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Symmetry of the Point Spectrum of Upper Triangular Infinite Dimensional Hamiltonian Operators

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Abstract In this paper, by using characterization of the point spectrum of the upper triangular infinite dimensional Hamiltonian operator H, a necessary and sufficient condition is obtained on the symmetry of $\sigma_p(A)$ and $\sigma_p^1(-A^*)$ with respect to the imaginary axis. Then the symmetry of the point spectrum of H is given, and several examples are presented to illustrate the results.

Keywords non-self-adjoint operator; infinite dimensional Hamiltonian operator; point spectrum; symmetry.

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1. Introduction

Infinite dimensional Hamiltonian operators are a kind of non-self-adjoint operators with deep mechanical background. Their spectral theory is the theoretical foundation of the separation of the variables method solving mechanical problems, and plays a significant role in elasticity mechanics and other related fields[1−4]. Recently, the research on infinite dimensional Hamiltonian operators is very active, and some interesting results are obtained in spectral theory. In [5,6], the authors got the characterizations of the point spectrum, residual spectrum, and continuous spectrum; Huang[7] studied the structure of the spectrum, and gave a necessary and sufficient condition on the symmetry with respect to the imaginary axis of the point spectrum of diagonal infinite dimensional Hamiltonian operators; and these make the spectrum of infinite dimensional Hamiltonian operators clearer.

The residual spectrum of many infinite dimensional Hamiltonian operators is empty. Then, which kind of operators has empty residual spectrum? By the structure of the spectrum of infinite dimensional Hamiltonian operators, we only need to consider the symmetry of the point

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spectrum with respect to the imaginary axis. In this paper, the point spectrum of a kind of upper triangular infinite dimensional Hamiltonian operators is investigated, and its symmetry is explicitly described. We begin with some definitions and lemmas.

Throughout this paper, we believe that the empty set $\emptyset$ is symmetric with respect to the imaginary axis, and the operators involved are always linear.

Definition 1.1 *Let X be a Hilbert space, and $H : \mathcal{D}(H) \subseteq X \times X \longrightarrow X \times X$ be a densely defined operator. The operator H is called an infinite dimensional Hamiltonian operator if $(JH)^* = JH$, where $J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$ with I being the identity operator on X, 0 the zero operator on X, and $(JH)^*$ the adjoint of (JH).*

It can be seen that the infinite dimensional Hamiltonian operator has the following matrix form:

$$H = \begin{pmatrix} A & B \\ C & -A^* \end{pmatrix} : \mathcal{D}(H) \subseteq X \times X \longrightarrow X \times X,$$

where A is a closed densely defined operator, and B, C are both self-adjoint operators.

Definition 1.2 *Let X be a complex Banach space, and A be a closed operator in X. The set*

$$\rho(A) = \{\lambda \in \mathbb{C} : \ \lambda I - A \text{ is a bijection}\}$$

is called the resolvent set of A, and the spectrum $\sigma(A)$ of A is the complement of $\rho(A)$ in $\mathbb{C}$. Then, we have $\sigma(A) = \sigma_p(A) \cup \sigma_r(A) \cup \sigma_c(A)$, where

$$\sigma_p(A) = \{\lambda \in \mathbb{C} : \ \lambda I - A \text{ is not an injection}\},$$

$$\sigma_r(A) = \left\{\lambda \in \mathbb{C} : \ \lambda I - A \text{ is injective}, \overline{\mathcal{R}(\lambda I - A)} \neq X\right\},$$

$$\sigma_c(A) = \left\{\lambda \in \mathbb{C} : \ \lambda I - A \text{ is injective}, \overline{\mathcal{R}(\lambda I - A)} = X, \mathcal{R}(\lambda I - A) \neq X\right\}$$

are called the point spectrum, residual spectrum and continuous spectrum of A, respectively.

Recall that if M is the collection of some complex numbers, then $\overline{M}$ stands for the set consisting of the complex conjugates of its members, otherwise stands for the closure of M; a subset $S \subseteq \mathbb{C}$ is symmetric with respect to the imaginary axis if $-\overline{\lambda} \in S$ for any $\lambda \in S$.

