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Coincidence Points and Common Fixed Points for Compatible Maps of Type (A) on Saks Spaces *

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Abstract In this paper, we introduce the concepts of compatible and compatible maps of type (A) in Saks spaces and prove a coincidence and common fixed point theorem for these maps. Our theorem includes several fixed point theorems for three and four maps.

Keywords compatible and compatible of type (A), conicidence and common fixed points.

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1. Introduction

Goebel[16] proved a remarkable coincidence theorem in 1968. After a wide gap, Okado[34], Singh-Virendra[52], Kulshrestha[28] and Nimpally-Singh and Whitfield[33] have extended Goebel's results to L-space, metric spaces, 2-metric spaces and multi-valued contraction maps on metric spaces, respectively.

In [19] Jungck comtraction principle (JCP) appeared for a pair of continuous and commuting self maps. After Jungck, a spate of research papers appeared using this concept in various ways with several contractive type, see [4]–[14],[18],[22]–[27],[29],[38],[39],[40],[41], [43],[45],[46],[48],[50]–[52]. On the other hand, in [21], Jungck, Murthy and Cho introduced the concept of compatible mappings of type (A) in metric spaces and gave some fixed point theorems for these mappings.

In this paper, we first prove a coincidence point theorem for four self maps of Saks spaces and then we derive a common fixed point theorem for two pairs of compatible mappings of type (A) in Saks spaces which are not necessarily continuous. Our theorem extend the theorems of Gregušš[16], Fisher and Sessa[15], Diviccaro etal[10], Mukherjee etal [29], Jungck[20] and others.

We need the following definitions and Lemmas for our main theorems:

Definition (1.1) *Let X be a linear space. A real-valued function f defined on X is called a B-norm if it satisfies the following conditions.*

(1) $f(x) = 0$ *if and only if* $x = 0$,

(2) $f(x + y) \leq f(x) + f(y)$,

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(3) $f(ax) = |a|f(x)$, *where a is any real number.*

Definition (1.2) *Let X be a linear space. A real valued- function f defined on X is called an F-norm if it satisfies*

(1) and (2) of definition (1.1) and the following:

(I) If the sequence $\{a_n\}$ of real numbers converges to a and then $f(a_n x_n - ax) \to 0$ as $n \to \infty$.

A two-norm space is a linear space X with two norms, a B-norm N_1 and F-norm N_2, and denoted by (X, N_1, N_2). If we let N_1 and N_2 be two norms defined on X and $x_n \in x, N_1(x_n) \to 0$ as $n \to \infty$ implies $N_2(x_n) \to 0$ as $n \to \infty$, then the norm N_1 is called non-weaker than N_2 in X and is denoted by $N_2 \leq N_1 1$. The two norms N_1 and N_2 of X are equivalent if $N_1 \leq N_2$ and $N_2 \leq N_1$.

Definition (1.3) *Let (X, N_1, N_2) be a two-norm space. A sequence $\{x_n\}$ in X is said to be an r-convergent to a point x in X if $\sup_n N_1(x_n) < \infty$ and $\lim_n N_2(x_n - x) = 0$ (denoted by $x_n \to x$).*

Definition (1.4) *Let (X, N_1, N_2) be a two norm space. A sequence $\{x_n\}$ in X is said to be an r-Cauchy if $N_2(x_{p_n} - x_{q_n}) \to 0$ as $p_n, q_n \to \infty$.*

A two norm space (X, N_1, N_2) is said to be r-complete if every r-Cauchy sequence in X is a r-convergent sequence in X.

Definition (1.5) *Let X be a linear set with two norms β-norm N_1 and F-norm N_2 on X, respectively. Let $X_s = \{x \in X : N_1(x) < 1\}$ and $d(x, y) = N_2(x - y)$ for an $x, y \in X_s$. Then d is a metric on X_s and the metric space (X_s, d) is called a Saks set.*

Definition (1.6) *A complete Saks set (X_s, d) is called a Saks space and denoted by (X, N_1, N_2)*

The following lemma due to Orlicz[35] is useful for the proof of our main theorem:

Lemma (1.1) *Let $(X_s, d) = (X, N_1, N_2)$ be a Saks space. Then the following statements are equivalent:*

(I) N_1 is equivalent to N_2 on X.

