

Generators, Relations and Coverings of ${}^2B_2(K)^*$

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Introduction

R. Steinberg^[1] has given a system of generators and relations of simple connected Chevalley groups, and constructed the universal covering of the group.

Using BN-pair technique, C.W. Curtis [5] and J. Grover [6] have also constructed the covering groups for more general groups such as the twisted groups over a field except the groups of type ${}^2A_{2n}$. On the other hand, Liu Chen [9] has generalized the results to the simple groups of Lie type with rank more than 1 but the group of type 2F_4 .

In this note, we generalize the method of Steinberg to the twisted Chevalley groups of type 2B_2 with rank 1, and give some results on Schur multiplier.

Notice: We adopt the notations and the results of [8] without any exposition.

I Preliminaries

A central extension of a group G is a couple (π, \tilde{G}) , where \tilde{G} is a group, π is a homomorphism of \tilde{G} onto G and $\ker \pi \subseteq \text{center of } \tilde{G}$. Furthermore, a central extension (π, \tilde{G}) is universal if for any central extension (π', E) of G , there exists a unique homomorphism $\varphi: \tilde{G} \rightarrow E$ such that $\pi' \varphi = \pi$. i.e. the following diagram is commutative:

$$\begin{array}{ccc}
 & \varphi & \\
 G & \longrightarrow & E \\
 \pi \searrow & & \nearrow \pi' \\
 & G &
 \end{array}$$

We abbreviate universal central extension by u.c.e.

A group G is perfect if $G = DG$, where DG is the commutator subgroup of G .

If (π, \tilde{G}) is a u.c.e of a perfect group G then we call the kernel of π the Schur multiplier of G .

(1.1) If (π, \tilde{G}) is a central extension of a group G which covers all others, i.e. for any central extension (π', E) of G , there exists a homomorphism $\varphi: \tilde{G} \rightarrow E$ such that $\pi' \varphi = \pi$ and $\tilde{G} = D\tilde{G}$ then (π, \tilde{G}) is a u.c.e of G .

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We assume that field K is a perfect field of characteristic 2 and that K admits an automorphism $t \rightarrow t^\theta$ such that $2\theta^2 = 1$. Let $\Phi = \{\pm a, \pm b; \pm(a+b), \pm(2a+b)\}$ be a root system of type B_2 and $S = \{a, b, a+b, 2a+b\}$ be an equivalence class in Φ , then ${}^2B_2(K)$ consists of the following elements:

$$\begin{aligned} X_S(t, u) &= x_a(t^\theta) x_b(t) x_{a+b}(t^{\theta+1} + u) x_{2a+b}(u^{2\theta}) \\ X_{-S}(t, u) &= x_{-a}(t^\theta) x_{-b}(t) x_{-(a+b)}(t^{\theta+1} + u) x_{-(2a+b)}(u^{2\theta}) \end{aligned}$$

for any $t, u \in K$.

$$X_R(t_1, u_1) X_R(t_2, u_2) = X_R(t_1 + t_2, u_1 + u_2 + t_1^\theta t_2) \quad (1.2)$$

where $t_i, u_i \in K$ ($i = 1, 2$) and $R = \pm S$.

We write $n_S = n_a n_{a+b}$ where $n_r = x_r(1) x_{-r}(-1) x_r(1)$ for any $r \in \Phi$.

$$n_S \in {}^2B_2(K) \quad (1.3)$$

Since $w_a w_{a+b} = w_a w_b w_a w_b$ is an element of length 4 in Weyl group W , and then it maps all the positive roots to negative roots.

Let $H^1 = \langle h_S(\lambda) \mid \lambda \in K^* \rangle$ (where $h_S(\lambda) = h_a(\lambda) h_b(\lambda^{2\theta})$),

$$N^1 = \langle n_S, H^1 \rangle; \quad U^1 = \langle X_S(t, u) \mid t, u \in K \rangle; \quad B^1 = U^1 H^1, \quad (1.4)$$

then K^1, N^1, U^1, B^1 are subgroups of ${}^2B_2(K)$ and ${}^2B_2(K)$ is a group with a splits BN-pair of rank 1. Thus, N^1/H^1 is isomorphic to the cyclic group of order 2.

