in \mathbb{N} \). \( S \cap \mathbb{N} \).

3. \( L_2 \in \mathbb{E} \). \( B_1 = \{ u_n \} , B_2 = \{ \_ \} .\]

4. \( L_2 \in \mathbb{E} \). \( u \in \mathbb{E} \). \( \mathcal{A} \) is a Banach space.

5. \( \mathcal{A} \) is a Banach space.

6. \( \frac{1}{\sqrt{M}} \left( e^{2 \sqrt{M}} - 1 \right) \left( L_1 + M + L_2 + L_3 + L_4 + L_5 \right) < 1. \)

7. \( u \in \mathbb{E} \). \( \tau \) is a Banach space.

8. \( \mathcal{A} \) is a Banach space.

9. \( 0 \leq G ( t , s ) \leq \frac{1}{2} \sqrt{M} e^{2 \sqrt{M}} \left( e^{2 \sqrt{M}} - 1 \right) , \forall ( t , s ) \in I \times I. \)

--- 426 ---
\( A : [\mathcal{T}_0, \omega_0] \times [\mathcal{T}_0, \omega_0] \to E, \quad \mathcal{T}_0 \leq A(\tau_0, \omega_0), A(\omega_0, \mathcal{T}_0) \leq \omega_0; \quad (i) \ A \cap \mathbb{R} \\
\end{aligned} \)

\( \tau_n = A(\tau_{n-1}, \omega_{n-1}), \ \omega_n = A(\omega_{n-1}, \tau_{n-1}), \ n = 1, 2, \ldots. \)  

\( \tau_0 \leq \tau_1 \leq \ldots \leq \tau_n \leq \ldots \leq \omega_n \leq \ldots \leq \omega_1 \leq \omega_0. \)  

\( \mathcal{T}_0 \leq \mathcal{T}_1 \leq \ldots \leq \mathcal{T}_n \leq \ldots \leq \omega_n \leq \ldots \leq \omega_1 \leq \omega_0. \)  

\( \begin{aligned}
\text{P}. \quad & f(t, \tau_{n-1}(t), \omega_{n-1}(t), (\mathcal{T}_{n-1})(t), (\omega_{n-1})(t)) + \mathcal{M}_{n-1}(t) \\
\leq & f(t, \tau_0(t), \omega_0(t), (\mathcal{T}_0)(t), (\omega_0)(t)) + \mathcal{M}_0(t) , \\
\leq & f(t, \omega_0(t), \tau_0(t), (\mathcal{T}_0)(t), (\mathcal{T}_0)(t)) + \mathcal{M}_0(t) , \\
\leq & f(t, \omega_0(t), \tau_0(t), (\mathcal{T}_0)(t), (\omega_0)(t)) + \mathcal{M}_0(t) . \\
\end{aligned} \)

\( m(t) \leq \frac{1}{\sqrt{M}} e^{\frac{2}{\sqrt{M}} - 1} (L_1 + M + L_2 + L_3) k_0 + 2L_4 h_0) \max(m(I), n(I)) , \)

\( n(t) \leq \frac{1}{\sqrt{M}} e^{\frac{2}{\sqrt{M}} - 1} (L_1 + M + L_2 + L_3) k_0 + 2L_4 h_0) \max(m(I), n(I)) , \)

\( p(t) = \max(m(t), n(t)), p(t) \leq \frac{1}{\sqrt{M}} e^{\frac{2}{\sqrt{M}} - 1} (L_1 + M + L_2 + L_3) k_0 + 2L_4 h_0) \)
The Solution of Mixed Type Integro-Differential Equation of Two Points Boundary Value Problem in Banach Space

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Abstract

In this paper, a solution of two points boundary value problem of differential equation is obtained. The result in this paper generalized the conclusion in [5].

Keywords normal cone, coupled quasi-solutions, measure of noncompactness.