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The Existence and Uniqueness of the Solution for Neutral Stochastic Functional Differential Equations with Infinite Delay

WANG Lin, HU Shi Geng
(School of Mathematics and Statistics, Huazhong University of Science and Technology,
Hubei 430074, China)
(E-mail: wanglinlin2009@yahoo.com.cn)

Abstract The main aim of this paper is to establish the existence-and-uniqueness theorem for neutral stochastic functional differential equations with infinite delay at phase space $BC((-\infty,0];R^n)$. An example is given for illustration.

Keywords neutral stochastic functional differential equations; infinite delay; existence; uniqueness; Burkholder-Davis-Gundy inequality; Borel-Cantelli lemma.

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1. Introduction and preliminaries

The existence, convergence, and stability of solutions for ordinary differential equations with infinite delay have been studied by many authors[1,4,5,8,9]. Recently, the existence and uniqueness theorem of solutions for stochastic functional differential equations with infinite delay has been established at phase space $BC((-\infty,0];R^n)$ by Wei et al.[2]. The stability of solutions for neutral stochastic functional differential equations with infinite delay has been derived by Luo[3] by using fixed theorem. The conditions of the existence and uniqueness of solutions for neutral stochastic functional differential equations with finite delay have been given by Mao[6,7]. However, the existence and uniqueness of solutions for neutral stochastic functional differential equations with infinite delay has not been studied until now. This paper is devoted to build the existence-and-uniqueness theorem of solutions for neutral stochastic functional differential equations with infinite delay (short for ISFDEs) at phase space $BC((-\infty,0];R^n)$. At last, we give an example to verify the conditions of Theorem 2.1.

Throughout this paper, unless otherwise specified, we let $\{\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},P\}$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all P-null sets). Let $W(t)=(W_1(t),W_2(t),\ldots,W_m(t))^{\mathrm{T}}$ be an m-dimensional Brownian motion defined on the probability space. Let $|\cdot|$ denote the Euclidean

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norm in R^n. If A is a vector or matrix, its transpose is denoted by A^{T}. If A is a matrix, its trace norm is denoted by $|A| = \sqrt{\mathrm{trace}(A^{\mathrm{T}}A)}$ while its operator norm is denoted by $\|A\| = \sup\{|Ax| : |x| = 1\}$. Let $BC((-\infty, 0]; R^n)$ denote the family of all bounded continuous R^n-valued functions φ on $(-\infty, 0]$ with the norm $\|\varphi\| = \sup_{-\infty<\theta\leq 0} |\varphi(\theta)|$. Moreover, denote by $L^2([a, b]; R^n)$ the family of Borel measurable functions $h : [a, b] \to R^n$ such that $\int_a^b |h(t)|^2 \mathrm{d}t < \infty$. Also, denote by $\mathcal{L}^2([a, b]; R^n)$ the family of R^n-valued $\mathscr{F}_t$-adapted processes $\{f(t)\}_{a\leq t\leq b}$ such that $\int_a^b |f(t)|^2 \mathrm{d}t < \infty$ a.s. Furthermore, denote by $\mathscr{M}^2((-\infty, T]; R^n)$ the family of processes $\{\phi(t)\}_{-\infty<t\leq T}$ such that $E[\sup_{-\infty<\theta\leq 0} |\varphi(\theta)|^2] + E\int_0^T |\phi(t)|^2 \mathrm{d}t < \infty$ a.s. In this paper, $a \vee b$ presents the maximum of a and b, while $a \wedge b$ shows the minimum of a and b.

