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On the Existence of Harmonic Solutions of the Forced Liénard Equation*

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In this paper, we present a method to study the existence of the harmonic solutions of the forced Liénard equation's equivalent system

$$x' = y - F(x) + P(t), \quad y' = -g(x), \qquad (*)$$

where $F(x) \in c^1$, $xg(x) > 0$, if $x \neq 0$ and $g(x)$ is locally Lipschitz continuous and $P(t) \in c^1$, $P(t) = P(t+T)$, $T > 0$.

Theorem 1. There exists at least one harmonic solution of (*) if

i) there exist C, $M > 0$ such that

$$F(x) - P + c > 0, \int_0^b (g(x)/(C + F(x) - P))dx < M, \quad 0 < x < b \leq +\infty; \ P = \sup P(t);$$

ii) $\inf\limits_{a<x<0} (F(x) - p) \leq -y_0$ or, $\sup\limits_{a<x<0} (\int_0^x (g(s)/(y_0 + F(s) - p))ds - (y_0 + F(x) - p) < 0$ if $\inf\limits_{a<x<0} (F(x) - p) > -y_0 (-\infty \leq a < 0)$, where $y_0 = C + M$, $p = \inf P(t)$;

iii) $\sup\limits_{0<x<b} (\Gamma(x) - P) \geq y_1$ or $G(b) > (y_0 + y_1)^{2/8}$, where $y_1 = A + \sqrt{2G(a)}$ with $A = \sup\limits_{a<x<0} (F(x) - p) < +\infty$, $G(x) = \int_0^x g(s)ds$ and $G(a) < +\infty$.

Theorem 2. There exists at least one harmonic solution of (*) if there exist constants C, $M > 0$ such that

i) $M - P + p > 0, C\int_0^x g(s)ds - F(x) < M - P + p$ if $x > 0$;

ii) $F(x)/g(x) < k$, $M + 1/C + F(\overline{x}) < 1/k$ for some $k > 0$, $\overline{x} < 0$ if $x < 0$;

iii) $\limsup\limits_{x \to +\infty} F(x) = +\infty$.

References

[1] Villari. G., Zanolin. F., On forced nonlinear oscillations of a second order equation with strong restoring term, (to appear).

[2] Chen Xiudong, Li Jiaxu, Fan Hongyi, The harmonic solutions of the equation $x'' + f(x)x' + g(x) = p(t)$, (submitted).

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