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The Koppelman-Leray-Norguet formula of type (p,q) on Stein manifolds*

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Abstract

Let M be a Stein manifold of complex dimension n, and let an open set $D\subset\subset M$ have a piecewise C^1-boundary. Using Chern metric and connection, we obtain an integral representation of (p,q) differential forms on D, which is a generalization of the Koppelman-Leray-Norgeut formula for $(0,q)$-forms on Stein manifolds. A integral formula for solving the $\bar{\partial}$-equation on D is also obtained. Finally, a formula for real non-degenerate strictly pseudoconvex polyhedron is given.

I. Introduction

The Koppelman-Leray-Norguet formula for analytic polyhedra in C^n was obtained in 1971 by G.M.Henkin [1] for $(0,1)$-forms and then by P.L.Poljakov [2] for $(0,q)$-forms by means of ideas of Henkin, Leray, Lieb, Koppelman and Norguet. Then R.H.Range and Y.T.Siu [3] and P.L.Poljakov [4] proved this formula for domains with piecewise smooth strictly pseudoconvex boundary in C^n. For the general case of strictly pseudoconvex polyhedra in C^n, this formula was obtained by G.M.Henkin (see E.M.Cirka and G.M.Henkin [5]), A.G.Sergeev [6] and see also N.Φvrelid [7]). Recently G.M.Henkin and J.Leiterer [8] obtained this formula for $(0,q)$-forms on Stein manifolds. The main purpose of this paper is to generalize this formula for (p,q)-forms on Stein manifolds by means of ideas of J.P.Demailly and Ch. Laurent-Thiebaut [9].

For simplicity, we use the notations from [3]、[8]、[9].

Definition I Let M be a Stein manifold of complex dimension n, an open set $D\subset\subset M$ is said to have a piecewise C^1-boundary if there exists a finite open covering $\{U_j\}_{j=1}^k$ of an open neighbourhood U of ∂D and C^1 functions $\rho_j: U_j\to R$ $(1\leq j\leq k)$ such that

1) $D\cap U=\{x\in U:$ for $1\leq j\leq k$, either $x\notin U_j$ or $\rho_j(x)<0\}$

2) for $1\leq i_1\leq\cdots\leq i_l\leq k$, the 1-forms $d\rho_{i_1},\cdots,d\rho_{i_l}$

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are linearly independent over R at every point of $\bigcap_{\nu=1}^{l} U_{i_\nu}$. $\{U_j, \rho_j\}_{j=1}^{k}$ is called a frame of D.

For every ordered subset $I=\{i_1, \cdots, i_l\}$ of $\{1, \cdots, k\}$, define

$$S_I=\{x\in\partial D\cap(\bigcap_{i\in I} U_i): \rho_i(x)=0 \text{ for } i\in I\}$$

and choose the orientation on S_I such that the orientation is skew symmetric in the components of I and the following two equations hold when D is given the natural orientation:

$$\partial D=\sum_{j=1}^{k} S_j \quad \text{and} \quad \partial S_I=\sum_{j=1}^{k} S_{I_j}$$

Let
$$\sigma=\{\lambda=(\lambda_0, \cdots, \lambda_k)\in R^{k+1}: \lambda_j\geq 0, \sum_{j=0}^{k}\lambda_j=1\}$$

be the standard k-simplex in R^{k+1} with the canonical orientation, for every ordered subset $J=\{j_1, \cdots, j_m\}$ of $\{0, 1, \cdots, k\}$ with strictly increasing components, set

$$\sigma_J=\{\lambda\in\sigma: \sum_{j\in J}\lambda_j=1\}$$

The orientation of each σ_J is chosen so that

$$\partial\sigma_J=\sum_{\nu=1}^{m}(-1)^{\nu+1}\sigma_{j_1\cdots\hat{j}_\nu\cdots j_m}$$

where $\hat{j}_\nu$ means that j_ν is omitted. With this orientation one has

$$\partial(S_I\times\sigma_J)=\partial S_I\times\sigma_J+(-1)^{|I|}S_I\times\partial\sigma_J$$

where $|I|$ is the number of components of I.