In this paper, we consider an n-dimensional stochastic functional differential equation of neutral type

$$\mathrm{d}[x(t) - D(x_t)] = f(x_t, t)\mathrm{d}t + g(x_t, t)\mathrm{d}W(t), \quad \text{on} \quad 0 \leq t \leq T, \tag{1.1}$$

where $x_t = x(t + \theta) : -\infty < \theta \leq 0$ can be regarded as a $BC((-\infty, 0]; R^n)$-valued stochastic process, where $f : BC((-\infty, 0]; R^n) \times [0, T] \to R^n$, $g : BC((-\infty, 0]; R^n) \times [0, T] \to R^{n\times m}$, and $D : BC((-\infty, 0]; R^n) \to R^n$ are all Borel measurable. By the definition of Itô's stochastic differential equation (1.1) means that for every $0 \leq t \leq T$,

$$x(t) - D(x_t) = x(0) - D(x_0) + \int_0^t f(x_s, s)\mathrm{d}s + \int_0^t g(x_s, s)\mathrm{d}W(s). \tag{1.2}$$

For the initial-value problem of this equation, we must specify the initial data on the entire interval $(-\infty, 0]$, and hence we impose the initial condition:

$$x_0 = \xi = \{\xi(\theta) : -\infty < t \leq 0\} \in \mathscr{M}^2((-\infty, 0]; R^n), \tag{1.3}$$

that is, ξ is an $\mathscr{F}_0$-measurable $BC((-\infty, 0]; R^n)$-valued random variable such that $E\|\xi\|^2 < \infty$. The initial-value problem for Eq.(1.1) is to find the solution of Eq.(1.1) satisfying the initial data (1.3). To be more precise, we give the definition of the solution.

Definition 1.1 *An R^n-valued stochastic process $x(t)$ on $-\infty < t \leq T$ is called a solution to Eq.(1.1) with the initial data (1.3) if it has the following properties:*

(i) It is continuous and $\{x_t\}_{0\leq t\leq T}$ is $\mathscr{F}_t$-adapted;

(ii) $f(x_t, t) \in \mathcal{L}^1([0, T]; R^n)$ and $g(x_t, t) \in \mathcal{L}^2([0, T]; R^{n\times m})$;

(iii) $x_0 = \xi$ and (1.2) holds for every $0 \leq t \leq T$.

A solution $x(t)$ is said to be unique if any other solution $\bar{x}(t)$ is indistinguishable from it, that is

$$P\{x(t) = \bar{x}(t), \quad \text{for all} \quad -\infty < t \leq T\} = 1.$$

Let us establish the theory of the existence and uniqueness of the solution. Obviously, the Lipschitz condition as well as the linear growth condition on the functionals f and g are required, for Eq.(1.1) reduces to the stochastic functional differential equation if $D(\cdot) \equiv 0$. The question is: What condition should be imposed on the functional D? It turns out that D should be uniformly Lipschitz continuous with the Lipschitz coefficient less than 1.

2. The existence-and-uniqueness theorem

Theorem 2.1 *Assume that there exist two positive constants $\bar{K}$ and K such that for all $\phi, \varphi \in BC((-\infty, 0]; R^n)$ and $t \in [0, T]$,*

$$|f(\phi, t) - f(\varphi, t)|^2 \vee |g(\phi, t) - g(\varphi, t)|^2 \le \bar{K}\|\phi - \varphi\|^2; \tag{2.1}$$

and for all $(\phi, t) \in BC((-\infty, 0]; R^n) \times [0, T]$,

$$\|f(\phi, t)\|^2 \vee \|g(\phi, t)\|^2 \le K(1 + \|\phi\|^2). \tag{2.2}$$

Assume also that there is a $\kappa \in (0, 1)$ such that for all $\phi, \varphi \in BC((-\infty, 0]; R^n)$,

$$|D(\phi) - D(\varphi)| \le \kappa\|\phi - \varphi\|. \tag{2.3}$$

Then there exists a unique solution $x(t)$ to Eq.(1.1) with the initial data (1.3). Moreover, the solution belongs to $\mathscr{M}^2((-\infty, T]; R^n)$.

In order to prove this theorem, let us present two useful lemmas.

