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Generalizations of Diagonal Dominance for Matrices and Its Applications*

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(一)

In [1] Feingold and Varga discussed the block diagonal dominance of matrices and its applications. In this paper we give the definition of the generalized block diagonally dominant matrix and whose determinations and applications, these results generalize and improve corresponding results of [1]—[4], respectively.

Let A be an $n\times n$ matrix with complex entries, which is partitioned in the following manner:

$$A=\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix} \qquad (1)$$

where the diagonal submatrices A_{ii} are square of order n_i, $1\leq i\leq N$.

Definition 1 Suppose $A\in C^{n\times n}$ is partitioned as in (1). If diagonal submatrices A_{ii} are nonsingular, $1\leq i\leq N$, and if

$$(\|A_{jj}^{-1}\|)^{-1}\geq \sum_{\substack{k=1\\k\neq j}} \|A_{jk}\|, \quad (1\leq j\leq N), \qquad (2)$$

then A is block diagonally dominant, relative to the partitioning (1), denoted as $A\in G_0$. If strict inequality in (2) is valid for all $1\leq j\leq N$, then A is block strictly diagonally dominant, relative to the partitioning (1), denoted as $A\in G$. If strict inequality in (2) is valid for at least one $j(1\leq j\leq N)$ and $B=(\|A_{ij}\|)_{N\times N}$ is an irreducible matrix, then A is block irreducible diagonally dominant, relative to the partitioning (1), denoted as $A\in I$.

Definition 2 Let $A\in C^{n\times n}$ be partitioned as in (1). If there exists a positive diagonal matrix D of order n such that $B=AD\in G_0$, $D=\mathrm{diag}\{D_1,D_2,\cdots,D_N\}$, where D_i are positive diagonal matrices of order n_i, $1\leq i\leq N$, then A is called generalized block diagonally dominant, relative to the partitioning (1), denoted as $A\in G_0^*$. In particular, if $B=AD\in G$, then A is called generalized block strictly diago-

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nally dominant, relative to the partitioning (1), denoted as $A \in G^*$.

Clearly, if $D=I$ then definition 2 becomes definilion 1, if all A_{ii} are 1×1 matrices and $\|x\|=|x|$, then definition (2) becomes definition of generatized diagonally dominant matrix, In [1] it is proved that, if $A\in G\cup I$, then $\det A \neq 0$. Clearly if $A\in G^*$ then $\det A\neq 0$. But $A\in G^*$ does not imply that $A\in G\cup I$. For instance, consider the case $n=4$, $N=2$ of

$$A=\left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ \hline 0 & \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 \end{array}\right]$$

where we choose the vector norms $\|x\|_\infty=\max\limits_j |x_j|$. In this case,

$$(\|A_{11}^{-1}\|_\infty)^{-1}=(\|A_{22}^{-1}\|_\infty)^{-1}=\frac{1}{3}<\frac{2}{3}=\|A_{12}\|_\infty=\|A_{21}\|_\infty,$$

so $A \bar{\in} G\cup I$. But take $D=\operatorname{diag}\{3,1,1,3\}$, then $B=AD$ satisfies

$$(\|B_{11}^{-1}\|_\infty)^{-1}=(\|B_{22}^{-1}\|_\infty)^{-1}=1>\frac{2}{3}=\|B_{12}\|_\infty=\|B_{21}\|_\infty,$$

so $A\in G^*$, thus A is nonsingular.

Let $A\in \mathbf{C}^{n\times n}$ be partitioned as in (1). Denote

$$\mu(A)=\begin{bmatrix} (\|A_{11}^{-1}\|)^{-1} & -\|A_{12}\| & \cdots & -\|A_{1N}\| \\ -\|A_{21}\| & (\|A_{22}^{-1}\|)^{-1} & \cdots & -\|A_{2N}\| \\ \cdots & \cdots & \cdots & \cdots \\ -\|A_{N1}\| & -\|A_{N2}\| & \cdots & (\|A_{NN}^{-1}\|)^{-1} \end{bmatrix} \qquad (3)$$

Lemma 1 Let $A\in \mathbf{C}^{n\times n}$ be partitioned as in (1). If $\mu(A)$ is an M-matrix, then $A\in G^*$, and $\det A\neq 0$.