Lemma 1[3] $\partial(\sum_I(-1)^{|I|}S_I\times\sigma_{OI})=\sum_I S_I\times\sigma_I-\partial D\times\sigma_0$

where the summation is over all ordered subsets I of $\{1, \cdots, k\}$ with strictly increasing components, and $OI=(0, i_1, \cdots, i_l)$.

2. The Koppelman-Leray-Norguet formula

Throughout this section, M is a Stein manifold of complex dimension n and $D\subset\subset M$ is an open set with a piecewise C^1-boundary.

Definition 2 Let $S(z,\xi)$, $\varphi(z,\xi)$, K be as in Lemma 4.2.4 in [8], a Leray-Norguet section for (D, S, φ) is by definition a collection $(S_1^*, \cdots, S_k^*, K^*)$, where $K^*\geq 0$ is an integer and, for every $1\leq j\leq k$, $S_j^*(z,\xi)$ is a C^1-map with values in the complex cotangent bundle T^*M of M defined for ξ in some neighbourhood of S_j and $z\in D$ such that the following conditions are fulfilled:

1) $S_j^*(z,\xi)\in T_z^*(M)$ for all $z\in D$ and ξ in some neighbourhood of S_j.

2) $\langle S_j^*(z,\xi), S(z,\xi)\rangle\neq 0$ if $z\in D$, $\xi\in S_j$ and $\varphi(z,\xi)\neq 0$, and the function

$$\varphi^{K^*}(z,\xi)/\langle S^*(z,\xi), S(z,\xi)\rangle$$

is C^1 for (z,ξ) in some neighbourhood $\subseteq D\times M$ of $D\times S_j$.

Let $(S_1^*, \cdots, S_k^*, K^*)$ be a Laray-Norguet section for (D, S, φ), and suppose that $\overline{T}^*(M\times M\times[0,1])$ is the pull-back of T^*M relative to the projection $M\times M\times[0,1]\to M$, $(z,\xi,\lambda)\to\xi$. Set

$$t^*(z,\xi,\lambda)=\lambda_0\hat{S}(z,\xi)/|S(z,\xi)|_\theta^2+\sum_{j=1}^k\lambda_j S_j^*(z,\xi)/\langle S_j^*(z,\xi),S(z,\xi)\rangle$$

where $\hat{S}(z,\xi)=\delta S\in\widetilde{T}^*(M\times M)$ is defined in [9], which possesses the same characters with $\bar{S}$ in [8], the map $\delta:\widetilde{T}(M\times M)\to\widetilde{T}^*(M\times M)$, $\xi\mapsto\langle\cdot,\xi\rangle_\theta$, θ is a Hermitian metric in $\widetilde{T}(M\times M)$, here, $\widetilde{T}(M\times M)$ and $\widetilde{T}^*(M\times M)$ are the pull-backs of TM and T^*M, respectively, relative to the projection $M\times M\to M$, $(z,\xi)\mapsto\xi$.

Wherever this is defined and $t^*\in\overline{T}^*(M\times M\times[0,1])$. In particular, for any strictly increasing collection $I=(i_1,\cdots,i_l)$ of integers $1\le i_1<\cdots<i_l\le k$, this section t^* is defined in a submanifold $W\subseteq M\times M\times[0,1]$

$$W=[\bigcup_I(D\times S_I\times\sigma_{0I})]\cup[((M\times M)\setminus\Delta(M))\times\sigma_0]$$

