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A Class of Generalized Hausdorff Means*

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In the research of divergent series the important idea is that choose a suitable trasformation have to be of regularity, of sequence of partial sum of the series. The Hausdorff transformation is $H=\delta\mu\delta$ in matrix form, in which μ is any diagonal transformation and δ is so-called δ-transformation. In this paper, we construct a class of generalized Housdorff transformation using the Gould-Hsu inverse series, and give necessary and sufficient condition of that it should be regular .

The Gould-Hsu inverse is

$$\begin{cases} t_n=\sum_{k=1}^{n}(-1)^k\binom{n}{k}\psi(k,\ n)s_k \\ s_n=\sum_{k=0}^{n}(-1)^k\binom{n}{k}\frac{c_{k+1}}{\psi(n,\ k+1)}t_k \end{cases} \qquad (1)$$

where $\{a_i\}$ and $\{b_i\}$ are two sequences of nonnegative number such that

$$\psi(x,\ n)=\prod_{i=1}^{n}(a_i+b_ix)\neq 0$$

for all nonnegative integers x, n, and $c_{k+1}=a_{k+1}+kb_{k+1}$. We have $t=\hat\delta s$, $s=\hat\delta^{-1}t$ in matrix form, where $\hat\delta$ is the generalized δ-transformation. In general, $\hat\delta$ isn't a self reciprocal transformation.

$\hat H=\hat\delta\mu\hat\delta^{-1}$ is called the generalized Hausdorff transformation, where μ is any diagonal transformation, It is not difficult prove that any two $\hat H$ transformations are commutable. Since H includ two sequence $\{a_i\}$, $\{b_i\}$ as the parameters, hence the freedom of a class of $\hat H$ is very large. A class of $\hat H$ considered by us is that let $a_i=1$, $i=1,2\cdots$, and suitable choose b_i such that $c_i=a_i+(i-1)b_i$ $=1+(i-1)b_i<B$ for all i, B is a constant independent of i.

Let $t=\hat Hs=\hat\delta\mu\hat\delta^{-1}s$. It can be written $t=\hat\delta v$, $v=\mu u$, $u=\hat\delta^{-1}s$. Then

$$t_m=\sum_{k=0}^{m}(-1)^k\binom{m}{k}\psi(k,m)v_k=\sum_{k=0}^{m}(-1)^k\binom{m}{k}\psi(k,m)\mu_ku_k$$

$$=\sum_{k=0}^{m}(-1)^k\binom{m}{k}\psi(k,\ m)\mu_k\ \sum_{n=0}^{k}(-1)^n\binom{k}{n}\frac{c_{n+1}}{\psi(k,\ n+1)}s_n\ . \qquad (2)$$

(to p47)

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(from p48)

Let $P^{(m)}(x)=\psi(x, m)/\psi(x, n+1)$, then

$$t_m=\sum_{n=0}^{m} C_{m,n} s_n \qquad (3)$$

$C_{m,n}=c_{n+1}\binom{m}{n}\Delta^{m-n}(p^{(m)}(n)\mu_n)$.

Theorem In order that the transformation (3) should be regular, it is necessary and sufficient that for every m sequence $(P^{(m)}(n)\mu_n)$ should be the difference of two totally monotone sequences, that

$$\Delta^m(P^{(m)}(0)\mu_0)\to 0 \quad (m\to\infty)$$

and that $\mu_0=1$.

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