(II) (X, N_1) is a Banach space and $N_1 \leq N_2$ on X.

(III) (X, N_2) is a Frechet space and $N_2 \leq N_1$ on X.

The general information for Saks spaces may be found in ([1],([35]-[37])).

2. Compatible Mapping of Type (A)

The concept of compatible mapping of type (A) was investigated by Jungck, Murthy and Cho[21] in metric spaces.

On the other hand, Murthy and Sharma[32] Cho and Singh[2],[3] and many authors have studied the aspects of conicidence and common fixed point theorems in the setting of Saks space. They have been motivated by various concepts already known in ordinary metric spaces and have thus introduced analogue of various concepts in the frame work of the Saks spaces. Especially, Cho and Singh[2] and Murty and Sharma[32] introduced the concepts of commuting and weakly uniformly contraction pair of mappings, respectively,

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and they have proved several fixed point theorems by using these concepts.

In this paper, we extend the concepts of weakly uniformly contraction, compatible and compatible pair of type (A) of metric spaces in the setting of Saks spaces and give some relationship between these mappings. Of course, any commuting mappings are weakly commuting but the converse is not true as shown in Example [45]. In turn, any weakly commuting mappings are compatible but the converse is not true [20].

Now, we shall give some definitions and propositions of compatible mappings and compatible mappings of type (A) on Saks spaces.

Throughout this paper, we shall define $\lim_{n\to\infty} = \lim$ as the situation demands and $(X_s, d) = (X, N_1, N_2)$ be a Saks space where N_1 is equivalent to N_2 on X. In short, we shall define X as a Saks space.

Definition (2.1) *Let S and T be mappings of Saks space X into itself. Then S and T are compatible mapping if*

$$\lim N_2(STx_n - TSx_n) = 0$$

whenever $\{x_n\}$ be a sequence in X such that $\lim sx_n = \lim Tx_n = t$ for some $t \in X$.

Definition (2.2) *Let S and T be mappings of a Saks space X onto itself. Then S and T are compatible mappings of type (A) if*

$$\lim N_2(STx_n - TTx_n) = 0 \quad \text{and} \quad \lim N_2(TSx_n - SSx_n) = 0,$$

whenever $\{x_n\}$ be a sequence in X such that $\lim Sx_n = \lim Tx_n = t$ for some t in X.

The proofs of the following propositions follow from the same lines in [21] and so we omit here.

Proposition (2.1) *Let S and T be continuous mappings of a Saks space X into itself. If a pair $\{S, T\}$ is compatible on X, then it is compatible of type (A) on X.*

Proposition (2.2) *Let S and T be mappings from a Saks space X into itself and let a pair $\{S, T\}$ be compatible of type (A) on X. If one of S and T is continuous, then the pair $\{S, T\}$ is compatible on X.*

The following proposition is a direct consequence of Prop. (2.1) and Prop. (2.2):

Proposition (2.3) *Let S and T be as in Proposition (2.1). Then a pair $\{S, T\}$ is compatible on X if and only if it is compatible of type (A) on X.*

Remark 2 In [21], we may find two examples to show the fact that Proposition (2.3) is not true if S and T are not continuous.

Now, we give some properties of compatible mappings of type (A) for our main theorems.

Proposition (2.4) *Let S and T be compatible of type (A) from a Saks space X into itself. If $S(t) = T(t)$ for some $t \in X$, then $ST(t) = TT(t) = TS(t) = SS(t)$.*

Proposition (2.5) *Let S and T be compatible mappings of type (A) from a Saks space X into itself. Suppose that $\lim Sx_n = \lim Tx_n = t$ for some $t \in X$. Then we have the following:*

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(1) $\lim TSx_n = St$ *if* S *is continuous at* t.
(2) $STt = TSt$ *and* $St = Tt$ *if* S *and* T *are continuous at* t.

