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A New Method for Riemann-Hilbert Problems of Generalized Analytic Functions—— Imbedding Method*

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In this paper, by using imbedding method, we consider Riemann-Hilbert problems for elliptic systems of first order.

Let G be a unit disc, Γ the boudary of G.

We consider following problem (E):

$$\begin{cases} \partial_{\bar z} w = Aw + B\overline{w} + f & \text{in } G \\ \operatorname{Re} w = 0 & \text{on } \Gamma \\ \operatorname{Im} w(z_0) = c \end{cases} \qquad (E)$$

where A, B, $f \in C^{\alpha}(G)$ are given functions. z_0 is a fixed point and c is a given constant.

We introduce a family of boundary value problems (F_λ):

$$\begin{cases} \partial_{\bar z} u = \lambda (Au + B\overline{u} + f) & \text{in } G \\ \operatorname{Re} u = 0 & \text{on } \Gamma \\ \operatorname{Im} u(z_0) = c \end{cases} \qquad (F_\lambda)$$

Obviously, (F_1) is (E).

In this paper, we get following results:

Theorem I $u(z,\lambda)$ is an infinite differential function with respect to $\lambda \in (-\infty, \infty)$.

Let us consider following Cauchy system (C_λ):

$$\frac{\partial \Gamma_1(z,s,\lambda)}{\partial \lambda} = \iint_G [\Gamma_1(z,s_1,\lambda)A(s_1)\Gamma_1(s_1,s,\lambda) + \Gamma_1(z,s_1,\lambda)B(s_1)\overline{\Gamma_2(s_1,s,\lambda)}$$
$$+ \Gamma_2(z,s_1,\lambda)\overline{A(s_1)}\,\overline{\Gamma_2(s_1,s,\lambda)} + \Gamma_2(z,s_1,\lambda)\overline{B(s_1)}\Gamma_1(s_1,s,\lambda)]\, d\xi_1 d\eta_1$$

$$\frac{\partial \Gamma_2(z,s,\lambda)}{\partial \lambda} = \iint_G [\Gamma_1(z,s_1,\lambda)A(s_1)\Gamma_2(s_1,s,\lambda) + \Gamma_1(z,s_1,\lambda)B(s_1)\overline{\Gamma_1(s_1,s,\lambda)}$$
$$+ \Gamma_2(z,s_1,\lambda)\overline{A(s_1)}\,\overline{\Gamma_1(s_1,s,\lambda)} + \Gamma_2(z,s_1,\lambda)B(s_1)\Gamma_2(s_1,s,\lambda)]\, d\xi_1 d\eta_1$$

$$\frac{\partial u(z,\lambda)}{\partial \lambda} = \iint_G \Gamma_1(z,s,\lambda)[Au(s,\lambda) + \overline{Bu(s,\lambda)} + f(s)]\, d\xi d\eta$$

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$$+\iint_G \Gamma_2(z, s, \lambda)[\overline{A(s)}\,\overline{u(s, \lambda)} + \overline{B(s)}\,u(s, \lambda) + \overline{f}(s)]\, d\xi\, d\eta$$

$$\Gamma_1(z, s, 0) = -\frac{1}{\pi}\frac{1}{s-z} + \frac{1}{2\pi}\frac{1}{s-z_0} - \frac{1}{2\pi}\frac{z_0}{1-\bar{z}_0 s}$$

$$\Gamma_2(z, s, 0) = -\frac{1}{\pi}\frac{z}{1-sz} + \frac{1}{2\pi}\frac{z_0}{1-z_0 s} - \frac{1}{2\pi}\frac{1}{\overline{s}-\overline{z_0}}$$

$$u(z, 0) = ic$$

Theorem 2 The problem (F_λ) is equivalent to the problem (C_λ).

In the last section, by using imbedding method, we give some numerical solutions for the problem (E). We also obtain the error estimate of the approximation solutions.

Remark This method can be used to treat nonlinear Riemann-Hilbeyt boundary value problem for elliptic systems of first odder. We think this method is an effective method.

References

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