Lemma 2 Let $A\in \mathbf{C}^{n\times n}$ be partitioned as in (1). If there exists a positive diagonal matrix $D=\operatorname{diag}\{D_i,\cdots,D_N\}$ such that $AD\in G$, where $D_i=d_iI_{n_i}$, $d_i>0$ $(1\leq i\leq N)$, then there exists at least one $1\leq j\leq N$ such that

$$(\|A_{jj}^{-1}\|)^{-1}>\sum_{\substack{k=1\\k\neq j}}\|A_{jk}\|.$$

Proof Obviously, $D\neq dI$, $d>0$, if $A\bar{\in} G$. Without loss of generality assume

$$d_1=\max_i d_i>\min_i d_i=d_N$$

If for all $1\leq j\leq N$, $(\|A_{jj}^{-1}\|)^{-1}\leq\sum\limits_{k\neq j}\|A_{jk}\|$, then $B=AD$ satisfies

$$\begin{aligned}(\|B_{NN}^{-1}\|)^{-1}&=(\|(A_{NN}D_N)^{-1}\|)^{-1}=d_N(\|A_{NN}^{-1}\|)^{-1}\\ &\leq d_N\sum_{k\neq N}\|A_{Nk}\|\leq\sum_{k\neq N}\|A_{Nk}\|d_k\\ &=\sum_{k\neq N}\|A_{Nk}D_k\|=\sum_{k\neq N}\|B_{Nk}\|,\end{aligned}$$

this contradicts the assumption $B=AD\in G$.

(二)

Theorem 1 Let $A\in \mathbf{C}^{n\times n}$ be partitioned as in (1). If

$$(\|A_{ii}^{-1}\|\|A_{jj}^{-1}\|)^{-1}>(\sum_{k\neq i}\|A_{ik}\|)(\sum_{k\neq j}\|A_{jk}\|),\quad (1\leq i,j\leq N,\ i\neq j)$$

then $A\in G^*$.

Proof By assumptions there exists at most one $1\leq i\leq N$ such that

$$(\|A_{ii}^{-1}\|)^{-1}\leq\sum_{k\neq i}\|A_{ik}\|.$$

So without loss of generality assume that

$$(\|A_{11}^{-1}\|)^{-1}\leq\sum_{k\neq 1}\|A_{1k}\|,\ (\|A_{jj}^{-1}\|)^{-1}>\sum_{k\neq 1}\|A_{jk}\|,\quad (1<j\leq N).$$

Since

$$(A_{11}^{-1}\|\|A_{jj}^{-1}\|)^{-1}>(\sum_{k\neq 1}\|A_{1k}\|)(\sum_{k\neq j}\|A_{jk}\|),\quad (1<j\leq N)$$

there exists a positive number ε such that

$$\|A_{11}^{-1}\|(\sum_{k\neq 1}\|A_{1k}\|)+\varepsilon<\min_{1<j\leq N}\{(\|A_{jj}^{-1}\|)^{-1}/(\sum_{k\neq j}\|A_{jk}\|)\}.$$

Denote $d_i=\|A_{11}^{-1}\|(\sum_{k\neq 1}\|A_{1k}\|)+\varepsilon$, and let $D=\mathrm{diag}(d_iI_{n_1},I_{n_2},\cdots,I_{n_r}\}$. Then $B=AD$ satisfies

$$(\|B_{11}^{-1}\|)^{-1}=(\|(A_{11}D_1)^{-1}\|)^{-1}=(\|A_{11}^{-1}\|)^{-1}d_1>\sum_{k\neq 1}\|A_{1k}\|=\sum_{k\neq 1}\|B_{1k}\|,$$

$$(\|B_{jj}^{-1}\|)^{-1}=(\|A_{jj}^{-1}\|)^{-1}>d_1\cdot(\sum_{k\neq j}\|A_{jk}\|)$$

$$\geq d_1\|A_{j1}\|+\sum_{\substack{k>1\\k\neq j}}\|A_{jk}\|=\sum_{k\neq j}\|B_{jk}\|,\quad (1<j\leq N).$$

So that $B=AD\in G$, and $A\in G^*$.

