Journal of Mathematical Research and Exposition
Vol.11 No.3 Aug. 1991

The Saturation Property of Cosine Operator Function*

Zhou Mi
(Xiamen Fishery College)

Let X be a Banach space and $B(X)$ the algebra of all bounded linear operators on X. $\{C(t); t\in R\}$ is a strongly continuous cosine operator function on X with infinitesimal generator A [1].

Lemma 1 For each f, X, one has:

i) $(C(X)-I)\int_0^t (t-u)C(u)f\,du = (C(t)-I)\int_0^x (x-s)C(s)f\,ds$,

ii) $\int_0^t (t-u)C(u)f\,du \in D(A)$ and $A\int_0^t (t-u)C(u)f\,du = C(t)f-f$,

iii) $\lim\limits_{t\to 0}\frac{2}{t}\int_0^t (t-u)C(u)f\,du = f$.

Lemma 2 If $f\in D(A)$ then

i) $C(t)f-f=\int_0^t (t-u)C(u)Af\,du = A\int_0^t (t-u)C(u)f\,du$,

ii) $\|\frac{2}{t^2}(C(t)-I)f\| \leqslant \frac{2M}{\omega^2 t^2}\|Af\|\,|e^{\omega|t|}-\omega|t|-1|$, where M,ω satisfying $\|C(t)\|\leqslant M\,e^{\omega|t|}$ as in [1].

Definition 1 $C(t)$ is said to be saturated in X with order $O[\varphi(\frac{1}{|t|})]$ $(t\to 0^+)$, $\varphi(t)$ a positive non-increasing function on $(0,\infty)$ satisfying $\lim\limits_{s\to+\infty} C(s)=0$, if:

i) for every $f\in X$ and $\|C(t)f-f\| = s_0(\varphi(\frac{1}{|t|}))$ $(t\to 0) \Rightarrow T(t)f=f$ for small t,

ii) there is a class of functions $F\subset X$ containing at least one element which is not invariant such that $\|C(t)f-f\| = O(\varphi(\frac{1}{|t|}))$ $(t\to 0) \Leftrightarrow f\in F$. F is called saturation class.

Definition 2 Let Y be a linear manifold of X, endowed with norm $\|\cdot\|_Y$, and the completion of Y related to X, denoted by $\widetilde{Y}$, is defined as follows:
$Y=\{f|f\in X$, there are $\{f_n\}\in Y$ and $M>0$ such that $\|f_n\|_Y\leqslant M$ and $\lim\limits_{n\to\infty}\|f_n-f\|_X=0\}$.

Theorem 1 i) If $f\in X$ and there is $g\in X$ such that

$$\lim_{t\to 0}\|\frac{2}{\tau^2}(C(\tau)f-f)-f)-g\| = 0\,;$$

then $f\in D(A)$ and $Af=g$, in particular, if $g=\theta$ then $Af=0$, a.e. $C(t)f=f$ for small t, f is an invariant element of $C(t)$. (to 371)

Received Dec. 27, 1989.

$$\sup_{2i+1>\max(1,1-k)} (\partial_0\partial_1\cdots\partial_{(2i+1)-1+k})/(\beta_0\beta_1\cdots\beta_{(2i+1)-1})<\infty \qquad (e)$$

由式（c），（e）得引理2中第一个不等式成立．同理由定理8中式（2），（3），（4）可推推得引理2中其余三个不等式成立，因此W_λ，W_β拟相似．

林辰教授详细审阅了本文原稿，特此致谢！

参 考 文 献

〔1〕孙善利，辽宁大学学报，1987年第一期，1－7．
〔2〕L·A·Fialkow, Acta Sci·Math., 39(1977), 65—85.
〔3〕孟南，数学年刊，9(A)，2:110—118.
〔4〕严子锟，科学通报，33(1988)，21:1677—1678.
〔5〕L·A·Fialkow, Tran.Amer.Math.Soc., 243(1978) 147—168.

Quasisimilarity of Weighted Shifts of Multiplicity N and Degree m

Yan Zikun
(Fujian Teachers University, Fuzhou)

Abstract

In this article, We show that quasisimilar injective bilateral weighted shifts of multiplicity N and degree m have equal essential spectra and show that quasisimilar injective unilateral weighted shifts of multiplicity N and degree m are similar. It is also gived that necessary and sufficient conditions for two injective bilateral weighted shifts of degree 2 to be quasisimilar.

(from 372)

ii) If X is a reflexive and $f\in X$ such that $\lim_{\tau\to 0}\left\|\frac{T(\tau)f-f}{\tau^2}\right\|<\infty$ then $f\in D(A)$.

Theorem 2 If X is a reflexive Banach space, then $C(t)$ is saturated in X with order $O(t^2)$ $(t\to 0)$ and $D(A)$ is its saturation class.

Theorem 3 For general case, $C(t)$ is saturated in X with order $O(t^2)$ $(t\to 0)$ and its saturation class is $D(A)$, here $D(A)$ is endowed with norm $\|f\|_{D(A)}=\|f\|+\|Af\|$, for $f\in D(A)$.

Referece

〔1〕D. Lutz, Strongly continuous operator function, in "Functional Analysisy" Proc, Conf., Dubrovnik, Yugoslavia, 1981, 73—97, Lect. Notes in Math. No. 948, 1982.
〔2〕Sen-Yen Shaw, On W*-Continuous Cosine Operator Functions J. Func. Anal. Vol. 66, 73-95 (1986).