Lemma 1.1[7] *Let A be a closed densely defined operator in a Hilbert space X. Then,*

(i) If $\lambda \in \sigma_p(A)$, then $\overline{\lambda} \in \sigma_p(A^*) \cup \sigma_r(A^*)$;

(ii) If $\lambda \in \sigma_r(A)$, then $\overline{\lambda} \in \sigma_p(A^*)$;

(iii) $\lambda \in \sigma_c(A) \Longleftrightarrow \overline{\lambda} \in \sigma_c(A^*)$.

Lemma 1.2[7] *Let H be an infinite dimensional Hamiltonian operator. Then*

(i) $\lambda \in \sigma_p(H) \Longleftrightarrow -\lambda \in \sigma_p(H^*)$;

(ii) $\lambda \in \sigma_c(H) \Longleftrightarrow -\lambda \in \sigma_c(H^*)$;

(iii) $\lambda \in \sigma_r(H) \Longleftrightarrow -\lambda \in \sigma_r(H^*)$.

By the structure of the spectrum of infinite dimensional Hamiltonian operators, we have

Lemma 1.3[7] *Let H be an infinite dimensional Hamiltonian operator. Then*

(i) The union of the point spectrum and residual spectrum $\sigma_{pr}(H)$, the continuous spectrum $\sigma_c(H)$ and spectrum $\sigma(H)$ of H are all symmetric with respect to the imaginary axis;

(ii) If $\lambda \in \sigma_r(H)$, then $-\bar{\lambda} \notin \sigma_r(H)$;

(iii) $\sigma_r(H) = \{\lambda \in \mathbb{C} : \lambda \notin \sigma_p(H), -\overline{\lambda} \in \sigma_p(H)\}$.

2. Main results

In this section, we give a necessary and sufficient condition on the symmetry of the point spectrum of upper triangular infinite dimensional Hamiltonian operators

$$H = \begin{bmatrix} A & C \\ 0 & -A^* \end{bmatrix}. \tag{2.1}$$

In [5], the point spectrum of H is described as

$$\sigma_p(H) = \sigma_p(A) \cup \{\lambda \in \mathbb{C} : \lambda \in \sigma_p(-A^*), \mathcal{R}(C_\lambda) \cap \mathcal{R}(\lambda I - A) \neq \emptyset\},$$

where $C_\lambda = C|_{\mathcal{N}(\lambda I + A^*) \cap D(C) \setminus \{0\}}$. Write $\sigma_p^1(-A^*) = \{\lambda \in \mathbb{C} : \lambda \in \sigma_p(-A^*), \mathcal{R}(C_\lambda) \cap \mathcal{R}(\lambda I - A) \neq \emptyset\}$, then $\sigma_p(H) = \sigma_p(A) \cup \sigma_p^1(-A^*)$, and $\sigma_p^1(-A^*) \subseteq \sigma_p(-A^*)$.

In the following, we assume that $\sigma_p(A)$ and $\sigma_p^1(-A^*)$ are both nonempty. First of all, we discuss the symmetry of $\sigma_p(A)$ and $\sigma_p^1(-A^*)$.

Lemma 2.1 *$\sigma_p(A)$ and $\sigma_p^1(-A^*)$ are symmetric with respect to the imaginary axis each other if and only if*

(i) $\sigma_r(A^*) = \emptyset$;

(ii) For each $\lambda \in \sigma_p(A)$, if $-\overline{\lambda} \in \sigma_p(-A^*)$, then $\mathcal{R}(C_{-\overline{\lambda}}) \cap \mathcal{R}(-\overline{\lambda} I - A) \neq \emptyset$;

(iii) $\sigma_p^1(-A^*) \cap \overline{\sigma_r(-A)} = \emptyset$.

Proof Suppose that (i), (ii), (iii) hold. For $\lambda \in \sigma_p(A)$, we have $\overline{\lambda} \in \sigma_p(A^*) \cup \sigma_r(A^*)$ by Lemma 1.1. Then, by (i), $\overline{\lambda} \in \sigma_p(A^*)$, i.e., $-\overline{\lambda} \in \sigma_p(-A^*)$; and by (ii), $-\overline{\lambda} \in \sigma_p^1(-A^*)$. Similarly, from Lemma 1.1 and (iii), it follows that if $\lambda \in \sigma_p^1(-A^*)$, then $-\overline{\lambda} \in \sigma_p(A)$. Thus, $\sigma_p(A)$ and $\sigma_p^1(-A^*)$ are symmetric with respect to the imaginary axis each other.

