Journal of Mathematical Research and Exposition
Vol. 11, No.4 Nov. 1991

Note on the RMI Method and RMI Solver*

L. C. Hsu

(Dalian University of Technology)

In a booklet entitled "Relation-Mapping-Inversion (RMI) Method" (Jiangsu Education Publishing House (China), Nanking 1989) by the present author and the co-author Y. X. Zheng, the general RMI method and more than 20 examples involving various problems (solved by RMI) have been expounded. The RMI method used as a working principle applies to every branch of mathematical science and it may be formulated briefly:

$$(S, \underline{x}) \xrightarrow{\varphi} (S^*, \underline{x}^*) \xrightarrow{\psi} x^* \xrightarrow{\varphi^{-1}} x$$

where $(S, \underline{x}) \in \Sigma$ (a given class of problems with unknowns $\underline{x}$ to be found), $\varphi \in \Phi$ (proposed family of invertible mappings), $\{S^* = \varphi(S), \underline{x}^* = \varphi(\underline{x})\}$ is the image system with the unknown image $\underline{x}^*$ to be determined, $\psi \in \Psi$ (proposed family of image-determining procedures or solving operators), x^* is the image of x obtained via ψ, φ^{-1} denotes the inverse mapping, and $x = \varphi^{-1}(x^*)$ is the desired answer/solution obtained via φ^{-1}.

Generally, Σ is called a RM I solvable class with reespect to Φ and Ψ, and denoted by (Σ, Φ, Ψ). Some inverse problems have also been investigated (cf. J. Qufu Normal Univ., 15(1989), No.2, p. 1-9). Here we propose that various types of computerized "RMI Solvers" may be designed, which should in general consist of 5 machinery units, namely (1) a sorting machine used for deciding $S \in \Sigma$ and clarifying $\underline{x}$, (2) a machine used for choosing suitable mappings φ's from Φ (proposed store of mappings and compound mappings), (3) a machine used for dealing with S^* and for determining x^* by use of suitable $\psi \in \Psi$, (4) a machine of inverting x^* by use of φ^{-1}, (5) a machine for checking the result.

For examples, some simpler RMI Solvers (Ⅰ)(Ⅱ)(Ⅲ), etc., may be designed, where (Ⅰ) is that used for summing numerical series and power series by use of differential operators, etc., (Ⅱ) is for finding closed forms of various algebraic/combinatorial sums by means of Gould-Hsu inversion and the hypergeometric series method, and (Ⅲ) is for solving (IVP) of ODE using Laplace transforms, etc.

* Received Nov. 3, 1991