Let θ^* and $\bar{\theta}^*$ be Hermitian metrics in $\widetilde{T}^*(M\times M)$ and $\overline{T}^*(M\times M\times[0,1])$, which are induced by the metric θ, respectively, and let D. ∇ and $\triangle$ be Chern connections of $\widetilde{T}(M\times M)$、$\widetilde{T}^*(M\times M)$ and $\overline{T}^*(M\times M\times[0,1])$ relative to θ、θ^* and $\bar{\theta}^*$ respectively. Suppose that $(e_j)_{j=1}^n$ are holomorphic trivial frames of $\widetilde{T}(M\times M)$, under these frames, the metric θ is expressed by a C^∞、positive definite Hermitian matrix H which does not depend on the variable z. Set $\Delta=\Delta'+\Delta''$, where $\Delta't^*=\overline{H}(\partial_{z,\xi}(\overline{H}^{-1}v^*))$, $\Delta''t^*=(\bar{\partial}_{z,\xi}+d_\lambda)v^*$, v^* is the coordinate expression of t^*, then, for every integer $v\ge\max\{nK^*,nK\}$ and every $z\in D$, the form

$$\overline{\Omega}^k(\varphi^v,\hat{S},S_1^*,\cdots,S_k^*,S)(z,\xi,\lambda)$$
$$=(-1)^{n-1}/(2\pi)^n\varphi^v(z,\xi)\times\langle t^*,DS\rangle\cap(\langle\Delta''t^*,DS\rangle)^{n-1}$$

is continuous for all (z,ξ,λ) in submanifold W. Write

$$\overline{\Omega}^k(\varphi^v,\hat{S},S_1^*,\cdots,S_k^*,S)(z,\xi,\lambda)=\sum_{\substack{1\le p\le n\\1\le q\le n-1}}\Omega_{p,q}^k(z,\xi,\lambda)$$

and let

$$\Omega_{p,-1}^k=\Omega_{p,n}^k=0$$

where $\Omega_{p,q}^k(z,\xi,\lambda)$ is the component of $\overline{\Omega}^k(\varphi^v,\hat{S},S_1^*,\cdots,S_k^*,S)(z,\xi,\lambda)$, which is of type (p,q) with respect to z.

Lemma 2

$$\overline{\Omega}^k(\varphi^v,\hat{S},S_1^*,\cdots,S_k^*,S)|_{\sigma_{0I}}=\overline{\Omega}^k(\varphi^v,S_1^*,\cdots,S_k^*,S)$$
$$\overline{\Omega}^k(\varphi^v,\hat{S},S_1^*,\cdots,S_k^*,S)|_{\sigma_0}=\overline{\Omega}^k(\varphi^v,\hat{S},S)=\widetilde{\Omega}^0(\varphi^v,\hat{S},S)$$

where

$$\widetilde{\Omega}^0(\varphi^v,\hat{S},S)=(-1)^{n-1}/(2\pi)^n\varphi^v(z,\xi)\langle\hat{S},DS\rangle\wedge(\langle\nabla''\hat{S},DS\rangle^{n-1}/|S|_\theta^{2n}.$$

Proof It suffices to show

$$\overline{\Omega}^k(\varphi^v,\hat{S},S)=\widetilde{\Omega}^0(\varphi^v,\hat{S},S)\qquad(1)$$

In fact

$$\overline{\Omega}^k(\varphi^v,\hat{S},S)=(-1)^{n-1}/(2\pi)^n\varphi^v\langle\hat{S}/|S|_\theta^2,DS\rangle$$
$$\wedge(\langle\Delta''(\hat{S}/|S|_\theta^2),DS\rangle)^{n-1}$$
$$\Delta''(\hat{S}/|S|_\theta^2)=(\Delta''(1/|S|_\theta^2))\hat{S}\quad(1/|S|_\theta^2)\Delta''\hat{S}$$

(1) follows from

$$\langle \hat{S}, DS\rangle \wedge \langle \hat{S}, DS\rangle = 0 \quad \text{and} \quad \Delta'' \hat{S} = \nabla'' \hat{S}.$$

