Journal of Mathematical Research & Exposition
Vol.15, No.3, 410, Aug. 1995

A Homotopy Interior Point Method for Nonlinear Complementarity Problems *

Wang Yu
(Inst. of Eng. Mech., Dalian Univ. of Tech., Dalian 116024)

Kojima et al proposed a series of algorithms and results for nonlinear complementarity problems(NCP) ([1]–[3] and theirs references). In this note a homotopy interior point algorithm with global linear rate of convergence for uniform P-NCP which does not satisfy the scale Lipschitz condition [3] is given.

Under the symbols and definitions of [1], and NCP is equivalent to the system of equations $F(x,y) = (Xy, y - f(x)) = 0, (x,y) \geq 0$, and for $(a,b) \in R^n_{++} \times R^n$, the system of homotopy equations $H(x,y,t) = F(x,y) - t(a,b) = 0, (x,y) \in R^{2n}_{+}$ has a unique solution $(x(t), y(t))$ for every $t \in R_+$, and the set $C = \{x(t), y(t), t; t \in R_+\}$ forms a continus curve (path of homotopy). Let $b = 0$ and suppose that we know in advance an initial point (x^0, y^0) on path such that $(x^0, y^0) \in S_{++} = \{(x,y) \in R^{2n}_{++}; y = f(x)\}$. The following is a numerical method for tracing the path of homotopy from $(x^0, y^0, t_0 = 1)$ to $(x^*, y^*, t = 0)$, which is the solution of $F(x,y) = 0$.

Algorithm Step 1: let $(x^0, y^0) \in S_{++}, t_0 = 1, a = X^0 y^0, 0 < \delta < \sqrt{n}$ and $\varepsilon > 0$ be given, $k = 0$; Step 2: if $(x^k)^T y^k \leq \varepsilon$ stop; Step 3: let $t_{k+1} = (1 - \delta/\sqrt{n}) t_k, \Delta x^k = [Y^k + X^k f'(x^k)]^{-1}(X^k y^k - t_{k+1} a), x^{k+1} = x^k - \Delta x^k, y^{k+1} = f(x^{k+1}), k = k+1$, goto 2.

Lemma *There exist constant β with $0 < \beta \leq 1$ and positive diagonal matrix $D = diag(d_1, \cdots, d_n)$ such that $\|D\| = 1, \min d_i \geq \beta$ and $u^T D f'(x) u \geq \alpha \|u\|^2, \forall u \in R^n$.*

Let $Q = \max a_i, q = \min a_i$ and θ is a given constant with $0 \leq \theta < 1$ depending on paremeters above. By the results of [4], we have

Theorem *All points (x^k, y^k) and t_k generated by Algorithm satisfy: (a) For all $k = 1, 2, \cdots, (x^k, y^k) \in S_{++}$ and $\|X^k y^k - t_k a\| \leq \theta q t_k$; (b) For all $k = 1, 2, \cdots, (x^k)^T y^k \leq (1+\theta) n Q t_k$, where $t_k = (1 - \delta/\sqrt{n})^k$.*

It is easy to see $t_k \to t^* = 0$ as $k \to \infty$ with linear rate, hence $(x^k)^T y^k \to 0$ and $(x^k, y^k) \to (x^*, y^*)$ with linear rate.

References

[1] M.Kojima, S.Mizuno and T.Noma, Math. Prog., **43**(1989), 107–113.
[2] M.Kojima, N.Megiddo and T.Noma, Math. of Oper. Res., **16**(1991), 754–774.
[3] M.Kojima, T.Noma and M.Sato, Res. Reports B251, Tokyo Tns. of Tech, 1992.
[4] Y.Wang, G.Feng and T.Liu, Numer. Math. A J. of Chinese Univ., **1**(1992), 1–8.

*Received Sep.21, 1993, Project supported by the National Natural Science Foundation of China.