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A Note on Continuity of $\partial_{\circledast} f(\cdot)$

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Supposa that $f(x)$ is a quasidifferentiable function, defined on $S \subset R^n$ where S is an open set, with a $\circledast$-equivalent bounded quasidifferential subfamily.

Lenma 1 (a) $u \in \partial_{\circledast} f(x) \Leftrightarrow \underline{\mathscr{M}}(x, u) \neq \phi$

(b) $w \in \underline{\mathscr{M}}(u) \Leftrightarrow \phi(u \circledast d) \geq \langle w, d \rangle, \quad \forall d \in R^n$.

(c) $u \in \partial_{\circledast} f(x) \Leftrightarrow u \in \underline{\mathscr{M}}(x, u)$.

(d) $u \in \partial_{\circledast} f(x) \Leftrightarrow f'(x; d) \leq \max_{w \in (x, u)} \langle w, d \rangle = \delta(d \mid \partial_c \underline{\varphi}(u \circledast \cdot)(O)), \ \forall d \in R^n$.

Lemma 2 If $\underline{\varphi}(x, u \circledast d)$ is upper semicontinuous in $(x, u) \in S \times \partial_{\circledast} f(x)$ for each $d \in R^n$, then the mapping $\partial_{\circledast} f(\cdot)$ and $(\cdot, \cdot)$ are closed, *i.e.*, $u \in \partial_{\circledast} f(x)$ and $w \in \underline{\mathscr{M}}(x, u)$ (or $\partial_c \underline{\varphi}(u \circledast)(0)$) whenever $x_i \to x$, $u_i \to u$, $w_i \to w$, and $u_i \in \partial_{\circledast} f(x_i)$, $w_i \in \underline{\mathscr{M}}(x_i, u_i)$, $i \to \infty$.

Theorem 3 Suppose $\mathscr{D}_M f(x)$ is boundod uniformly in a neighborhood of x, $N_x(\delta)$, where δ is a positive number. If $\underline{\varphi}(x, u \circledast d)$ is upper semicontinuous in $(x, u) \in S \times \partial_{\circledast} f(x)$ for each $d \in R^n$, then $\partial_{\circledast} f(\cdot)$ is upper semicontinuous

Lemma 4 Suppose $\underline{f}'(x; d)$ is lower semi-conlinuous in $x \in S$ for each $d \in R^n$ and the mapping $\underline{\mathscr{M}}(x, u)$ is upper semicontinuous in $(x, u) \in S \times \partial_{\circledast} f(x)$. Then the mapping $\partial_{\circledast} f(\cdot)$ is closed.

Theorem 5 Suppose $\underline{f}'(x, d)$ is lower semi-continuous in $x \in S$ for each $d \in R^n$, and $D_m f(x)$ is bounded uniformly in a neighbourhood of x, $N_x(\delta)$, and $\mathscr{M}(x, u)$ is upper semicontinuous in $(x, u) \in S \times \partial_{\circledast} f(x)$. Then $\partial_{\circledast} f(\cdot)$ is a upper **Simicontinuous mapping.**

References

[1] Demyanov, V. F. and Rubinov, A.M.(1980), On quasidifferentiable functionals, Soviet Math. Dokl., Vol. 2, No. 1, pp. 14—17.

[2] Demyanov. V.F. and Rubinov, A. M.(1985/1986), Quasidifferential calculus, IIASA, Laxenburg Austria/Springer-Verlag.

[3] Xia, Z. Q.(1986), The ⊛-Kernel for a Quasidifferentiable Function, Report, SDS, IIASA. Laxenburg, Austria.

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