Lemma 3 For every $(z, \xi, \lambda) \in W \subset D \times M \times [0,1]$, we have

$$\begin{aligned} d\overline{\Omega}^k(\varphi^v, \hat{S}, S_1^*, \cdots, S_k^*, S) &= (\overline{\partial}_{z,\xi} + d_\lambda)\overline{\Omega}^k(\varphi^v, \hat{S}, S_1^*, \cdots, S_k^*, S) \\ &= (-1)^{n-1}/(2\pi)^n \varphi^v [\langle t^*, C(\widetilde{T}(M\times M)) \wedge S\rangle \wedge (\langle \Delta'' t^*, DS\rangle)^{n-1} \\ &\quad + (n-1)\langle t^*, DS\rangle \wedge \langle \Delta'' t^*, C(\widetilde{T}(M\times M)) \wedge S\rangle \wedge (\langle \Delta'' t^*, DS\rangle)^{n-2} \\ &=: Q^k(\varphi^v, \hat{S}, S_1^*, \cdots, S_k^*, S)(z, \xi, \lambda) \end{aligned}$$

and therefore

$$(\overline{\partial}_\xi + d_\lambda)\Omega^k_{p,q}(z, \xi, \lambda) = Q^k_{p,q}(z, \xi, \lambda) - \overline{\partial}_z \Omega^k_{p,q-1}(z, \xi, \lambda)$$

where $Q^k_{p,q}(z,\xi,\lambda)$ is the component of $Q^k(\varphi^v, \hat{S}, S_1^*, \cdots, S_k^*, S)(z, \xi, \lambda)$, which is of type (p, q) with respect to z, and $C(\widetilde{T}(M\times M)) = D^2$.

Proof Similar to the case of Koppelman-Leray formula of (p, q)-forms (cf. [9]).

Lemma 4[9] (Bochner-Martinelli-Koppelman formula) Let f be a continuous (p,q)-form on $\overline{D}$ such that $\overline{\partial} f$ is also continuous on $\overline{D}$, and let $v \geq 2nK$, then [9]

$$\begin{aligned} &(-1)^{p+q} f(z) - \int_{\partial D} f(\xi) \wedge \Omega^0_{p,q}(z,\xi) - \int_D \overline{\partial}_\xi f(\xi) \wedge \Omega^0_{p,q}(z,\xi) \\ &+ \overline{\partial}_z \int_D f(\xi) \wedge \Omega^0_{p,q-1}(z,\xi) + (-1)^{p+q+1} \int_D f(\xi) \wedge P^0_{p,q}(z,\xi), \quad z \in D \end{aligned}$$

where $\Omega^0_{p,q}(z,\xi)$ and $P^0_{p,q}(z,\xi)$ are the components of $\widetilde{\Omega}^0(\varphi^v, \hat{S}, S)(z,\xi)$ and $\overline{\partial}_{z,\xi}\widetilde{\Omega}^0(\varphi^v, \hat{S}, S) =: P^0(z,\xi)$, respectively, which are types of (p, q) with respect to z.

Under the hypotheses of Lemma 4, it follows immediately from Lemma 3 that

Lemma 5 Under the hypotheses of Lemma 4, we have

$$\begin{aligned} d_{\xi,\lambda}[f(\xi) \wedge \Omega^k_{p,q}(z, \xi, \lambda)] &= \overline{\partial}_\xi f(\xi) \wedge \Omega^k_{p,q}(z, \xi, \lambda) \\ &+ (-1)^{p+q} f(\xi) \wedge Q^k_{p,q}(z, \xi, \lambda) + (-1)^{p+q+1} f(\xi) \wedge \overline{\partial}_z \Omega^k_{p,q-1}(z, \xi, \lambda) \end{aligned}$$