3. Coincidence Points

Let A, B, S and T be mappings from a Saks space X into itself such that

$$A(x) \cup B(x) \subset S(x) \cap T(x), \tag{3.1}$$

$$\begin{aligned} &N_2(Ax - By) \le \alpha N_2(Sx - Ty) + \beta \max\{N_2(Ax - Sx), \\ &N_2(By - Ty), 1/2[N_2(Ax - Ty) + N_2(By - Sx)]\} \end{aligned} \tag{3.2}$$

for all $x, y \in X$, where $\alpha, \beta > 0$ and $\alpha + \beta < 1$. Then by (3.1), since $A(X) \subset T(X)$, for any arbitrary point $x_0 \in X$, there exists a point $x_1 \in X$, such that $Ax_0 = Tx_1$. Since $B(X) \subset S(X)$, for this point x_1. We can choose a point $x_2 \in X$ such that $Bx_1 = Sx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that.

$$y_{2n} = Sx_{2n} = Bx_{2n-1} \text{ and } y_{2n+1} = Tx_{2n+1} = Ax_{2n} \text{ for } n = 0, 1, 2, \cdots \tag{3.3}$$

Then we have the following lemma for our main theorem:

Lemma (3.1) *Let* A, B, S *and* T *be mappings from a Saks space* X *into itself satisfying the conditions (3.1) and (3.2). Then the sequence* $\{y_n\}$ *defined by (3.3) is a Cauchy sequence in* X.

Proof By (3.2), we have

$$\begin{aligned} &N_2(y_{2n+1} - y_{2n}) = N_2(Ax_{2n} - Bx_{2n-1}) \le \alpha N_2(Sx_{2n} - Tx_{2n-1}) \\ &+\beta \max\{N_2(Sx_{2n} - Ax_{2n})N_2(Tx_{2n-1} - Bx_{2n-1}), 1/2N_2(Ax_{2n} - Tx_{2n-1}) \\ &+N_2(Bx_{2n-1} - Sx_{2n})\} = \alpha N_2(y_{2n} - y_{2n-1}) \\ &+\beta \max\{N_2(y_{2n} - y_{2n+1})N_2(y_{2n-1} - y_{2n}), 1/2N_2(y_{2n+1} - y_{2n-1}) \\ &+N_2(y_{2n} - y_{2n})\}. \end{aligned} \tag{3.4}$$

If $N_2(y_{2n+1} - y_{2n}) > N_2(y_{2n} - y_{2n-1})$ in (3.4), then we have

$$\begin{aligned} N_2(y_{2n+1} - y_{2n}) &\le (\alpha + \beta)N_2(y_{2n+1} - y_{2n}) \\ &< N_2(y_{2n+1} - y_{2n}), \end{aligned}$$

which is a contradiction since $\alpha + \beta < 1$ and so

$$N_2(y_{2n+1} - y_{2n}) \le (\alpha + \beta)N_2(y_{2n} - y_{2n-1}).$$

Similary, we have

$$N_2(y_{2n} - y_{2n-1}) \le (\alpha + \beta)N_2(y_{2n-1} - y_{2n-2}).$$

Therefore, we have

$$N_2(y_{n+1} - y_n) \le (\alpha + \beta)N_2(y_n - y_{n-1}) \le \cdots \le (\alpha + \beta)^n N_2(y_1 - y_0). \tag{3.5}$$

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If $m \geq n$, then the repeated use of (3.5) yields

$$\begin{aligned} N_2(y_m - y_n) &\leq N_2(y_m - y_{m-1}) + N_2(y_{m-1} - y_{m-2}) + \cdots + N_2(y_{n+1} - y_n) \\ &\leq \{(\alpha+\beta)^{m-2} + (\alpha+\beta)^{m-3} + \cdots + (\alpha+\beta)^{n-1})\} N_2(y_1 - y_0) \\ &= \frac{(\alpha+\beta)^{n-1}}{1-(\alpha+\beta)} N_2(y_1 - y_0) \end{aligned}$$

Therefore, since $0 < \alpha + \beta < 1$, the sequence $\{y_n\}$ is a Cauchy sequence in $S(X) \cap T(X)$ with respect to N_1 since N_1 is equivalent to N_2 on X.