Lemma 2.2 *For any $a, b \ge 0$ and $0 < \alpha < 1$, we have*

$$(a + b)^2 \le \frac{a^2}{\alpha} + \frac{b^2}{1 - \alpha}.$$

Proof Note that for any $\varepsilon > 0$,

$$(a + b)^2 = a^2 + b^2 + 2ab \le (1 + \varepsilon)a^2 + (1 + \varepsilon^{-1})b^2.$$

Let $\varepsilon = (1 - \alpha)/\alpha$. We obtain the required inequality. □

Lemma 2.3 *Let (2.2) and (2.3) hold. Let $x(t)$ be a solution to Eq.(1.1) with the initial data (1.3). Then*

$$E\Big(\sup_{-\infty<t\le T} |x(t)|^2\Big) \le \Big[1 + \frac{4 + \kappa\sqrt{\kappa}}{(1 - \kappa)(1 - \sqrt{\kappa})E\|\xi\|^2}\Big] \times \exp\Big[\frac{3KT(T + 4)}{(1 - \kappa)(1 - \sqrt{\kappa})}\Big]. \tag{2.4}$$

In particular, $x(t)$ belongs to $\mathscr{M}^2((-\infty, T]; R^n)$.

Proof For every integer $n \ge 1$, define the stopping time

$$\tau_n = T \wedge \inf\{t \in [0, T] : \|x_t\| \ge n\}.$$

Clearly, $\tau_n \uparrow T$, a.s,. Set $x^n(t) = x(t \wedge \tau_n)$ for $t \in (-\infty, T]$. Then, for $0 \le t \le T$,

$$x^n(t) = D(x^n_t) - D(\xi) + J^n(t),$$

where

$$J^n(t) = \xi(0) + \int_0^t f(x^n_s, s)I_{[0,\tau_n]}(s)\mathrm{d}s + \int_0^t g(x^n_s, s)I_{[0,\tau_n]}(s)\mathrm{d}B(s).$$

Applying Lemma 2.2 twice, one derives that

$$\begin{aligned}|x^n(t)|^2 &\le \frac{1}{\kappa}|D(x^n_t) - D(\xi)|^2 + \frac{1}{1 - \kappa}|J^n(t)|^2 \\ &\le \kappa\|x^n_t - \xi\|^2 + \frac{1}{1 - \kappa}|J^n(t)|^2\end{aligned}$$

$$\leq \sqrt{\kappa}\|x_t^n\|^2 + \frac{\kappa}{1-\sqrt{\kappa}}\|\xi\|^2 + \frac{1}{1-\kappa}|J^n(t)|^2,$$

where the condition (2.3) has also been used. Hence

$$E\Big(\sup_{0\leq s\leq t}|x^n(s)|^2\Big) \leq \sqrt{\kappa}E\Big(\sup_{-\infty<s\leq t}|x^n(s)|^2\Big) + \frac{\kappa}{1-\sqrt{\kappa}}E\|\xi\|^2 + \frac{1}{1-\kappa}E\Big(\sup_{0\leq s\leq t}|J^n(s)|^2\Big).$$

Noting that $\sup_{-\infty<s\leq t}|x^n(s)|^2 \leq \|\xi\|^2 + \sup_{0\leq s\leq t}|x^n(s)|^2$, one sees that

$$E\Big(\sup_{-\infty<s\leq t}|x^n(s)|^2\Big) \leq E\Big(\sup_{-\infty<s\leq t}|x^n(s)|^2\Big) + \frac{1+\kappa-\sqrt{\kappa}}{1-\sqrt{\kappa}}E\|\xi\|^2 + \frac{1}{1-\kappa}E\Big(\sup_{0\leq s\leq t}|J^n(s)|^2\Big).$$

Consequently

$$E\Big(\sup_{-\infty<s\leq t}|x^n(s)|^2\Big) \leq \frac{1+\kappa-\sqrt{\kappa}}{(1-\sqrt{\kappa})^2}E\|\xi\|^2 + \frac{1}{(1-\kappa)(1-\sqrt{\kappa})}E\Big(\sup_{0\leq s\leq t}|J^n(s)|^2\Big). \tag{2.5}$$

On the other hand, by Hölder's inequality, Doob's martingale inequality and the linear growth condition (2.2), one can show that

$$E\Big(\sup_{0\leq s\leq t}|J^n(s)|^2\Big) \leq 3E\|\xi\|^2 + 3K(T+4)\int_0^t (1+E\|x_s^n\|^2)\mathrm{d}s.$$