Theorem 1 (Koppelman-Leray-Norguet formula) Let f be a continuous (p, q)-form on $\overline{D}$ such that $\overline{\partial} f$ is also continuous on $\overline{D}$, and let $v \geq \max(2nK, nK^*)$, $0 \leq p,\ q \leq n$, then

$$\begin{aligned} (-1)^{p+q} f(z) = &\sum_{|I| \leq n-p-q} \int_{S_I \times \sigma_I} f(\xi) \wedge \Omega^k_{p,q}(z, \xi, \lambda) \\ &+ \overline{\partial}_z \Big[\sum_{|I| \leq n-p-q} (-1)^{|I|} \int_{S_I \times \sigma_{0I}} f(\xi) \wedge \Omega^k_{p,q}(z, \xi, \lambda) \\ &+ \int_{D \times \sigma_0} f(\xi) \wedge \Omega^k_{p,q-1}(z, \xi, \lambda) \Big] \\ &- \Big[\sum_{|I| \leq n-p-q-1} (-1)^{|I|} \int_{S_I \times \sigma_{0I}} \overline{\partial}_\xi f(\xi) \wedge \Omega^k_{p,q}(z, \xi, \lambda) \\ &+ \int_{D \times \sigma_0} \overline{\partial}_\xi f(\xi) \wedge \Omega^k_{p,q}(z, \xi, \lambda) \Big] \\ &+ (-1)^{p+q+1} \Big[\sum_{|I| \leq n-p-q} (-1)^{|I|} \int_{S_I \times \sigma_{0I}} f(\xi) \wedge Q^k_{p,q}(z, \xi, \lambda) \\ &+ \int_{D \times \sigma_0} f(\xi) \wedge P^0_{p,q}(z, \xi) \Big], \quad z \in D, \end{aligned} \tag{2}$$

where the summation is over all strictly increasing subsets I of $\{1,\cdots,k\}$ with $|I|\leq n-p-q$ and $|I|\leq n-p-q-1$, respectively.

Remark 1) If $p=0$, notice that $C(\widetilde{T}(M\times M))$ is of type (1.1) with respect to z, then $P^0_{p,q}(z,\xi)=0$, $Q^k_{p,q}(z,\xi,\lambda)=0$, and therefore we obtain the Koppelman-Leray-Norguet formula for $(0,q)$-forms (cf. (4.6.4) in [8] and (3.7.53) in [10]).

2) If the metric θ satisfies $C(\widetilde{T}(M\times M))=0$, then $P^0_{p,q}(z,\xi)=0$, $Q^k_{p,q}(z,\xi,\lambda)=0$, and we obtain a formula, which is a generalization of the classical Koppelman-Leray-Norguet formula to Stein manifolds.

3) If $k=0$, (2) agrees with Theorem 4.3 in [9]. which is the Koppelman-Leray formula of type (p,q) on Stein manifolds. Especially when $p=0$, (2) agrees with Theorem 4.5.3 in [8].

The Proof of Theorem 1. Applying Stokes' formula to the form $f(\xi)\wedge\Omega^k_{p,q}(z,\xi,\lambda)$ on the submanifold $\sum_I(-1)^{|I|}S_I\times\sigma_{0I}$, we obtain from Lemma 1 and Lemma 5, that

$$\begin{aligned}&\sum_I(-1)^{|I|}\int_{S_I\times\sigma_{0I}}\bar\partial_\xi f(\xi)\wedge\Omega^k_{p,q}(z,\xi,\lambda)\\&+(-1)^{p+q+1}\sum_I(-1)^{|I|}\int_{S_I\times\sigma_{0I}}f(\xi)\wedge\bar\partial_z\Omega^k_{p,q-1}(z,\xi,\lambda)\\&+(-1)^{p+q}\sum_I(-1)^{|I|}\int_{S_I\times\sigma_{0I}}f(\xi)\wedge Q^k_{p,q}(z,\xi,\lambda)\\=&\sum_I\int_{S_I\times\sigma_{0I}}f(\xi)\wedge\Omega^k_{p,q}(z,\xi,\lambda)\\&-\int_{\partial D\times\sigma_0}f(\xi)\wedge\Omega^k_{p,q}(z,\xi,\lambda),\qquad z\in D,\end{aligned}\tag{3}$$

where the summation is over all ordered subsets I of $\{1,\cdots,k\}$ with strictly increasing components.