Conversely, assume that $\sigma_p(A)$ and $\sigma_p^1(-A^*)$ are symmetric with respect to the imaginary axis each other. Then, for each $\lambda \in \sigma_p(A)$, we have $-\overline{\lambda} \in \sigma_p^1(-A^*)$, i.e., $-\overline{\lambda} \in \sigma_p(-A^*)$ and $\mathcal{R}(C_{-\overline{\lambda}}) \cap \mathcal{R}(-\overline{\lambda} I - A) \neq \emptyset$, and hence (ii) is valid. Note that $\overline{\lambda} \notin \sigma_r(A^*)$, since $\overline{\lambda} \in \sigma_p(A^*)$. This shows that $\sigma_p(A) \cap \overline{\sigma_r(A^*)} = \emptyset$. Also, by Lemma 1.1, $\overline{\sigma_r(A^*)} \subset \sigma_p(A)$. Therefore, $\sigma_r(A^*) = \emptyset$. Analogously, (iii) can be proven. □

Since $\sigma_p(H) = \sigma_p(A) \cup \sigma_p^1(-A^*)$, if $\sigma_p(A)$ and $\sigma_p^1(-A^*)$ are symmetric with respect to the imaginary axis each other, then so is $\sigma_p(H)$. Thus,

Theorem 2.2 *If H satisfies the following conditions, then $\sigma_p(H)$ is symmetric with respect to the imaginary axis:*

(i) $\sigma_r(A^*) = \emptyset$;

(ii) *For each* $\lambda \in \sigma_p(A)$, *if* $-\overline{\lambda} \in \sigma_p(-A^*)$, *then* $\mathcal{R}(C_{-\overline{\lambda}}) \cap \mathcal{R}(-\overline{\lambda}I - A) \neq \emptyset$;

(iii) $\sigma_p^1(-A^*) \cap \overline{\sigma_r(-A)} = \emptyset$.

Next, we investigate the symmetry of $\sigma_p(H)$ when $\sigma_p(A)$ and $\sigma_p^1(-A^*)$ are not symmetric with respect to the imaginary axis each other. There are three cases to consider.

When $\sigma_r(A^*) \neq \emptyset$, write $M_1 = \overline{\sigma_r(A^*)} \setminus \sigma_p^1(-A^*)$, $M_2 = \sigma_p(A) \setminus \overline{\sigma_r(A^*)}$, then

$$\sigma_p(H) = \sigma_p(A) \cup \sigma_p^1(-A^*) = M_1 \cup M_2 \cup \sigma_p^1(-A^*).$$

Theorem 2.3 *If H satisfies the following conditions, then $\sigma_p(H)$ is symmetric with respect to the imaginary axis if and only if so is M_1:*

(i) $\sigma_r(A^*) \neq \emptyset$;

(ii) *For each* $\lambda \in \sigma_p(A)$, *if* $-\overline{\lambda} \in \sigma_p(-A^*)$, *then* $\mathcal{R}(C_{-\overline{\lambda}}) \cap \mathcal{R}(-\overline{\lambda}I - A) \neq \emptyset$;

(iii) $\sigma_p^1(-A^*) \cap \overline{\sigma_r(-A)} = \emptyset$.

Proof Assume that $\sigma_p(H)$ is symmetric with respect to the imaginary axis. Without loss of generality, let $M_1 \neq \emptyset$ and $M_2 \neq \emptyset$. If $\lambda \in M_1$, i.e., $\lambda \in \overline{\sigma_r(A^*)} \setminus \sigma_p^1(-A^*)$, then $-\overline{\lambda} \in \sigma_r(-A^*)$. So, $-\overline{\lambda} \notin \sigma_p(-A^*)$, and further $-\overline{\lambda} \notin \sigma_p^1(-A^*)$. Thus, by assumption, $-\overline{\lambda} \in M_1 \cup M_2$.