Substituting this into (2.5) yields that

$$E\Big(\sup_{-\infty<s\leq t}|x^n(s)|^2\Big) \leq \frac{4+\kappa\sqrt{\kappa}}{(1-\kappa)(1-\sqrt{\kappa})}E\|\xi\|^2 + \frac{3K(T+4)}{(1-\kappa)(1-\sqrt{\kappa})}\int_0^t (1+E\|x_s^n\|^2)\mathrm{d}s.$$

Therefore

$$1+E\Big(\sup_{-\infty<s\leq t}|x^n(s)|^2\Big) \leq 1+\frac{4+\kappa\sqrt{\kappa}}{(1-\kappa)(1-\sqrt{\kappa})}E\|\xi\|^2 + \frac{3K(T+4)}{(1-\kappa)(1-\sqrt{\kappa})}\int_0^t \Big[1+E\Big(\sup_{-\infty<r\leq s}|x^n(r)|^2\Big)\Big]\mathrm{d}s.$$

Now the Gronwall inequality yields that

$$1+E\Big(\sup_{-\infty<t\leq T}|x^n(t)|^2\Big) \leq \Big(1+\frac{4+\kappa\sqrt{\kappa}}{(1-\kappa)(1-\sqrt{\kappa})}E\|\xi\|^2\Big)\exp\Big[\frac{3K(T+4)}{(1-\kappa)(1-\sqrt{\kappa})}\Big].$$

Finally the required inequality (2.4) follows by letting $n\to\infty$. The proof is completed. □

Proof of Theorem 2.1 Existence. We divide the whole proof of the existence into two steps:

Step 1. We impose an additional condition: T is sufficiently small so that

$$\delta := \kappa + \frac{2\bar{K}T(T+4)}{1-\kappa} < 1. \tag{2.6}$$

Define $x_0^0 = \xi$ and $x^0(t) = \xi(0)$ for $0 \leq t \leq T$. For each $n = 1, 2, \ldots$, set $x_0^n = \xi$ and define, by the Picard iterations,

$$x^n(t) - D(x_t^{n-1}) = \xi(0) - D(\xi) + \int_0^t f(x_s^{n-1}, s)\mathrm{d}s + \int_0^t g(x_s^{n-1}, s)\mathrm{d}W(s) \tag{2.7}$$

for $t \in [0, T]$. It is not difficult to show that $x^n(\cdot) \in \mathscr{M}^2((-\infty, T]; R^n)$. Note that for $0 \le t \le T$,

$$x^1(t) - x^0(t) = x^1(t) - \xi(0) = D(x_t^0) - D(\xi) + \int_0^t f(x_s^0, s)\mathrm{d}s + \int_0^t g(x_s^0, s)\mathrm{d}W(s).$$

By Lemma 2.2 and the condition (2.3), one easily derives that

$$|x^1(t) - x^0(t)|^2 \le \kappa \|x_t^0 - \xi\|^2 + \frac{2}{1-\kappa}\Big[\Big|\int_0^t f(x_s^0, s)\mathrm{d}s\Big|^2 + \Big|\int_0^t g(x_s^0, s)\mathrm{d}W(s)\Big|^2\Big].$$

Then by Holder'inequality, Burkholder-Davis-Gundy inequality, and the condition (2.2), we can derive

$$\begin{aligned}
&E\Big[\sup_{0\le t\le T} |x^1(t) - x^0(t)|^2\Big] \\
&\le \kappa E \sup_{0\le t\le T} \|x_t^0 - \xi\|^2 + \frac{2}{1-\kappa}\Big[TE\int_0^T |f(x_s^0, s)|^2\mathrm{d}s + 4E\int_0^T |g(x_s^0, s)|^2\mathrm{d}s\Big] \\
&\le 2\kappa E\|\xi\|^2 + \frac{2KT(T+4)}{1-\kappa}(1 + E\|\xi\|^2) := C.
\end{aligned} \tag{2.8}$$