We denote

$$N^*=\{1,2,\cdots,N\},$$

$$J=\{i\in N^*|(\|A_{ii}^{-1}\|)^{-1}>\sum_{k\neq i}\|A_{ik}\|\},$$

$$Z^{n\times n}=\{A=(a_{ij})\in \mathbf{R}^{n\times n}|a_{ij}\leq 0,\ i\neq j\}.$$

By Lemma 2 if there exists $D=\mathrm{diag}\{d_1I_{n_1},\cdots,d_NI_{n_N}\}$ such that $AD\in G$, where $d_i>0$ $(1\leq i\leq N)$, then $J\neq\phi$.

Lemma 3 Suppose $A=(a_{ij})\in \mathbf{R}^{n\times n}$ is an M-matrix. Let $L(U)$ be any unit lower (upper) triangular matrix, if LA $(UA)\in \mathbf{Z}^{n\times n}$, then $LA(UA)$ is also an M-matrix.

Lemma 4 ([5]) Suppose $A\begin{bmatrix}A_1 & B_1\\ 0 & C_1\end{bmatrix}\in \mathbf{Z}^{n\times n}$, where A_1 and C_1 are square matrices. Then A is an M-matrix iff A_1 and C_1 are M-matrices.

Theorem 2 Let $A\in \mathbf{C}^{n\times n}$ be partitioned as in (1). If A satisfies:

1) $J\neq\phi$, $\{i_1<i_2<\cdots<i_k\}=N^*-J$;

2) $$A_k=\begin{bmatrix}(\|A_{i_1i_1}^{-1}\|)^{-1} & -\|A_{i_1i_2}\| & \cdots & -\|A_{i_1i_k}\|\\ -\|A_{i_2i_1}\| & (\|A_{i_2i_2}^{-1}\|)^{1} & \cdots & -\|A_{i_2i_k}\|\\ \cdots & \cdots & \cdots & \cdots\\ -\|A_{i_ki_1}\| & -\|A_{i_ki_2}\| & \cdots & (\|A_{i_ki_k}^{-1}\|)^{-1}\end{bmatrix}$$

is an M-matrix;

3) $\min\limits_{j\in J} a_j = a > \beta = \max\limits_{s\in N^*-J} \beta_s$, where

$$a_j = \frac{(\|A_{jj}^{-1}\|)^{-1} - \sum\limits_{\substack{t\in J\\ t\neq j}} \|A_{jt}\|}{\sum\limits_{t\notin J} \|A_{jt}\|}, \qquad P_k = \begin{bmatrix} \sum\limits_{t\in J} \|A_{i_1 t}\| \\ \sum\limits_{t\in J} \|A_{i_2 t}\| \\ \vdots \\ \sum\limits_{t\in J} \|A_{i_k t}\| \end{bmatrix},$$

$\beta_s = (A_k^{-1} P_k)_s$ denotes the s-th component of $A_k^{-1} P_k$. When $\sum\limits_{t\notin J} \|A_{jt}\| = 0$ take $a_j = +\infty$. Then $A\in G^*$.

Proof If $J = N^*$ then conclusion holds. So that suppose $N^* - J \neq \phi$ and without loss of generality suppose $i_1 = 1, \cdots, i_k = k$ $(k \geq 1)$. Moreover Let $\mu(A)$ be partitioned as in the following manner:

$$\mu(A) = \begin{bmatrix} A_k & A_1 \\ A_2 & A_{n-k} \end{bmatrix}, \tag{4}$$

where A_k and A_{n-k} are squares of order k and $n-k$, respectively. Construct unit lower triangular matrix

$$L = \begin{bmatrix} I_k & 0 \\ -A_2 A_k^{-1} & I_{n-k} \end{bmatrix} \geq 0, \text{ as } A_2 \leq 0,\ A_k^{-1} \geq 0$$

then

$$L\mu(A) = \begin{bmatrix} A_k & A_1 \\ 0 & A_{n-k} - A_2 A_k^{-1} A_1 \end{bmatrix} \in \mathbf{Z}^{n\times n}$$

Denote by $e_{n-k} = (1, \cdots, 1)^T$ a column vector of order $n-k$, then by assumption 3) we deduce that

$$(A_{n-k} - A_2 A_k^{-1} A_1) e_{n-k} = A_{n-k} e_{n-k} + A_2 A_k^{-1} P_k > 0.$$