If $-\overline{\lambda} \in M_2(\subset \sigma_p(A))$, then $\lambda \in \sigma_p(-A^*) \cup \sigma_r(-A^*)$. By the definition of M_2, $-\overline{\lambda} \notin \overline{\sigma_r(A^*)}$, i.e., $\lambda \notin \sigma_r(-A^*)$, and hence $\lambda \in \sigma_p(-A^*)$. Note that $\mathcal{R}(C_\lambda) \cap \mathcal{R}(\lambda I - A) \neq \emptyset$ by (ii), thus $\lambda \in \sigma_p^1(-A^*)$. This contradicts $\lambda \in M_1$. Therefore, if $\lambda \in M_1$, then $-\overline{\lambda} \in M_1$, i.e., M_1 is symmetric with respect to the imaginary axis.

Conversely, suppose that M_1 is symmetric with respect to the imaginary axis. For each $\lambda \in \sigma_p(H)$, we have $\lambda \in M_1 \cup M_2 \cup \sigma_p^1(-A^*)$. The discussions are as follows:

(a) If $\lambda \in M_1$, then $-\overline{\lambda} \in M_1$ by the assumption, and so $-\overline{\lambda} \in \sigma_p(H)$;

(b) If $\lambda \in M_2$, similarly to the second paragraph of the proof, $-\overline{\lambda} \in \sigma_p^1(-A^*)$, hence $-\overline{\lambda} \in \sigma_p(H)$;

(c) If $\lambda \in \sigma_p^1(-A^*)$, then $-\overline{\lambda} \in \sigma_p(A) \cup \sigma_r(A)$. Note that $-\overline{\lambda} \notin \sigma_r(A)$ by (iii), then $-\overline{\lambda} \in \sigma_p(A)$. Thus, by $\sigma_p(H) = \sigma_p(A) \cup \sigma_p^1(-A^*)$, $-\overline{\lambda} \in \sigma_p(H)$. □

When $\sigma_p^1(-A^*) \cap \overline{\sigma_r(-A)} \neq \emptyset$, write $M_3 = (\sigma_p^1(-A^*) \cap \overline{\sigma_r(-A)}) \setminus \sigma_p(A)$, $M_4 = \sigma_p^1(-A^*) \setminus (\sigma_p^1(-A^*) \cap \overline{\sigma_r(-A)})$, then $\sigma_p(H) = \sigma_p(A) \cup M_3 \cup M_4$. As the analogue of Theorem 2.3, we have

Theorem 2.4 *If H satisfies the following conditions, then $\sigma_p(H)$ is symmetric with respect to the imaginary axis if and only if so is M_3:*

(i) $\sigma_r(A^*) = \emptyset$;

(ii) *For each* $\lambda \in \sigma_p(A)$, *if* $-\overline{\lambda} \in \sigma_p(-A^*)$, *then* $\mathcal{R}(C_{-\overline{\lambda}}) \cap \mathcal{R}(-\overline{\lambda}I - A) \neq \emptyset$;

(iii) $\sigma_p^1(-A^*) \cap \overline{\sigma_r(-A)} \neq \emptyset$.

When $\sigma_r(A^*) \neq \emptyset$, $\sigma_p^1(-A^*) \cap \overline{\sigma_r(-A)} \neq \emptyset$, write $M_5 = \sigma_p^1(-A^*) \cap \overline{\sigma_r(A^*)} \cap \overline{\sigma_r(-A)}$, then

$$\sigma_p(H) = \sigma_p(A) \cup \sigma_p^1(-A^*) = M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5.$$

Theorem 2.5 *If H satisfies the following conditions, then $\sigma_p(H)$ is symmetric with respect to the imaginary axis if and only if M_1 and M_3 are both symmetric with respect to the imaginary axis, and $M_5 = \emptyset$:*

(i) $\sigma_r(A^*) \neq \emptyset$;

(ii) *For each* $\lambda \in \sigma_p(A)$, *if* $-\overline{\lambda} \in \sigma_p(-A^*)$, *then* $\mathcal{R}(C_{-\overline{\lambda}}) \cap \mathcal{R}(-\overline{\lambda}I - A) \neq \emptyset$;

(iii) $\sigma_p^1(-A^*) \cap \overline{\sigma_r(-A)} \neq \emptyset$.