Note also that for $n \ge 1$ and $0 \le t \le T$,

$$\begin{aligned}
&x^{n+1}(t) - x^n(t) \\
&= D(x_t^n) - D(x_t^{n-1}) + \int_0^t [f(x_s^n, s) - f(x_s^{n-1}, s)]\mathrm{d}s + \int_0^t [g(x_s^n, s) - g(x_s^{n-1}, s)]\mathrm{d}W(s).
\end{aligned}$$

Now by Lemma 2.2, Holder'inequality, Burkholder-Davis-Gundy inequality, the conditions (2.1) and (2.3), one can easily show that

$$\begin{aligned}
&E\Big(\sup_{0\le t\le T} |x^{n+1}(t) - x^n(t)|^2\Big) \le \kappa E\Big(\sup_{0\le t\le T} |x^n(t) - x^{n-1}(t)|^2\Big) + \\
&\quad \frac{2\bar{K}(T+4)}{1-\kappa}\int_0^T E\Big(\sup_{0\le s\le t} |x^n(s) - x^{n-1}(s)|^2\Big)\mathrm{d}t \\
&\le \delta E\Big(\sup_{0\le t\le T} |x^n(t) - x^{n-1}(t)|^2\Big) \\
&\le \delta^n E\Big(\sup_{0\le t\le T} |x^1(t) - x^0(t)|^2\Big) \le C\delta^n,
\end{aligned} \tag{2.9}$$

where (2.8) has been used. Hence

$$P\Big\{\sup_{0\le t\le T} |x^{n+1}(t) - x^n(t)|^2 > \delta^{\frac{n}{4}}\Big\} \le C\delta^{\frac{n}{2}}.$$

Since $\sum_{n=0}^{\infty} C\delta^{\frac{n}{2}} < \infty$, using the additional condition (2.6) and the Borel-Cantelli lemma[7] yield that for almost all $\omega \in \Omega$ there exists a positive integer $n_0 = n_0(\omega)$ such that

$$\sup_{0\le t\le T} |x^{n+1}(t) - x^n(t)|^2 \le \delta^{\frac{n}{4}} \text{ whenever } n \ge n_0.$$

It follows that, with Probability 1, the partial sums

$$x^0(t) + \sum_{i=0}^{n-1}[x^{i+1}(t) - x^i(t)] = x^n(t)$$

are convergent uniformly in $t \in [0,T]$. Denote the limit by $x(t)$. Clearly, $x(t)$ is continuous and $\mathscr{F}_t$-adapted. On the other hand, one sees from (2.9) that for every $t \in [0,T]$, $\{x^n(t)\}_{n\geq 1}$ is a Cauchy sequence in L^2 as well. Hence we also have $x^n(t) \to x(t)$ in L^2 as well. Letting $n \to \infty$, using (2.9) as well as $\xi \in BC((-\infty,0];R^n)$, one easily derives that $x(\cdot) \in \mathscr{M}^2([0,T];R^n)$. It remains to show that $x(t)$ satisfies the equation (1.1). Note that

$$\begin{aligned}&E|D(x_t^n) - D(x_t)|^2 + E\Big|\int_0^t f(x_s^n,s)\mathrm{d}s\Big|^2 + E\Big|\int_0^t g(x_s^n,s)\mathrm{d}W(s)\Big|^2\\ &\leq \kappa^2 E\sup_{0\leq t\leq T}|x^n(t) - x(t)|^2 + \bar{K}(T+4)\int_0^T E|x^n(t) - x(t)|^2\mathrm{d}s \text{ as } n \to \infty.\end{aligned}$$

Hence we let $n \to \infty$ in (2.7) to obtain that

$$x(t) - D(x_t) = \xi(0) - D(\xi) + \int_0^t f(x_s,s)\mathrm{d}s + \int_0^t g(x_s,s)\mathrm{d}W(s) \text{ on } 0 \leq t \leq T$$

as desired.

