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On the Truth of a Conjecture Concerning the q-Derangement Numbers *

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Let $D_n(q)=[n]!\sum_{k=0}^{n}(-1)^k q^{\binom{k}{2}}/[k]!$ be q-derangement numbers, where $[n]=1+q+q^2+\cdots+q^{n-1}$. W.Y.C.Chen and G.C.Rota have proved that $D_n(q)$ is unimodal for all n and stated the following conjecture on $D_n(q)$: the maximum coefficient appearing in $D_n(q)$ is that of $q^{\lceil n(n-1)/4\rceil}$, where $\lceil x\rceil$ is the usual notation for smallest integer not less than x.

Here is an announcement about the truth of the conjecture. Let us denote

$$D_n(q)=\sum_{i=1}^{\binom{n}{2}}a_i^n q^i,\, N_0(n)=\lceil n(n-1)/4\rceil.$$

Theorem 1 *We have*

$$a^n_{N_0(n)-t+1}\ge a^n_{N_0(n)+t}\ge a^n_{N_0(n)-t},\ \textit{if } n=4k \textit{ or } 4k+1;$$

$$a^n_{N_0(n)+t-1}\ge a^n_{N_0(n)-t}\ge a^n_{N_0(n)+t},\ \textit{if } n=4k+2 \textit{ or } 4k+3,$$

where $k=0,1,2,\cdots,t=1,2,\cdots$.

Paying attention to $D_n(q)=(1+q+q^2+\cdots+q^{n-1})D_{n-1}(q)+(-1)^n q^{\binom{n}{2}}$, we can prove the theorem by mathematical induction. Details will appear elsewhere.

Theorem 2 *The conjecture of Chen and Rota is true.*

In fact, Theorem 1 has not only claimed the truth of the conjecture but also given an elementary proof of the unimodality of $D_n(q)$.

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References

[1] W.Y.C.Chen and G.C.Rota, Discrete, Math., 104(1992), 7-22.
[2] X.D.Zhang, PhD. Thesis, Dalian Univ. of Technology, China, 1994.

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