Thus $A_{n-k} - A_2 A_k^{-1} A_1 \in \mathbf{Z}^{(n-k)\times(n-k)}$ and is strictly diagonally dominant matrix, so that $A_{n-k} - A_2 A_k^{-1} A_1$ is an M-matrix. Again by assmption 2) and Lemma 4 $L\mu(A)$ is an M-matrix. By Lemma 3 and [5] and notice that $L\mu(A) \in \mathbf{Z}^{n\times n}$, $\mu(A)$ is also an M-matrix. Therefore there exists a positive diagonal matrix $D = \mathrm{diag}\{d_1, \cdots, d_N\}$ such that $\mu(A)D$ is strictly diagonally dominant. Moreover let $D = \mathrm{diag}\{d_1 I_{n_1}, d_2 I_{n_2}, \cdots, d_N I_{n_N}\}$, then $AD \in G$, i.e., $A\in G^*$.

Similarly, we can prove the following.

Theorem 3 Let $A\in \mathbf{C}^{n\times n}$ be partitioned as in (1). If A satisfies:

1) $J \neq \phi$, $N^* - J = \{1, 2, \cdots, k\}$;

2) $\min\limits_{j\in N^*-J} \hat{a}_j = \hat{a} > \hat{\beta} = \max\limits_{s\in J} \beta_s$, where

$$A_{n-k}=\begin{bmatrix} (\|A_{k+1,k+1}^{-1}\|)^{-1} & -\|A_{k+1,k+2}\| & \cdots & -\|A_{k+1,N}\| \\ -\|A_{k+2,k+1}\| & (\|A_{k+2,k+2}^{-1}\|)^{-1} & \cdots & -\|A_{k+2,N}\| \\ \cdots & \cdots & \cdots & \cdots \\ -\|A_{N,k+1}\| & -\|A_{N,k+2}\| & \cdots & (\|A_{NN}^{-1}\|)^{-1} \end{bmatrix}$$

$$a_j=\frac{(\|A_{jj}^{-1}\|)^{-1}-\sum\limits_{\substack{t\leq k\\ t\neq j}}\|A_{jt}\|}{\sum\limits_{t>k}\|A_{jt}\|} \qquad \hat{P}=\begin{bmatrix} \sum\limits_{t\leq k}\|A_{k+1\,t}\| \\ \sum\limits_{t\leq k}\|A_{k+2\,t}\| \\ \vdots \\ \sum\limits_{t\leq k}\|A_{Nt}\| \end{bmatrix}$$

$\hat{\beta}_s=(A_{n-k}^{-1}\hat{P})_s$ denotes s-th component of $A_{n-k}^{-1}\hat{P}$. Then $A\in G^*$.

Lemma 5 Let $A\in C^{n\times n}$ be partitioned as in (1). If $A\in G_0$ and $J\neq\phi$, and for $i\notin J$ there exists a nonzero element chain $A_{ii_1}\neq 0, A_{i_1i_2}\neq 0,\cdots,A_{i_rj}\neq 0$ such that $j\in J$ Then $A\in G^*$.

Proof Without loss of generality we suppose $\{1,2,\cdots,k\}=N^*-J$, $k\geq 1$, and let $\mu(A)$ be partitioned as in (4). Obviously, $A_1\neq 0$, so that A is diagonally dominant with nonzero element chains, A_k is an M-matrix ([2]). By similar proof of Therem 2 we show that matrix $A_{n-k}-A_2A_k^{-1}A_1$ is also an M-matrix, By assumption $A_ke_k=-A_1e_{n-k}$, so that $e_k=-A_k^{-1}A_1e_{n-k}$. Thus

$$(A_{n-k}-A_2A_k^{-1}A_1)e_{n-k}=A_{n-k}e_{n-k}-A_2A_k^{-1}A_1e_{n-k}=A_{n-k}e_{n-k}+A_2e_k>0.$$

Notice that the leask $n-k$ rows of $\mu(A)$ are strictly diagonally dominant, so that $A_{n-k}-A_2A_k^{-1}A_1\in Z^{(n-k)\times(n-k)}$ and is an M-matrix.