Proof Without loss of generality, assume that M_1 and M_3 are both nonempty. If $\lambda \in M_1$, then $-\overline{\lambda} \notin \sigma_p^1(-A^*) \cup M_2$ by the proof of Theorem 2.3. Note that $M_3 \cup M_4 \cup M_5 \subset \sigma_p^1(-A^*)$, so $-\overline{\lambda} \notin M_2 \cup M_3 \cup M_4 \cup M_5$. Similarly, if $\lambda \in M_3$, then $-\overline{\lambda} \notin M_1 \cup M_2 \cup M_4 \cup M_5$.

We claim that M_2 and M_4 are symmetric with respect to the imaginary axis each other. In fact, if $\lambda \in M_2$, then, similarly to the proof of Theorem 2.3, $-\overline{\lambda} \in \sigma_p^1(-A^*)$. Also, $\lambda \in \sigma_p(A)$ by the definition of M_2, which implies $\lambda \notin \sigma_r(A)$, i.e.,$-\overline{\lambda} \notin \overline{\sigma_r(-A)}$, and so $-\overline{\lambda} \in M_4$. On the other hand, if $\lambda \in M_4 (\subset \sigma_p^1(-A^*))$, then $-\overline{\lambda} \in \sigma_p(A) \cup \sigma_r(A)$. By the definition of M_4, we see that $\lambda \notin \overline{\sigma_r(-A)}$, i.e., $-\overline{\lambda} \notin \sigma_r(A)$, and then $-\overline{\lambda} \in \sigma_p(A)$. Moreover, note that $\lambda \in \sigma_p(-A^*)$. We know that $-\overline{\lambda} \notin \overline{\sigma_r(A^*)}$. Thus, $-\overline{\lambda} \in M_2$.

Now we prove that if $\lambda \in M_5$, then $-\overline{\lambda} \notin \sigma_p(H)$. In fact, if $\lambda \in M_5$, then $-\overline{\lambda} \notin M_1 \cup M_3$ by the first paragraph of the proof. Note that $M_5 \cap (M_2 \cup M_4) = \emptyset$, and M_2 and M_4 are symmetric with respect to the imaginary axis, so $-\overline{\lambda} \notin M_2 \cup M_4$. Again, if $\lambda \in M_5$, then $\lambda \in \overline{\sigma_r(-A)}$ by the definition of M_5. Thus, $-\overline{\lambda} \notin \sigma_p(A)$. By $M_5 \subset \sigma_p(-A^*) \cap \sigma_p(A)$, we have $-\overline{\lambda} \notin M_5$. Therefore, if $\lambda \in M_5$, then $-\overline{\lambda} \notin M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5$ $(= \sigma_p(H))$.

From the above discussions, it can be seen that $\sigma_p(H)$ is symmetric with respect to the imaginary axis if and only if M_1 and M_3 are symmetric with respect to the imaginary axis, and $M_5 = \emptyset$. □

If $\sigma_r(A^*) = \emptyset$ and $\sigma_p^1(-A^*) \cap \overline{\sigma_r(-A)} = \emptyset$, then $M_1 = \emptyset$ and $M_3 = \emptyset$, respectively. And in both cases, $M_5 = \emptyset$. Thus, Theorems 2.2–2.5 can be rewritten as

Theorem 2.6 *Assume that for each* $\lambda \in \sigma_p(A)$, *if* $-\overline{\lambda} \in \sigma_p(-A^*)$, *then* $\mathcal{R}(C_{-\overline{\lambda}}) \cap \mathcal{R}(-\overline{\lambda}I - A) \neq \emptyset$. *Then,* $\sigma_p(H)$ *is symmetric with respect to the imaginary axis if and only if* M_1 *and* M_3 *are both symmetric with respect to the imaginary axis, and* $M_5 = \emptyset$.

3. Examples

To illustrate the main theorems, we give some examples.