Step 2. We need to remove the additional condition (2.6). Let $\sigma > 0$ be sufficiently small for

$$\kappa + \frac{2\bar{K}\sigma(\sigma+4)}{1-\kappa} < 1.$$

By Step 1, there is a solution to equation (1.1) on $(-\infty,\sigma]$. Now consider equation (1.1) on $[\sigma,2\sigma]$ with initial data x_σ. By Step 1 again, there is a solution to equation (1.1) on $[\sigma,2\sigma]$. Repeating this procedure, we see that there is a solution to equation (1.1) on the entire interval $(-\infty,T]$. The existence has been proved.

Uniqueness. Let $x(t)$ and $\bar{x}(t)$ be the two solutions. By Lemma 2.3, both of them belong to $\mathscr{M}^2((-\infty,T];R^n)$. Note that

$$\begin{aligned}x(t) - \bar{x}(t) =& D(x_t) - D(\bar{x}_t) + \int_0^t [f(x_s,s) - f(\bar{x}_s,s)]\mathrm{d}s+\\ &\int_0^t [g(x_s,s) - g(\bar{x}_s,s)]\mathrm{d}W(s).\end{aligned}$$

In the same way as in the proof of the existence one derives that

$$E(\sup_{0\leq s\leq t}|x(s) - \bar{x}(s)|^2) \leq \frac{2\bar{K}(T+4)}{(1-\kappa)^2}\int_0^t E(\sup_{0\leq r\leq s}|x(r) - \bar{x}(r)|^2)\mathrm{d}s.$$

The Gronwall inequality then yields that

$$E(\sup_{0\leq t\leq T}|x(t) - \bar{x}(t)|^2) = 0.$$

This implies that $x(t) = \bar{x}(t)$ for $0 \leq t \leq T$, hence for all $-\infty < t \leq T$, almost surely. The uniqueness has been proved. The proof is completed. □

The Lipschitz condition (2.1) means that the coefficients $f(x_t,t)$ and $g(x_t,t)$ do not change faster than a linear function of x as change in x. This implies in particular the continuity of

$f(x_t, t)$ and $g(x_t, t)$ in x for all $t \in [0, T]$. Thus, functions that are discontinuous with respect to x are excluded as the coefficients. Besides, functions like $\sin x^2$ do not satisfy the Lipschitz condition. These indicate that the Lipschitz condition is too restrictive. The next theorem is a generalization of Theorem 2.1 in which this (uniform) Lipschitz condition is replaced by the local Lipchitz condition.

Theorem 2.4 *Let (2.2) and (2.3) hold, but replace the condition (2.1) with the following local Lipschitz condition: For every integer $n \geq 1$, there exists a positive constant K_n such that, for all $t \in [0, T]$ and those $\phi, \varphi \in BC((-\infty, 0]; R^n)$ with $\|\phi\| \vee \|\varphi\| \leq n$,*

$$|f(\phi, t) - f(\varphi, t)|^2 \vee |g(\phi, t) - g(\varphi, t)|^2 \leq K_n \|\phi - \varphi\|^2. \tag{2.10}$$

Then there exists a unique solution $x(t)$ to the initial-value problem (1.1) and (1.3), and the solution belongs to $\mathscr{M}^2((-\infty, T]; R^n)$.

The proof of Theorem 2.4 is left to the reader.

Now an example is given to verify the conditions of Theorem 2.1.

Example 2.5 Consider the following linear scalar neutral stochastic delay differential equation

$$\begin{aligned} &\mathrm{d}[x(t) - c_1 x(t - \tau_1(t))] \\ &\quad = [-2x(t) + c_2 x(t - \tau_2(t))]\mathrm{d}t + [c_3 x(t) + c_4 x(t - \tau_3(t))]\mathrm{d}W(t), \quad t \geq 0, \end{aligned} \tag{2.11}$$

where $\tau_1(t) = 2\mu t - \mu|\sin t|$, $\tau_2(t) = 2\mu t$, $\tau_3(t) = 2\mu t - \mu|\cos t|$. To be specific, let $\mu = 0.0025$, $t \geq 0$, $0 < |c_1| < 1$, and c_2, c_3, c_4 be constants. For any bounded initial data, by Theorem 2.1, there exists a unique solution $x(t)$ to Eq.(2.11).

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