Corollary Let $A\in C^{n\times n}$ be partitioned as in (1). If $A\in I$, then $A\in G^*$.

Theorem 4 Let $A\in C^{n\times n}$ be partitioned as in (1). If $A\in G_0$, then there exists a $D=\mathrm{diag}\{d_1I_{n_1},d_2I_{n_2},\cdots,d_NI_{n_N}\}$, $d_i>0$, $i\in N^*$ such that $AD\in G$ iff for each $i\notin J$ there exists an nonzero element chain $A_{ii_1}\neq 0, A_{i_1i_2}\neq 0,\cdots,A_{i_rj}\neq 0$ such that $j\in J$.

The proof of necessity is similar to Lemma 3, 4 in [2].

Lemma 6 Let $A\in C^{n\times n}$ be block irreducible and partitioned as in (1). If A satisfies assumptions 1)、2) of Theorem

3) either $\min\limits_{j\in J}a_j\geq\max\limits_{s\in N^*-J}\beta_s$ when $\max\limits_{j\in J}a_j>\min\limits_{j\in J}a_j$ or $\min\limits_{j\in J}a_j>\max\limits_{s\in N^*-J}\beta_s$ when $\max\limits_{j\in J}a_j=\min\limits_{j\in J}a_j$, where definitions of a_j and β_s are as in 3) of Theorem 2. Then $A\in G^*$.

Proof Without loss of generality assume that $J=\{k+1,\cdots,N\}$ and $k\geq 1$, $a_{k+1}=\max\limits_{j\in J}a_j>\min\limits_{j\in J}a_j$. Let $D_1=\mathrm{diag}\{\beta_1I_{n_1},\beta_2I_{n_2},\cdots,\beta_kI_{n_k},I_{n_{k+1}},\cdots,I_{n_N}\}$ Then we can prove that $B=AD_1\in I$ by similar proof of Theorem 2. Again by Lemma 5 there exists a positive diagonal matrix D_2 such that $BD_2\in G$. Denote $D=D_1D_2$, then $AD\in G$, i.e., $A\in G^*$.

Theorem 5 Let $A\in C^{n\times n}$ be partitioned as in (1). If A satisfies all assumptions of Lemma 6 except that A is block irreducible, and suppose $J=\{k+1,\cdots,N\}$, and for each $j\in N^*-J+\{t_1,\cdots,t_l\}$, there exists an nonzero element chain $A_{jr_1}\neq 0$, $A_{r_1r_2}\neq 0,\cdots,A_{r_uj}\neq 0$, $\{j^*\in J-\{t_1,\cdots,t_l\}\neq\phi$, where $t_1,\cdots,t_l\in J$ and $a_{t_1}=\cdots=a_{t_l}=\min_{j\in J}a_j=\max_{s\in N^*-J}\beta_s$. Then $A\in G^*$.

Proof By Lemma 6 there exists a positive diagonal matrix D_1 such $B=AD_1\in G$. and for each $j\in J-\{t_1,\cdots,t_l\}$, the j-th block-row of B is block strictly diagonally dominant. By Lemma 5 there exists a positive diagonal matrix D_2 such that $BD_2\in G$. Let $D=D_1D_2$ then $AD\in G$, i.e., $A\in G^*$.

Clearly, Lemma 6 and Theorem 5 are generalizations of Lemma 5 and Theorem 4 in the same direction, respectively.

Theorem 6 Let $A\in C^{n\times n}$ be partitioned as in (1). If there exists N positive numbers $a_1,a_2,\cdots,a_N$ such that

$$a_N\leq\sum_{j\leq N}\|A_{Nj}\|\|A_{NN}^{-1}\|<1;$$

$$a_{N-1}\leq\|A_{N-1N}\|\|A_{N-1,N-1}^{-1}\|a_N+\sum_{j<N-1}\|A_{N-1,j}\|\|A_{N-1,N-1}^{-1}\|<1,$$

$$\cdots\cdots$$

$$a_1\leq\sum_{j>1}\|A_{1j}\|\|A_{11}^{-1}\|a_j<1.$$

Then $A\in G^*$.

