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Use Cesàro Means To Describe Two Classes of Functions*

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Let $H(D)$ be the collection of functions which are analytic in the unit disc D. we call $B_0=\{f\in H(D), \lim_{|z|\to 1}(1-|z|^2)|f'(z)|=0\}$ litlle Bloch space. Let $f\in H(D), 0<p\leq\infty$. $M_p(r,f)=\{\frac{1}{2\pi}\int_0^{2\pi}|f(re^{i\theta})|^p d\theta\}^{\frac{1}{p}}$. If $\|f\|_p=\sup_{0\leq r<1}M_p(r,f)<\infty$, We say $f\in H^p$. If $\|f\|_{B^p}=(\int_0^1 M_p(r,f)^p dr)^{\frac{1}{p}}<\infty$, we say $f\in B^p$. Let $f(z)=\sum_{n=0}^{\infty}a_n z^n\in H(D)$, We call $\sigma_n(f)(z)=\sum_{k=0}^{n}(1-\frac{k}{n+1})a_k z^k$, $n=0,1,2,\cdots$ the Cesàro Means of f.

In this paper, we use Cesàro Means to describe B_0 and B^p, and obtain the following results.

Theorem 1 A function f is in B_0 if and only if $\|\sigma_n'(f)\|_\infty=o(n), n\to\infty$.

Theorem 2 If $f\in B_0$, then $\|\sigma_n(f)\|_\infty=o(\log n)$, $n\to\infty$.

Theorem 3 Let $f\in H(D)$, $\sigma_n(f)\in H^p$, $1\leq p<\infty$ and $\|\sigma_n(f)\|_p=O(n^a)$. If $-\frac{2}{p}<a<\frac{1}{p}$, then $f\in B^p$.

Theorem 4 Let $f\in H(D)$, $\sigma_n(f)\in H^p$, $0<p<1$ and $\|\sigma_n(f)\|_p=O(n^a)$. If $-1-\frac{1}{p}<a<1$, then $f\in B^p$.

Corollary 1 Let $f\in H(D)$, $a>0$, $\beta>0$, $1\leq p<\infty$.

(1) If $\|\sigma_n(f)\|_p=O(n^a)$ and $a+\beta<\frac{1}{p}$, then $f^{(\beta)}\in B^p$.

(2) If $\|\sigma_n(f)\|_p=O(n^{a+\beta})$ and $a<\frac{1}{p}$, then $f_{(\beta)}\in B^p$.

Corollary 2 Let $f\in H(D)$, $a>0$, $\beta>0$, $0<p<1$

(1) If $\|\sigma_n(f)\|_p=O(n^a)$ and $a+\beta<1$, then $f^{(\beta)}\in B^p$.

(2) If $\|\sigma_n(f)\|_p=O(n^{a+\beta})$ and $a<1$, then $f_{(\beta)}\in B^p$.

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