When $\lambda\in\sigma_0$, by Lemma 2, $\Omega^k_{p,q}(z,\xi,\lambda)=\Omega^0_{p,q}(z,\xi)$. It follows from Lemma 4 that

$$\begin{aligned}\int_{\partial D\times\sigma_0}f(\xi)\wedge\Omega^0_{p,q}(z,\xi)=&(-1)^{p+q}f(z)\\&+\int_{D\times\sigma_0}\bar\partial_\xi f(\xi)\wedge\Omega^0_{p,q}(z,\xi)-\bar\partial_z\int_{D\times\sigma_0}f(\xi)\wedge\Omega^0_{p,q-1}(z,\xi)\\&+(-1)^{p+q}\int_{D\times\sigma_0}f(\xi)\wedge P^0_{p,q}(z,\xi),\quad z\in D,\end{aligned}\tag{4}$$

Notice that

$$\int_{S_I\times\sigma_{0I}}f(\xi)\wedge\Omega^k_{p,q-1}(z,\xi,\lambda)=0,\text{ if }|I|>n-p-q.$$

The result follows from the fact that the form $f(\xi)\wedge\Omega^k_{p,p-1}(z,\xi,\lambda)$ is of degree $\geq n+p+q-1$ in ξ, whereas $\dim_R S_I=2n-|I|-1<n+p+q-1$. Similar

$$\int_{S_I\times\sigma_I}f(\xi)\wedge\Omega^k_{p,q}(z,\xi,\lambda)=0,\quad\text{if }|I|>n-p-q,$$

$$\int_{S_I\times\sigma_{0I}}f(\xi)\wedge Q^k_{p,q}(z,\xi,\lambda)=0,\quad\text{if }|I|>n-p-q,$$

$$\int_{S_I\times\sigma_{0I}}\bar\partial_\xi f(\xi)\wedge\Omega^k_{p,q}(z,\xi,\lambda)=0,\text{ if }|I|>n-p-q-1.$$

Formula (2) follows from (3) and (4).

Corollary Under the hypotheses of Theorem 1, if the sections $S_1^*(z,\xi),\cdots,$

$S_k^*(z,\xi)$ are holomorphic in $z\in D$ and if $C(\widetilde{T}(M\times M))=0$, $1\le q\le n$, then, for every continuous (p,q)-form f on $\overline{D}$ such that $\overline{\partial}f=0$ in D,

$$g(z)=(-1)^{p+q}\sum_{|I|\le n-p-q}(-1)^{|I|}\int_{S_I\times\sigma_{0I}}f(\xi)\wedge\Omega_{p,q}^k(z,\xi,\lambda)$$
$$+\int_{D\times\sigma_0}f(\xi)\wedge\Omega_{p,q-1}^k(z,\xi,\lambda) \quad (5)$$

is a continuous solution of $\overline{\partial}g=f$ in D.

Proof If $S_1^*(z,\xi),\cdots,S_k^*(z,\xi)$ are holomorphic in z, then, for every collection $I=(i_1,\cdots,i_l)$ of strictly increasing integers $1\le i_1<\cdots<i_l\le k$, the form $\overline{\Omega}^k(\varphi^v,\hat{S},S_1^*,\cdots,S_k^*,S)(z,\xi,\lambda)$ is of degree zero in z. Therefore, $\Omega_{p,q}^k(z,\xi,\lambda)=0$, if $q>0$. In view of Remark 2) following Theorem 1, the proof is complete.

3. Formula for real non-degenerate strictly pseudoconvex polyhedron

Definition 3 Let M be a Stein manifold of complex dimension n, an open set $D\subset\subset M$ is called a strictly pseudoconvex polyhedron if there is a neighbourhood U_D of $\overline{D}$, finitely many Stein manifolds $M_1,\cdots,M_k$ of complex dimention $\le n$, holomorphic maps $F_j\colon U_{\overline{D}}\to M_j$, $j=1,\cdots,k$, as well as strictly pseudoconvex open sets $D_j\subset\subset M_j$, $j=1,\cdots,k$, such that $D=F_1^{-1}(D_1)\cap\cdots\cap F_k^{-1}(D_k)$.

