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The Equivalence of Medium Propositional Calculus MP^* and 3-Valued Łukasiewicz Propositional Calculus $Ł_3$*

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In this paper it is proved that the medium propositional calculus MP^* is equivalent to the functionally compete 3-valued Łukasiewicz propositional calculus $Ł_3$.

Definition 1 In MP^*, $D(\top)$: $\top A =_{df} \sim\sim A \prec \dashv \sim A$

Theorem 1 In MP^*,

$ŁA_1: \vdash A \prec (B \prec A)$

$ŁA_2: \vdash A \prec B \cdot \prec \cdot (B \prec C) \prec (A \prec C)$

$ŁA_3: \vdash (A \prec \dashv A) \prec A \cdot \prec A$

$ŁA_4: \vdash \dashv B \prec \dashv A \cdot \prec \cdot A \prec B$

$ŁA_5: \vdash \top A \prec \dashv \top A$

$ŁA_6: \vdash \dashv \top A \prec \top A$

$ŁMP$: $A \prec B,\ A \vdash B$

It is clear that if we take $ŁA_1 - ŁA_6$ as axioms schemes and ŁMP as rule of inference, we can obtain a logical system which is just the functionally complete 3-valued Łukasiewicz propositional calculus $Ł_3$, here "$\dashv$" corresponds to "N", "$\prec$" to "C" and "$\top$" to Slupecki operator "$\top$". (See [3]). Thus, theorem 1 shows that the system MP^* implies the system $Ł_3$.

Definition 2 In $Ł_3$,

$D(\Rightarrow)$: $A \Rightarrow B =_{df} A \prec (A \prec B)$

$D(\rightarrow)$: $A \rightarrow B =_{df} (\dashv A \prec B) \prec B$

$D(\sim)$: $\sim A =_{df} ((A \prec \top A) \prec \dashv(\top A \prec A))$

$D(\neg)$: $\neg A =_{df} A \prec \top A$

From the above definition we have

Theorem 2 In $Ł_3$,

$(\in)$: $A_1 A_2, \cdots, A_n \vdash A_i$

(τ): If $\Gamma \vdash \Delta \vdash A$, then $\Gamma \vdash A$

$(\neg)$: If $\Gamma, \neg A \vdash B, \neg B$, then $\Gamma \vdash A$

$(\rightarrow_-)$: $A \rightarrow B, A \vdash B$; $A \rightarrow B, \sim A \vdash B$

$(\rightarrow_+)$: If $\Gamma, A \vdash B$, and $\Gamma, \sim A \vdash B$, then $\Gamma \vdash A \rightarrow B$

(Y): $A \dashv\vdash \neg\dashv A,\ \neg \sim A$

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$(Y_{\sim})$: $\sim A \vDash \neg A, \neg \exists A$

(Y): $\exists A \vDash \neg A, \neg \sim A$

$(\exists\exists_{+}), (\exists\exists_{-})$: $A \vDash \exists\exists A$

$(\exists_{\rightarrow})$: $A, \exists B \vDash \exists(A \rightarrow B)$

$(\sim\sim)$: $A \rightarrow A \vDash \sim\sim A$

$(\prec)$: $A \prec B \vDash (A \rightarrow B) \vee (\sim A \wedge B)$

$(\sim\prec)$: $\sim(A \prec B) \vDash (\sim A \wedge \exists B) \vee (A \wedge \sim B)$

$(\exists\prec)$: $\exists(A \prec B) \vDash A \wedge \exists B$

Theorem 3 In $Ł_3$, for any wff $f(P)$,

$$f(\neg A) \vDash f(A \rightarrow \sim A)$$

From theorem 2, 3 we know that the system $Ł_3$ implies the system MP^*, therefore, we can conclude that $Ł_3$ is equivalent to MP^*.

Refernces

[1] Zhu Wujia, Xiao Xian, JMRE, Vol.8(1988) No.2,3,4.

[2] Xiao Xian, Zhu Wujia, Nature Journal, 8 (1985) 601, 681.

[3] R. Ackermann, An Introduction to Many-valued Logics, London, 1967.