Example 3.1 Let X be a Hilbert space and C be a self-adjoint operator with non-zero domain. We consider the infinite dimensional Hamiltonian operator

$$H = \begin{bmatrix} I & C \\ 0 & -I \end{bmatrix}.$$

It is clear that $\sigma_p(I) = 1$, $\sigma_r(I) = \emptyset$. For $\lambda = 1 \in \sigma_p(I)$, we have $-\overline{\lambda} = -1 \in \sigma_p(-I)$, thus $\mathcal{R}(-\overline{\lambda}I - I) = X$, so $\mathcal{R}(C_{-\overline{\lambda}}) \cap \mathcal{R}(-\overline{\lambda}I - I) \neq \emptyset$. Therefore, by Theorem 2.2, $\sigma_p(H)$ is symmetric with respect to the imaginary axis.

Example 3.2 Define the operators A and C on ℓ^2 by $Ax = (-x_1 - x_2, -2x_1 - x_3, -x_4, -x_5, \ldots)$ and $Cx = (x_1, x_2, 0, 0, \ldots)$ for each $x = (x_1, x_2, \ldots, x_n, \ldots)$, respectively. Then, the infinite dimensional Hamiltonian operator

$$H=\begin{bmatrix} A & C \\ 0 & -A^* \end{bmatrix}$$

satisfies the conditions of Theorem 2.3. We assert that M_1 is not symmetric with respect to the imaginary axis, nor is $\sigma_p(H)$.

In fact, direct calculations show that $\sigma_p(A)=\{1,-2\}\cup\{\lambda\in\mathbb{C}:|\lambda|<1\}$, $\sigma_p(-A^*)=\{2\}$, $\sigma_r(A)=\emptyset$, and $\sigma_r(-A^*)=\{-1\}\cup\{\lambda\in\mathbb{C}:|\lambda|<1\}$, then (i), (iii) are satisfied.

For $\lambda=-2\in\sigma_p(A)$, we have that $-\overline{\lambda}=2\in\sigma_p(-A^*)$ and there exist $x=(\frac{3a}{8},-\frac{a}{8},0,0,\ldots)\in\ell^2$ and $y=(a,\frac{a}{2},\frac{a}{2^2},\frac{a}{2^3},\frac{a}{2^4},\ldots)\in\mathcal{N}(2I+A^*)\setminus\{0\}$ such that $(2I-A)x=Cy$, i.e., $\mathcal{R}(C_{-\overline{\lambda}})\cap\mathcal{R}(-\overline{\lambda}I-A)\neq\emptyset$. This proves that (ii) holds. Also, $2\in\sigma_p^1(-A^*)$, so $\sigma_p(-A^*)=\sigma_p^1(-A^*)$. Thus, $M_1=\overline{\sigma_r(A^*)}\setminus\sigma_p^1(-A^*)=\overline{\sigma_r(A^*)}$, which is not symmetric with respect to the imaginary axis.

Example 3.3 Define $B:\ell^2\to\ell^2$ by $Bx=(x_1+2x_2,x_1,x_2,\ldots)$ for each $x=(x_1,x_2,\ldots,x_n,\ldots)$. Then $B^*x=(x_1+x_2,2x_1+x_3,x_4,x_5,\ldots)$. Let

$$A=\begin{bmatrix} B & 0 \\ 0 & -B^* \end{bmatrix}.$$

Then $\sigma_r(A)=\{-1\}$. Consider the infinite dimensional Hamiltonian operator

$$H=\begin{bmatrix} A & 0 \\ 0 & -A^* \end{bmatrix}.$$

Then $\sigma_p^1(-A^*)=\sigma_p(-A^*)$. Hence for each $\lambda\in\sigma_p(A)$, if $-\overline{\lambda}\in\sigma_p(-A^*)$, then $\mathcal{R}(C_{-\overline{\lambda}})\cap\mathcal{R}(-\overline{\lambda}I-A)\neq\emptyset$. Note that $\sigma_p^1(-A^*)\cap\overline{\sigma_r(-A)}=\overline{\sigma_r(-A)}$, and $\sigma_r(A^*)=\sigma_r(-A)$ by Lemma 1.2. We deduce that $M_5=\overline{\sigma_r(-A)}=\{1\}$ is nonempty. Therefore, $\sigma_p(H)$ is not symmetric with respect to the imaginary axis by Theorem 2.6.

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