Journal of Mathematical Research & Exposition
Vol.16, No.3, 338, Aug. 1996

Note on a Paper of Deutsch *

Pang Mingxian
(Jilin Teacher's College, Jilin 132011)

Let $A = (a_{ij})_{n\times n}$ be a nonnegative irreducible matrix and have the following partition form:

$$A = \begin{bmatrix} A_{11} & \cdots & A_{1m} \\ \cdots & \cdots & \cdots \\ A_{m1} & \cdots & A_{mm} \end{bmatrix} = (A_{ij})^{m\times m}, \tag{1}$$

where $A_{ij} \in \mathbf{R}^{n_i\times n_j}, 1 \le i,j \le m, \sum_{i=1}^m n_i = n$. If $n = mk$, i.e., $A_{ij} \in \mathbf{R}^{k\times k}, 1 \le i,j \le m$, we denote $R_i(A) = \sum_{j=1}^m A_{ij}, 1 \le i \le m$, and

$$(\bigwedge_{i=1}^m R_i(A))_{st} = \min_{1\le i\le m}(R_i(A))_{st}, \quad (\bigvee_{i=1}^m R_i(A))_{st} = \max_{1\le i\le m}(R_i(A))_{st}, 1 \le s,t \le k,$$

where $B = (b_{ij})_{n\times n}, b_{st} = (B)_{st}$. In general, Let $A = (a_{ij})_{n\times n}$ be nonnegative irreducible and have the partition form (1), we denote $k = \max_{1\le i\le m} n_i$, for each A_{ij} denote $\tilde{A}_{ij} = \begin{pmatrix} A_{ij} & 0 \\ 0 & 0 \end{pmatrix} \in \mathbf{R}^{k\times k}, 1 \le i,j \le m, \tilde{A} = (\tilde{A}_{ij})^{m\times m}$.

Theorem *Let $A = (a_{ij})_{n\times n}$ be nonnegative irreducible and have the partition form (1). Then*

$$\rho(\bigwedge_{i=1}^m R_i(\tilde{A})) \le \rho(A) \le \rho(\bigvee_{i=1}^m R_i(\tilde{A})). \tag{2}$$

where $P(A)$ denotes the spectral radius of matrix A.

Corollary *Let $A = (a_{ij})_{n\times n}$ be nonnegative irreducible and have the partition form (1). For each A_{ij} denote $\hat{A} = \begin{pmatrix} 0 & 0 \\ 0 & A_{ij} \end{pmatrix} \in \mathbf{R}^{k\times k}, 1 \le i,j \le m, \hat{A} = (\hat{A}_{ij})_{m\times m}$. Then*

$$\rho(\bigwedge_{i=1}^m R_i(\hat{A})) \le \rho(A) \le \rho(\bigvee_{i=1}^m R_i(\hat{A})). \tag{3}$$

The above results generalize corresponding results of [1].

References

[1] E.Deutsch, Pacific. J. Math., 92:2(1981), 49–56.

*Received Feb.17, 1993.