Theorem (3.1) *Let $(X_s, d) = (X, N_1, N_2)$ be a Saks space in which N_1 is equivalent to N_2 on X. Let A, B, S and T be self mappings of X satisfying the conditions (3.1), (3.2) and the following:*

$$S(X) \cap T(X) \text{ is a closed subspace of} X \text{ with respect to } N_1 \tag{3.6}$$

Then

(i) A and S have a coincidence point in X, and
(ii) B and T have a coincidence point in X.

Proof From Lemma (3.1) the sequence $\{Y_n\}$ is a Cauchy sequence in $S(X) \cap T(X)$ with respect to N_1. Since N_1 is equivalent to N_2 on X. So, by Lemma (1.1), (X, N_1) is Banach space and hence converges to a point w in $S(X) \cap T(X)$. The subsequences $\{Y_{2n}\}$ and $\{Y_{2n+1}\}$ of $\{Y_n\}$ are also Cauchy sequences and converge to z. Thus there exist two points u and v in X such that $Su = w$, and $Tv = w$ respectively.

Putting $x = u$ and $y = x_{2n+1}$ in (3.2). Then, we have

$$\begin{aligned} &N_2(Au - Bx_{2n+1}) \leq \alpha N_2(Su - Tx_{2n+1}) \\ &+\beta \max\{N_2(Su - Au), N_2(Tx_{2n+1} - Bx_{2n+1}), 1/2[N_2(Su - Bx_{2n+1}) \\ &+N_2(Tx_{2n+1} - Au)]\}. \end{aligned} \tag{3.7}$$

Taking $n \to \infty$ in (3.7), $N_2(Au - w) \leq \beta N_2(Au - w)$, a contradiction. Hence $Au = w = Su$. Similarly, we have $Bv = w = Tv$.

As an immediate consequence of Theorem (3.1) we have the following corollary:

Corollary (3.2) *Let $A = B, S$ and T be self maps of a Saks space X in which N_1 is equivalent to N_2 on X satisfying the condition (3.1),(3.2) and (3.6):*
Then

(i) A and S have a coincidence point,
(ii) A and T have a coincidence point.

Indeed A, S and T have a concidence point if and only if A is One-to- One.

4. Common Fixed Points

Theorem (4.1) *Let $(X_s, d) = (X, N_1, N_2)$ be a Saks space in which N_1 is equivalent to N_2 on X. Let A, B, S and T be self mappings of a Saks space X satisfying the condition (3.1),(3.2),(3.6) and (4.1):*

$$\{A, S\} \text{ and } \{B, T\} \text{ are compatible pair of type (A) on } X \tag{4.1}$$

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Then A, B, S had T have a unique common fixed point on X.

Proof In Theorem (3.1), we have shown that $Su = Au = w$ and $Tv = Bv = w$. Now suppose that A and S are compatible maps of type (A). Hence by Proposition (2.4), we have

$$ASu = SSu = AAu \Rightarrow Aw = Sw.$$

Now suppose $Aw \neq w$,

$$\begin{aligned}&N_2(Aw - Bx_{2n+1} \leq \alpha N_2(Sw - Tx_{2n+1}\\&+\beta \max\{N_2(Sw - Aw), N_2(Tx_{2n+1} - Bx_{2n+1}), 1/2[N_2(Sw - Bx_{2n+1}\\&+N_2(Tx_{2n+1} - Aw)]\},\end{aligned} \tag{4.2}$$