If $\rho_1,\cdots\rho_k$ are strictly plurisubharmonic C^2-functions in some neighbourhoods $\theta_1,\cdots,\theta_k$ of $\partial D_1,\cdots,\partial D_k$, respectively, such that

$$D_j\cap\theta_j=\{z\in\theta_j:\rho_j(z)\le 0\},\quad j=1,\cdots,k,$$

then $\partial D\subseteq F_1^{-1}(\theta_1)\cap\cdots\cap F_k^{-1}(\theta_k)$ and a point $z\in F_1^{-1}(\theta_1)\cup\cdots\cup F_k^{-1}(\theta_k)$ belongs to D if and only if, for every $1\le j\le k$ with $z\in F_j^{-1}(\theta_j)$, $\rho_j(F_j(z))<0$.

Definition 4 D is called real non-degenerate if these functions F_j and ρ_j can be chosen so that For every subset of indices $1\le i_1<\cdots<i_l\le k$, we have

$$d(\rho_{i_1}\circ F_{i_1})\cap\cdots\cap d(\rho_{i_l}\circ F_{i_l})\ne 0$$

for all $z\in\partial D$ with $\rho_{i_1}(F_{i_1}(z))=\cdots=\rho_{i_l}(F_{i_l}(z))=0$.

Remark 1) Special cases of real non-degenerate strictly pseuroconvex polyhedron are Weil analytic polyhedra as well as intersections of finitely many strictly pseudoconvex open sets.

2) The boundary of a real non-degenerate strictly pseudoconvex polyhedron is piecewise C^1 in the sense of Definition 1.

Let $S(z,\xi),\varphi(z,\xi)$, K be as in Lemma 4.2.4 in [8]. By Theorem 4.8.3 and Lemma 4.8.2 in [8], after shrinking θ_j, we can find numbers $\varepsilon,a>0$ and C^1-functions $\Phi_j(z,\xi)$ defined for $z\in D_j\cup\theta_j$ and $\xi\in\theta_j$ such that the following conditions are satisfied:

1) $\Phi_j(z,\xi)$ is holomorphic in $z\in D_j\cup\theta_j$.

2) $\Phi_j(z,\xi)\neq 0$ for $z\in D_j\cup\theta_j$, $\xi\in\theta_j$ with $\operatorname{dist}(z,\xi)\geq\varepsilon$. (6)

3) $|\Phi_j(z,\xi)|\geq a[\rho_j(\xi)-\rho_j(z)+(\operatorname{dist}(z,\xi))^2]$ (7)

for $z\in D_j\cup\theta_j$, $\xi\in\theta_j$ with $\operatorname{dist}(z,\xi)\leq 2\varepsilon$.

4) $\Phi_j(z,z)=0$ for all $z\in\theta_j$.

Set $\overline{\Psi}_j(z,\xi)=\Phi_j(F_j(z),F_j(\xi))$ for $z\in F_j^{-1}(D_j\cup\theta_j)$, $\xi\in F_j^{-1}(\theta_j)$. Then, by Corollary 4.9.4 in [8], after shrinking θ_j, we can find T^*M-valued C^1-maps $S_j^*(z,\xi)$ defined for $z\in F_j^{-1}(D_j\cup\theta_j)$ and $\xi\in F_j^{-1}(\theta_j)$ such that the following conditions are fulfilled:

a) $S_j^*(z,\xi)\in T_z^*(M)$ for $z\in F_j^{-1}(D_j\cup\theta_j)$, $\xi\in F_j^{-1}(\theta_j)$.

b) $S_j^*(z,\xi)$ is holomorphic in $z\in F_j^{-1}(D_j\cup\theta_j)$.