We suppose that $n_S(\lambda) = h_S(\lambda) n_S$ and choose a 4-dimensional representation of $B_2(K)$.

$$\begin{aligned} x_a(t) &= e + t(e_{12} + e_{43}); & x_b(t) &= e + te_{24}; \\ x_{a+b}(t) &= e + t(e_{14} + e_{23}); & x_{2a+b}(t) &= e + te_{13}, \end{aligned}$$

here e is the identity matrix and e_{ij} are matrix units.

Since ${}^2B_2(K)$ has a Bruhat decomposition, a simple calculation shows that

$$n_S(\lambda) = X_S^{-1}(\lambda^{2\theta}(t^{2\theta+1} + u^{2\theta}), \lambda^{2\theta}u) X_{-S}(t, u) X_S(\lambda^{2\theta}u^{2\theta}, \lambda t^\theta + \lambda^{2\theta+1}u^{2\theta+1}) \quad (1.5)$$

where $\lambda^{-1} = f(t, u) = t^{2\theta+1} + t^\theta u + u^{2\theta}$ for any $t, u \in K$ and $t \neq 0$ or $u \neq 0$.

$$h_S(\lambda) X_{\pm S}(t, u) h_S(\lambda)^{-1} = X_{\pm S}(\lambda^{\pm(4\theta-2)}t, \lambda^{\pm 2\theta}u). \quad (1.6)$$

$$X_R(t, u)^{-1} = X_R(t, u + t^{\theta+1}) \quad (R = \pm S) \quad (1.7)$$

$$n_S(\lambda) = n_S(\lambda)^{-1} \quad (1.8)$$

$$n_S(\lambda) X_{\pm S}(t, u) n_S(\lambda)^{-1} = X_{\mp S}(\lambda^{\mp(4\theta-2)}t, \lambda^{\mp 2\theta}u) \quad (1.9)$$

$$n_S(\lambda^{-1}) = X_{-S}(\lambda^{2\theta}(t^{2\theta+1} + u^{2\theta}), \lambda^{2\theta}u) X_S(t, u) X_{-S}(\lambda^{2\theta}u^{2\theta}, \lambda t^\theta) \quad (1.10)$$

where $\lambda^{-1} = f(t, u)$.

$$n_S(v) n_S(\lambda) n_S(v)^{-1} = n_S(v^2 \lambda^{-1}) \quad (1.11)$$

$$n_S(v) h_S(\lambda) n_S(v)^{-1} = h_S(v^2 \lambda^{-1}) h_S(v^2)^{-1} \quad (1.12)$$

II The Main Theorems

Definition 2.1 Suppose that \tilde{G} is a group generated by the collection of

symbols $\{\tilde{X}_R(t, u) \mid t, u \in k, R = \pm S\}$ subjecte to the following relations, taken from the corresponding group ${}^2B_2(K)$.

$$(A) \tilde{X}_S(t_1, u_1) \tilde{X}_S(t_2, u_2) = \tilde{X}_S(t_1 + t_2, u_1 + u_2 + t_1^{\theta} t_2) \quad (t_i, u_i \in K, i = 1, 2)$$

$$(B) \tilde{n}_S(\lambda) \tilde{X}_S(t, u) \tilde{n}_S(\lambda)^{-1} = \tilde{X}_{-S}(\lambda^{-4\theta+2} t, \lambda^{-2\theta} u),$$

where $\tilde{n}_S(\lambda)$ has the same expression as $n_S(\lambda)$ in (1.5).

If G also satisfies the following relation

$$(C) h_S(\lambda_1) h_S(\lambda_2) = \tilde{h}_S(\lambda_1 \lambda_2), \quad \lambda_1, \lambda_2 \in K^*,$$

Where $\tilde{h}_S(\lambda) = \tilde{n}_S(\lambda) \tilde{n}_S(1)$ then we use G instead of \tilde{G} .

Let $n_S = n_S(1)$ then the properties (1.2), (1.6) - (1.12) also hold with \tilde{x} (lies in \tilde{G}) in place of x (lies in ${}^2B_2(K)$)

In order to prove the main theorem, we introduce the following lemma.

Lemma 2.2 let K be a field and $|K| > 2$ then $\tilde{G} = D\tilde{G}$.