Taking $n \to \infty$ in (4.2), we have

$$N_2(Aw - w) \leq (\alpha + \beta) N_2(Aw - w) < N_2(Aw - w) \quad (\text{ since } \alpha + \beta < 1)$$

which implies that $Aw = w$. Hence, w is a common fixed point of A and S. If we argue for B and T by assuming that they are as a compatible maps of type (A), then we must have $Tw = Bw = w$.
Now suppose that w is common fixed point of A, B, S and T. If $Aw = Sw = w$ and $Bz = Tz = z$, then from condition (3.2), we have

$$\begin{aligned}&N_2(w - z) = N_2(Aw - Bz) \leq \alpha N_2(Sw - Tz)\\&\quad +\beta \max\{N_2(Sw - Aw), N_2(Tz - Bz) + 1/2(Sw - Bz) + N_2(Tz - Aw)]\}\\&= (\alpha + \beta) N_2(w - z) < N_2(w - z) \quad (\text{ since } \alpha + \beta < 1),\end{aligned}$$

which is a contradiction. Hence, w is a common fixed point of A, B, S and T. Uniqueness of w follows easily from (3.2).

Remark 3 (1) Compatibility of type (A) for B and T can be replaced from the condition (4.11) by the following condition:

$$N_2(x - Tx) \leq N_2(x - Sx)$$

for some x in X.

Since it is possible to replace the condition of weak commutativity or compatibility by compatibility of type (A), our theorem is an extension, generalization and improvement of several theorems already known in ordinary metric space.

(2) Our theorem includes Cho and Singh[2] for $S = T$ and the condition (3.2) is replaced by the following condition.

$$N_2(Ax - By) \leq f(N_2(Sx - Ax), N_2(Sy - By), N_2(Sx - Sy), N_2(Sy - Ax), N_2(Sx - By) \tag{4.3}$$

for every x, y in X where $f : (R^+)^5 \to R^+$, which is nondecreasing in each coordinate variable and $f(t, t, t, a_1 t, a_2 t) < t$ for $t > 0$ where $a_1 \in \{0, 1, 2\}$ with $a_1 + a_2 = 2$.

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(3) Our theorem includes Kang, Cho and Jungck[31] if X is a metric space and condition (3.2) is replaced by

$$d(Ax,By) \leq \phi(d(Sx,Ty),d(Sx,Ax),d(Ty,By),d(Sx,By),d(Ty,Ax)) \tag{4.4}$$

for all x,y in X where $\phi:[0,\infty)^5 \to [0,\infty)$ is a function such that

(i) ϕ is non-decreasing and upper-semi continuous in each coordinate variable,

(ii) for each $t>0, \Gamma(t)=\max\{\phi(0,0,t,t,t),\phi(t,t,t,2t,0),\phi(t,t,t,0,2t)\}<t$.

(4) Our theorem also includes Murthy and Sharma[32] if condition (3.2) in Theorem (4.1) is replaced by the following condition.

$$\begin{aligned} &N_2^2(Ax,By)\\ &\leq \phi(\max\{N_2^2(Sx-Ty),N_2(Sx-Ax)N_2(Ty-By),N_2(Sx-By)N_2(Ty-Ax),\\ &\quad N_2(Sx-Ax)N_2(Ty-Ax),N_2(Sx-By)N_2(Ty-By)\}) \end{aligned} \tag{4.5}$$

for all x,y in X where $\phi:R^+ \to R^+$ satisfying

(i) ϕ is non decreasing

(ii) $\phi(t)<t$ for each $t>0$.

As an immediate consequence of theorem (4.1) we have the following corollary:

Corollary (4.2) *Let $(X_s,d)=(X,N_1,N_2)$ be a Saks space in which N_1 is equivalent to N_2 on X. Let A,B,S and T be self maps of X satisfying (3.1),(3.6),(4.1) and (4.6):*

$$(N_2(Ax-By))^p \leq \alpha(N_2(Sx-Ty))^p \tag{4.6}$$

for all x,y in X, where $\alpha\in(0,1), p\geq 1$.
Then A,B,S and T have a unique common fixed point in X.

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