c) $\varphi(z,\xi)\overline{\Psi}_j(z,\xi)=\langle S_j^*(z,\xi),S(z,\xi)\rangle$ for $z\in F_j^{-1}(D_jU\theta_j)$, $\xi\in F_j^{-1}(\theta_j)$. (8)

For every ordered subset I of $\{1,\cdots,k\}$ with strictly increasing component, set $S_I=\{z\in\partial D:\rho_i(F_i(z))=0,\ i\in I\}$. Since, by (8),

$$\varphi(z,\xi)S_j^*(z,\xi)/\langle S_j^*(z,\xi),S(z,\xi)\rangle=S_j^*(z,\xi)/\Phi_j(F_j(z),F_j(\xi)) \text{ for } z\in D \text{ and } \xi\in S_j,$$

where, by (6) and (7), $\Phi_j(F_j(z),F_j(\xi))\neq 0$, we see that $(S_1^*,\cdots,S_k^*,1)$ is a Leray-Norguet section for (D,S,φ). Therefore, by the proof of Corollary in Section 2, we obtain the following

Theorem 2 Let $D\subset\subset M$ be a real non-degenerate strictly pseudoconvex polyhedron, and let $v\geq 2nK$, $1\leq q\leq n$. Then for every continuous (p,q)-form f on $\overline{D}$ such that $\overline{\partial}f$ is also continuous on $\overline{D}$, we have

$$\begin{aligned}(-1)^{p+q}f(z)=\;&\overline{\partial}_z\Big[\sum_{|I|\leq n-p-q}(-1)^{|I|}\int_{S_I\times\sigma_{0I}}f(\xi)\wedge\Omega^k_{p,q}(z,\xi,\lambda)\\&+\int_{D\times\sigma_0}f(\xi)\wedge\Omega^k_{p,q-1}(z,\xi,\lambda)\Big]\\&-\Big[\sum_{|I|\leq n-p-q-1}(-1)^{|I|}\int_{S_I\times\sigma_{0I}}\overline{\partial}_\xi f(\xi)\wedge\Omega^k_{p,q}(z,\xi,\lambda)\\&+\int_{D\times\sigma_0}\overline{\partial}_\xi f(\xi)\wedge\Omega^k_{p,q}(z,\xi,\lambda)\Big]\\&+(-1)^{p+q+1}\Big[\sum_{|I|\leq n-p-q}(-1)^{|I|}\int_{S_I\times\sigma_{0I}}f(\xi)\wedge Q^k_{p,q}(z,\xi,\lambda)\\&+\int_{D\times\sigma_0}f(\xi)\wedge P^0_{p,q}(z,\xi)\Big],\qquad z\in D,\end{aligned}\tag{9}$$

In particular, if $C(\widetilde{T}(M\times M))=0$, then, for every continuous (p,q)-form f on $\overline{D}$ such that $\overline{\partial}f=0$ in D,

$$\begin{aligned}g=(-1)^{p+q}\Big[&\sum_{|I|\leq n-p-q}(-1)^{|I|}\int_{S_I\times\sigma_{0I}}f(\xi)\wedge\Omega^k_{p,q}(z,\xi,\lambda)\\&+\int_{D\times\sigma_0}f(\xi)\wedge\Omega^k_{p,q-1}(z,\xi,\lambda)\Big].\end{aligned}\tag{10}$$

is a continuous solution of $\overline{\partial}g=f$ in D.

Remark If $p=0$, then formulas (9) and (10) agree with formulas (4.9.6) and (4.9.7) in [8].

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Stein流形上(*p*, *q*)型Koppelman-Leray-Norguet公式

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摘 要

设M是复n维Stein流形；并设开集$D\subset\subset M$具有逐块C^1边界．本文利用陈度量和陈联络，把Stein流形上$(0,q)$形式的Koppelman-Leray-Norguet公式推广到(p,q)形式，并得到D上$\bar{\partial}$-方程的解．最后，还给出了Stein流形上实非退化强拟凸多面体的Koppelman-Leray-Norguet公式及其$\bar{\partial}$-方程的解．