Proof $[\tilde{h}_S(\lambda), \tilde{X}_S(t, u)] = \tilde{X}_S((\lambda^{4\theta-2} + 1)t, (\lambda^{2\theta} + 1)u + (\lambda^{2-2\theta} + 1)t^{\theta+1})$, for any $t, u \in K$, there exists a $\lambda \in K^*$ such that $\lambda^{4\theta-2} + 1 \neq 0, \lambda^{2\theta} + 1 \neq 0$ since $|K| > 2$. Hence, $\tilde{G} = D\tilde{G}$.

Theorem 2.3 (i) (π, \tilde{G}) is a central extension of group ${}^2B_2(K)$ where π is the natural homomorphism ;

(ii) G is isomorphic to ${}^2B_2(K)$, denoted by $G = {}^2B_2(K)$.

Proof (i) (a) There is a homomorphism $\pi, \tilde{G} \rightarrow {}^2B_2(K)$ such that

$$\pi(\tilde{X}_R(t, u)) = X_R(t, u),$$

for any t, u, K and $R = \pm S$.

Let $C(\tilde{G})$ be the center of \tilde{G} , we shall show that $\ker \pi \subset C(\tilde{G})$.

As usual, let $\tilde{U}^1 = \langle \tilde{X}_S^4(t, u) \mid t, u \in K \rangle$ then $\pi: \tilde{U}^1 \rightarrow U^1$ is obviously an isomorphism.

(b) Let

$$\tilde{H}^1 = \langle \tilde{h}_S(\lambda) \mid \lambda \in K^* \rangle, \tilde{N}^1 = \langle \tilde{n}_S(\lambda) \mid \lambda \in K^* \rangle, \tilde{B}^1 = \tilde{U}^1 \tilde{H}^1,$$

then H^1 is in the normalizer of \tilde{U}^1 , also a normal subgroup of \tilde{N}^1 and $\tilde{N}^1 / \tilde{H}^1 = N^1 / H^1$.

(c) We now show that $\tilde{B}^1 \cup \tilde{B}^1 \tilde{n}_S \tilde{B}^1$ is a subgroup of \tilde{G} . It is sufficient to show that

$$\tilde{n}_S \tilde{B}^1 \tilde{n}_S \subseteq \tilde{B}^1 \cup \tilde{B}^1 \tilde{n}_S \tilde{B}^1.$$

Since

$$\tilde{n}_S \tilde{B}^1 \tilde{n}_S = \tilde{n}_S \tilde{U}^1 \tilde{n}_S^{-1} \tilde{n}_S \tilde{H}^1 \tilde{n}_S.$$

It follows from relation (B) that

$$\tilde{X}_{-S}(t, u) \in \tilde{B}^1 \tilde{n}_S \tilde{B}^1,$$

then

$$\tilde{n}_S \tilde{U}^1 \tilde{n}_S^{-1} \subseteq \tilde{B}^1 \tilde{n}_S \tilde{B}^1.$$

Hence, $\tilde{B}^1 \cup \tilde{B}^1 \tilde{n}_s \tilde{B}^1$ is a subgroup of \tilde{G} .

(d) It is evident that $\tilde{G} = \tilde{B}^1 \cup \tilde{B}^1 \tilde{n}_s \tilde{B}^1$ since \tilde{G} is generated by \tilde{B}^1 and \tilde{N}^1 which are contained in $\tilde{B}^1 \cup \tilde{B}^1 \tilde{n}_s \tilde{B}^1$.

(e) We show that each element of \tilde{G} has a unique expression of the form

$$\tilde{g} = \tilde{u}' \tilde{h} \tilde{n}_s \tilde{u} \quad \text{or} \quad \tilde{g} = \tilde{u}' \tilde{h}$$

with $\tilde{u}, \tilde{u}' \in U^1$ and $\tilde{h} \in \tilde{H}^1$.

