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On a Result of Ahuja (Ⅱ)*

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In [1], O. P. Ahuja established the following result:

If F is an element of $R_n(a)$ for $n>0$ and $0<a<1$,

$$F(z)=\frac{c+1}{z^c}\int_0^z f(t)t^{c-1}dt$$

with $|z|<1$, $\operatorname{Re}c>-a$, and $0<\beta<1$, then the function f is an element of $R_n(\beta)$ for $|z|<r_0$, where r_0 the smallest positive root in $(0,1)$ of the equation

$$(c+2a-1)(2a-\beta-1)r^2+2((c+a)(a-\beta)-(1-a)(2-a))r+(c+1)(1-\beta)=0. \quad (1)$$

Ahuja claimed that the above result covers both $\beta>a$ and $\beta<a$. It is easy to find that his conclusion is in general wrong even if c is a real number satisfying $c\geqslant -a$. For example, if $a=\frac{1}{2}$, $\beta=0$ and $c>0$, then the equation (1) reduces to $(c-1)r+(c+1)=0$ which has not any root in $(0,1)$. In this note we give a corrected version of the Ahuja result when c is a real number such that $c>-a$. We have:

If F is an element of $R_n(a)$ for $n>0$ and $0<a<1$,

$$F(z)=\frac{c+1}{z^c}\int_0^z f(t)t^{c-1}dt$$

with $|z|<1, c>-a$, and $0<\beta<1$, then the function f is an element of $R_n(\beta)$ for $|z|<r_0$, where r_0 is the smallest positive root of the equation

$$(c+2a-1)(2a-\beta-1)r^2+2((c+a)(a-\beta)-(1-a)(2-a))r+(c+1)(1-\beta)=0 \quad \text{if } u_0(r)<\frac{1}{1+r},$$

and of the equation

$$\rho^2(r)+2((c+\beta)(1-a)-(c+a)(3-2a))\rho(r)+((c+\beta)(1-a)+(c+a))^2=0 \quad \text{if } u_0(r)\geqslant\frac{1}{1+r},$$

where

$$u_0(r)=\frac{1}{2(1-a)}\left\{\left(\frac{(c+a)(c+1-(c+2a-1)r^2}{(2-a)(1-r^2)}\right)^{1/2}-(c+2a-1)\right\}$$

and

$$\rho(r)=\frac{c+1-(c+2a-1)r^2}{1-r^2}.$$

The result is sharp.

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Unique Constrution of Formanek Central Polynomial

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Abstract

Formanek constructed the first central polynomial. i.e. $G_1+G_2+\cdots+G_n$ where $G_1=G(x,y_1,\cdots,y_n)$, $G_2=G(x,y_2,y_3,\cdots,y_n,y_1)$ etc. Are called Formanek's polynomials. Rosset in his nota [3] raised the question that whether all sgmmetic polynomials in G_i also give central polynomials. He showed that the basic sgmmetric polynomials in G_i are not all central. In this paper we shall show, for $n\geqslant 3$ each polynomial in G_i is not central, except $f(G_1+\cdots+G_n)$, where $f(x)$ is a polynomial at x. Hence Formanek's central polynomial is unique in some sense.

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