Proof We take suitable $\varepsilon_N>0$ and $\varepsilon_{N-1}>0$

$$a_N^{(1)}=a_N+\varepsilon_N\leq\sum_{j<N}\|A_{Nj}\|\|A_{NN}^{-1}\|+\varepsilon_N<1;$$

$$a_{N-1}^{(1)}=\|A_{N-1,N}\|\|A_{N-1,N-1}^{-1}\|a_N^{(1)}+\sum_{j<N-1}\|A_{N-1j}\|\|A_{N-1,N-1}^{-1}\|<1;$$

$$\cdots\cdots$$

$$a_2^{(1)}=\sum_{j>2}^{N-1}\|A_{2j}\|\|A_{22}^{-1}\|a_j+\|A_{21}\|\|A_{22}^{-1}\|+\|A_{2N}\|\|A_{22}^{-1}\|a_N^{(1)}<1;$$

$$a_1^{(1)}=\|A_{1N}\|\|A_{11}^{-1}\|a_N^{(1)}+\sum_{j>1}^{N-1}\|A_{1j}\|\|A_{11}^{-1}\|a_j<1;$$

and

$$a_{N-1}^{(2)}=a_{N-1}^{(1)}+\varepsilon_{N-1}<1;$$

$$a_{N-2}^{(2)}=\|A_{N-2N}\|\|A_{N-2,N-2}^{-1}\|a_N^{(1)}+\|A_{N-2,N-1}\|\|A_{N-2,N-2}^{-1}\|a_{N-1}^{(2)}+ + \sum_{j<N-2}\|A_{N-2j}\|\|A_{N-2,N-2}^{-1}\|<1;$$

$$\cdots\cdots$$

$$a_1^{(2)}=\|A_{1N}\|\|A_{11}^{-1}\|a_N^{(1)}+\|A_{1N-1}\|\|A_{11}^{-1}\|a_{N-1}^{(2)}+\sum_{j>1}^{N-2}\|A_{1j}\|\|A_{11}^{-1}\|a_j<1;$$

$\cdots$, take suitable $\varepsilon_2>0$ such that

$$a_2^{(N-1)}=a_2^{(N-2)}+\varepsilon_2<1;$$

$$a_1^{(N-1)}=\sum_{j>1}\|A_{1j}\|\|A_{11}^{-1}\|a_j^{(N-j+1)}<1;$$

and suitable $\varepsilon_1>0$ such that

$$a_1^{(N)}=a_1^{(N-1)}+\varepsilon_1<1.$$

and let $D=\mathrm{diag}\{I_{n_1}, a_2^{(N-1)}I_{n_2}, a_3^{(N-2)}I_{n_3}, \cdots, a_{N-1}^{(2)}I_{n_{N-1}}, a_N^{(1)}I_{n_N}\}$ then $B=AD$ satisfies

$$(\|B_{11}^{-1}\|)^{-1}=(\|A_{11}^{-1}\|)^{-1}>(\|A_{11}^{-1}\|)^{-1}a_1^{(N-1)}=\sum_{j>1}\|A_{1j}\|a_j^{(N-j+1)}=\sum_{j>1}\|B_{1j}\|,$$

$$\begin{aligned}(\|B_{ii}^{-1}\|)^{-1}&=(\|A_{ii}^{-1}\|)^{-1}a_i^{(N-i+1)}>(\|A_{ii}^{-1}\|)^{-1}a_i^{(N-i)}\\&=\sum_{j>i}\|A_{ij}\|a_j^{(N-j+1)}+\sum_{j<i}\|A_{ij}\|\\&\geq\sum_{j>i}\|A_{ij}\|a_j^{(N-j+1)}+\sum_{\substack{j<i\\j\neq 1}}\|A_{ij}\|a_j^{(N-j+1)}+\|A_{i1}\|\\&=\sum_{j\neq i}\|B_{ij}\|,\qquad (1<i\leq N).\end{aligned}$$

Thus $B=AD\in G$, i.e., $A\in G^*$.

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矩阵块对角占优性的推广及应用

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摘　要

在本文中，我们给出了一类块对角占优矩阵的定义，讨论了块对角占优矩阵的判定及应用，相应的结果改进和推广了[1]—[4]中的若干结论.