Suppose that

$$\tilde{u}'_1 \tilde{h}_1 \tilde{h}_s \tilde{u}'_1 = \tilde{u}'_2 \tilde{h}_2 \tilde{n}_s \tilde{u}'_2 \quad \text{or} \quad \tilde{u}'_1 \tilde{h}_1 = \tilde{u}'_2 \tilde{h}_2.$$

Applying π on both sides of the equations, then, by (a) and the uniqueness of expression in ${}^2\mathbf{B}_2(K)$, we have

$$\tilde{u}'_1 = \tilde{u}'_2, \quad \tilde{u}_1 = \tilde{u}_2 \quad \text{and} \quad \tilde{h}_1 = \tilde{h}_2.$$

(f) We consider finally the kernel of π .

Let $\tilde{u}' \tilde{h} \tilde{n}_s \tilde{u}$ be in $\ker \pi$, then $\pi(\tilde{u}') \pi(\tilde{h}) \pi(\tilde{n}_s) \pi(\tilde{u}) = 1$.

It follows a contradiction that

$$\pi(\tilde{n}_s) = n_s = 1.$$

Now, suppose that $\tilde{u}' \tilde{h} \in \ker \pi$, then $\pi(\tilde{u}') \pi(\tilde{h}) = 1$.

By (a), $\tilde{u} = 1$, hence, $\ker \pi \subset \tilde{H}^1$.

Let $\tilde{h} = \prod_{i=1}^n \tilde{h}_s(\lambda_i) \in \ker \pi$, where $\lambda_i \in K^* (i = 1, 2, \dots, n)$, then

$$\tilde{h} \tilde{X}_s(t, u) \tilde{h}^{-1} = \tilde{X}_s\left(\left(\prod_{i=1}^n \lambda_i\right)^{4\theta-2} t, \left(\prod_{i=1}^n \lambda_i\right)^{2\theta} u\right).$$

Applying π , we have

$$\tilde{X}_s(t, u) = \tilde{X}_s\left(\left(\prod_{i=1}^n \lambda_i\right)^{4\theta-2} t, \left(\prod_{i=1}^n \lambda_i\right)^{2\theta} u\right),$$

hence

$$\prod_{i=1}^n \lambda_i = 1$$

and so \tilde{h} commutes with $\tilde{X}_s(t, u)$.

Similarly, we obtain that \tilde{h} commutes with $\tilde{X}_{-s}(t, u)$, therefore, \tilde{h} must be contained in $C(\tilde{G})$ since $\tilde{X}_R(t, u) (R = \pm S)$ generate the group \tilde{G} . Thus, the kernel of π belongs to $C(\tilde{G})$.

(ii) It follows immediately from the relation (C) that

$$\tilde{h} \in \ker \pi \Leftrightarrow \tilde{h} = \tilde{h}(1) = 1,$$

then G is isomorphic to ${}^2\mathbf{B}_2(k)$.

From now on, we can identify the abstract group G with group ${}^2\mathbf{B}_2(k)$, and we have

Theorem 2.4 If K is an extension of finite field $GF(2^5)$ then (π, \tilde{G}) is a u.c.e of G .

Proof We need only to show that for any central extension (ψ, E) of G , there exists a homomorphism $\varphi: \tilde{G} \rightarrow E$ such that $\varphi\psi = \pi$.

Define $\varphi\tilde{X}_R(t, u) \in \psi^{-1}(X_R(t, u))$ for any $t, u \in K$ and $R = \pm S$, so that

$$(1) \quad [\varphi\tilde{h}_S(\lambda), \varphi\tilde{X}_S(t, u)] = \varphi\tilde{X}_S((\lambda^{4\theta-2}+1)t, (\lambda^{2\theta}+1)u + (\lambda^{2-2\theta}+1)t^{\theta+1}).$$

Note that this choice is not circular since the commutator $[x, y]$ with $x, y \in E$ depends only on the classes mod $C(E)$ to which x and y belong, where $C(E)$ is the center of E .

$$(2) \quad \varphi\tilde{h}\varphi\tilde{X}_S(t, u)\varphi\tilde{h}^{-1} = \varphi(\tilde{h}\tilde{X}_S(t, u)\tilde{h}^{-1}),$$

for any $\tilde{h} = \prod_{i=1}^n \tilde{h}_S(\lambda_i)$ in \tilde{H}^1 , where $\lambda_i \in K$ ($i=1, 2, \dots, n$)

