Journal of Mathematical Research and Exposition
Vol.11 No.3 Aug.1991

On Graph Having Clique Number a, Chromatic Number $a+1$

Xu Baogang

(Dept. Math. ShanDong University, Jinan)

Let G is a simple graph, $\omega(G)$、$\Delta(G)$、$\chi(G)$ are maximum clique number of G, maximum degree and chromatic number of G respectively. In [2], James defines that (a,b,c) (where a,b,c are positive integer) is graphical if there exists G which $\omega(G)=a$, $\chi(G)=b$, $\Delta(G)=c$. We say G is on (a,b,c) and set $P(a,b,$ $(a,b,c))=\min\{|V(G)| \mid G$ is on $(a,b,c)\}$. All other signs are from [1].

We get the following results.

Lemma 1 If $(a,a+1,a+1)$ is graphical, then $P(a,a+1,a+1)\geqslant 2a+1$.

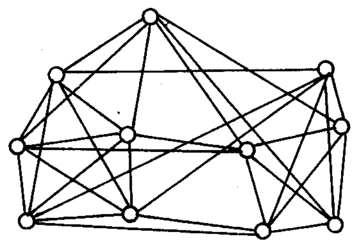
Lemma 2 Given graph G, $|V(G)|=2a+1, \omega(G)=a, \Delta(G)=a+1$, if G contains a subgraph $H\cong K_a$, and for $v_{i_0}\in V(H)$, $|I_{i_0}(G)|\geqslant 3$. Then $\chi(G)=a$ (Where $I_{i_0}(G)$ is a maximum independent set which contains v_{i_0}) $(a\geqslant 5)$.

Theorem 1 If G satisfies $\omega(G)=a$、$\Delta(G)=a+1$、$\alpha(G)\geqslant 3$ and $|V(G)|=2a+1$, then $\chi(G)=a$.

Theorem 2 For $a\geqslant 6$, if $(a,a+1,a+1)$ is graphical, then $P(a,a+1,a+1)\geqslant 2a+2$.

Corollary $P(5,6,6)=11$, $P(6,7,7)=14$.

In addition, we get a graph which is on $(5,6,6)$ and order 11.



References

[1] J.A.Bondy and U.S.R Murty, Graph theory with application, 1971.
[2] James.M.Benedict and Phyllis.Zweig Chinn, On graph having prescribed clique number, chromatic number and maximum degree, Lecture notes in mathematics 642.