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The Proof of a Theorem in DC Problem *

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In this short note the demonstration of a theorem in DC problem, [1], is given. The related notations can be referred to [1].

Theorem *Suppose* $N^+(x;Q) \neq \{0\}$ *and* $intCG(x;Q) \neq \emptyset$. *Then* $u \in int[CG(x;Q)+x] \Leftrightarrow \langle u-x, N^+(x;Q)\backslash\{0\}\rangle > 0$.

Proof $\Rightarrow$ (only if). Let $M(x;u-x,z-x)$ denote the linear manifold with dimension two, determined by $u-x$ and $z-x$, through x. $\forall u \in \mathrm{int}[CG(x;Q)+x]$ $\exists \lambda > 1 : u_G = \lambda_u + (1-\lambda)z \in M(x;u-x,z-x) \cap [CG(x;Q)+x]$. Clearly $u_G \neq u$ because $\lambda > 1$. Because $z-x \in N^+(x;Q)$, one has $M(x;u-x,z-x) \cap [N^+(x;Q)+x] \neq \emptyset$. Therefore, $\exists \mu \geq 1 : z_N = \mu z + (1-\mu)u \in M(x;u-x,z-x) \cap [N^+(x;Q)+x]$. The points u, u_G, z, z_N are in the same one-dimensional linear manifold. Since $\lambda > 1$, and $\mu \geq 1, u$ and z are included in the interval $\{w | w = \beta z_N + (1-\beta)u_G, \beta \in (0,1]\}$. Since $u_G - x \in CG(x;Q)$ and $z_N - x \in N^+(x;Q)$, we have $\langle u_G - x, z_N - x\rangle \geq 0$. As a result of $\lambda > 1, u = \beta z_N + (1-\beta)u_G$ for some $\beta \in (0,1)$. Thus $\xi(u)^T\xi(z) > \xi(u_G)^T\xi(z_N) \geq 0$, where $\xi(\eta) = (\eta - x)/\|\eta - x\|$. Hence, $\langle u-x, z-x\rangle > 0$, i.e., $\langle u-x, N^+(x;Q)\backslash\{0\}\rangle > 0$. That is the proof of necessity.

$\Leftarrow$(if). Prove by contradiction. Suppose $u \notin \mathrm{int}(CG(x;Q)+x)$. Then there exists a sequence $\{u^i\}_1^\infty$ convergent to u, but not included in $CG(x;Q)+x$. The sequence satisfies $\langle u^i - x, N^+(x;Q)\rangle \not> 0$ for any $i = 1, \cdots$. Correspondingly, there exists a sequence $\{z^i\} \subset N^+(x;Q)+x$ such that $\langle u^i - x, z^i - x\rangle \leq 0$. For the sake of simplicity, assume that $\{z^i\}_1^\infty$ is bounded and converges to $z \in N^+(x;Q)+x$ different from x. As a result of taking the limit of the last inequality, one have $\langle u-x, z-x\rangle \leq 0$. The above inequality contradicts $\langle u-x, N^+(x;Q)\rangle > 0$. The proof of sufficiency is completed. □

References

[1] Z.Q.Xia, J.J.Strodiot and V.H.Nguyen, *The C_M-embedded problem and optimality conditions*, Acta Mathematicae Applicatae Sinica, **6:1**(1990), 22–34.

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