Conjugating (1) by $\varphi\tilde{h}$, we obtain

$$\begin{aligned} & [\varphi\tilde{h}_S(\lambda), \varphi\tilde{X}_S(\prod_{i=1}^n \lambda_i^{4\theta-2}t, \prod_{i=1}^n \lambda_i^{2\theta}u)] \\ &= \varphi\tilde{h}\varphi\tilde{X}_S((\lambda^{4\theta-2}+1)t, (\lambda^{2\theta}+1)u + (\lambda^{2-2\theta}+1)t^{\theta+1})\varphi\tilde{h}^{-1} \\ &= \varphi\tilde{X}_S((\lambda^{4\theta-2}+1)\prod_{i=1}^n \lambda_i^{4\theta-2}t, (\lambda^{2\theta}+1)\prod_{i=1}^n \lambda_i^{2\theta}u + (\lambda^{2-2\theta}+1)\prod_{i=1}^n \lambda_i^{2\theta}t^{\theta+1}) \end{aligned}$$

Since $|K| \geq 2^5$, there exists $\lambda \in K^*$ such that

$$\lambda^{4\theta-2}+1 \neq 0 \quad \text{and} \quad \lambda^{2\theta}+1 \neq 0$$

so we can easily obtain (2).

Similarly we have

$$(3) \quad \varphi\tilde{n}\varphi\tilde{X}_S(t, u)\varphi\tilde{n}^{-1} = \varphi(\tilde{n}\tilde{X}_S(t, u)\tilde{n}^{-1}),$$

for any $\tilde{n} \in \tilde{N}$, hence φ preserves the relation (B).

(4) Let

$$[\varphi\tilde{X}_S(t_1, u_1), \varphi\tilde{X}_S(t_2, u_2)] = f(t_1, t_2, u_1, u_2)\varphi\tilde{X}_S(0, t_1^\theta t_2 + t_1 t_2^\theta) \quad (*)$$

then $f(t_1, t_2, u_1, u_2) = 1$.

It follows exactly as in Theorem 10 of [2] that

$$(a) \quad f(0, 0, u_1, u_2) = 1 \quad \text{and}$$

$$\varphi\tilde{X}_S(0, u_1)\varphi\tilde{X}_S(0, u_2) = \varphi\tilde{X}_S(0, u_1 + u_2).$$

Conjugating (*) by $\varphi h_S(\lambda)$ ($\lambda \in K^*$), we have

$$(b) \quad f(t_1, t_2, u_1, u_2) = f(\lambda^{4\theta-2}t_1, \lambda^{4\theta-2}t_2, \lambda^{2\theta}u_1, \lambda^{2\theta}u_2).$$

Considering $[\tilde{X}_S(t_1, u_1)\tilde{X}_S(t'_2, u'_2), \tilde{X}_S(t_2, u_2)]$, we have

$$\begin{aligned} (c) \quad & f(t_1 + t'_1, t_2, u_1 + u'_1 + t_1^\theta t'_1, u_2) \\ &= f(t_1, t_2, u_1, u_2) f(t'_1, t_2, u'_1, u_2) \cdot f(t_1, 0, u_1, t_1^\theta t_2 + t'_1 t_2^\theta). \end{aligned}$$

Similarly,

$$\begin{aligned} & f(t_1, t_2 + t'_2, u_1, u_2 + u'_2 + t_1^\theta t'_2) \\ &= f(t_1, t_2, u_1, u_2) f(t_1, t'_2, u_1, u'_2) \cdot f(t_2, 0, u_2, t_1^\theta t_2 + t_1 t_2^\theta). \\ (d) \quad & f(t_1, 0, u_1, u_2) f(t'_1, 0, u'_1, u_2) = f(t_1 + t'_1, 0, u_1 + u'_1 + t_1^\theta t'_1, u_2). \end{aligned}$$

$$f(t_1, 0, u_1, u_2) f(t_1, 0, u_1, u_2') = f(t_1, 0, u_1, u_2 + u_2').$$

$$(e) f(t_1, 0, u_1, u_2) = 1.$$

By the assumption of field K , there exists a $v \in K^*$ such that

$$\begin{cases} v^{\theta+1} + (1-v)^{\theta+1} + (v(1-v))^{\theta+1} \neq 0 \\ v^{2-2\theta} + (1-v)^{2-2\theta} + (v(1-v))^{2-2\theta} \neq 0 \\ v(1-v) \neq 0 \\ 1 + v(1-v) \neq 0. \end{cases}$$

This is possible since

$$\begin{aligned} v^{\theta+1} + (1-v)^{\theta+1} + (v(1-v))^{\theta+1} &= ((1-v)v^\theta - 1)(v \cdot v^\theta - (1-v)), \\ v^{2-2\theta} + (1-v)^{2-2\theta} + (v(1-v))^{2-2\theta} &= (v^\theta - v^2)^2 / v^{2\theta} (1-v)^{2\theta}, \end{aligned}$$

and at most 17 elements values of v are to be avoided.

Similarly, we can show

$$(f) f(t_1, t_2, u_1, u_2) = 1.$$

$$(5) \varphi \tilde{X}_S(t_1, u_1) \varphi \tilde{X}_S(0, u_2) = \varphi \tilde{X}_S(t_1, u_1 + u_2).$$

Suppose that

$$x = \varphi \tilde{X}_S(t_1, u_1) \varphi \tilde{X}_S(0, u_2) \varphi \tilde{X}_S(t_1, u_1 + u_2)^{-1},$$

then x is in the center of E since $\psi(x) = 1$.

Hence

$$\begin{aligned} x &= \varphi h_S(\lambda) x \varphi h_S(\lambda)^{-1} \\ &= \varphi \tilde{X}_S(\lambda_1 t_1, \lambda_2 u_1 + \lambda_1^\theta t_1^{\theta+1}) \varphi \tilde{X}_S(0, \lambda_2 u_2) \varphi \tilde{X}_S(\lambda_1 t_1, \lambda_2 (u_1 + u_2) + \lambda_1^\theta t_1^{\theta+1})^{-1} \cdot x \end{aligned}$$

where $\lambda_1 = \lambda^{4\theta-2} + 1$ and $\lambda_2 = \lambda^{2\theta} + 1$.

Thus,

$$\varphi \tilde{X}_S(\lambda_1 t_1, \lambda_2 u_1 + \lambda_1^\theta t_1^{\theta+1}) \varphi \tilde{X}_S(0, \lambda_2 u_2) = \varphi \tilde{X}_S(\lambda_1 t_1, \lambda_2 (u_1 + u_2) + \lambda_1^\theta t_1^{\theta+1}).$$

By (4) and (5), we have

$$(6) \quad \varphi \tilde{X}_S(t_1, u_1) \varphi \tilde{X}_S(t_2, u_2) = \varphi \tilde{X}_S(t_1 + t_2, u_1 + u_2 + t_1^\theta t_2),$$

then φ preserves the relation (A), thus we complete the proof.

Lemma 2.5. We assume that K is an algebraic extension of prime field $GF(2)$, let $h(t, u) = h_S(t) h_S(u) h_S(t, u)^{-1}$ then $h(t, u) = 1$. In another words, (A) and (B) suffice to define G abstractly.

Proof: Arguing as in Lemma 39 of [2], we then have

$$(i) \text{ If } t, u \text{ generated a cyclic subgroup of } K^* \text{ then } h(t, u) = h(u, t).$$

$$(ii) \text{ If } h(t, u) = h(u, t) \text{ then } h(t, u^2) = 1.$$

Since K is a perfect field of characteristic 2, then the elements in K^* are square.

For any $t, u \in K^*$, we denote the smallest subfield containing t, u of K as F , then F^* is a cyclic subgroup of K^* generated by t and u . Therefore,

Corollary 2.6. If the field K is an algebraic extension of field $GF(2^5)$,

then the Schur multiplier of ${}^2B_2(K)$ is trivial.

Remark If the field is finite then the group ${}^2B_2(K)$ is Suzuki group and the consequence which we just procured coincides with the corresponding result in [7].

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${}^2B_2(K)$ 的生成元、关系式和覆盖

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摘 要

本文用生成元、关系式构造了任意域 K 上扭 Chevalley 单群 ${}^2B_2(K)$ 的泛中心扩张. (域 $K = GF(2^3)$ 除外) 当域 K 为 $GF(2^5)$ 的代数扩域时, 证明了 ${}^2B_2(K)$ 的 Schur 乘子是平凡的.