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Interval Valued $(\in, \in \vee q)$-Fuzzy Filters of MTL-Algebras

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Abstract The concepts of interval valued $(\in, \in \vee q)$-fuzzy Boolean, MV- and G- filters in a MTL-algebra are introduced. The properties of these generalized fuzzy filters are studied and in particular, the relationships between these fuzzy filters in an MTL-algebra are investigated.

Keywords MTL-algebra; filter; interval valued $(\in, \in \vee q)$-fuzzy (Boolean, MV- and G-) filters.

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1. Introduction

Logic is always an essential tool in studying mathematics and computer science and is also a technique for laying foundation. The non-classical logic includes many-valued logic and fuzzy logic which takes the advantage of the classical logic to handle information with various facets of uncertainty (see [14] for the generalized theory of uncertainty) such as the fuzziness, randomness and so on. In particular, non-classical logic has become a formal and useful tool in computer science to deal with fuzzy information and uncertain information. Among all kinds of uncertainties, the incomparability is the most important one which is frequently encountered in our daily life.

On the other hand, the theory of fuzzy sets, introduced by Zadeh [13], can be applied to many branches of mathematics. For example, Rosenfeld [9] was inspired by using fuzzy sets to study fuzzy groups. A new type of fuzzy subgroups, namely (viz, $(\in, \in \vee q)$-fuzzy subgroup), were considered by Bhakat and Das [2] by using a combined notion of "belongingness" and "quasicoincidence" of fuzzy points and fuzzy sets. This idea was first introduced by Pu and Liu [8], and as a consequence, we can consider the $(\in, \in \vee q)$-fuzzy subgroup which is a generalization of the Rosenfeld's fuzzy subgroup. Recently, Davvaz [3] applied this concept to study near-rings and obtained some useful results.

It is noteworthy that the concept of "filters" plays an important role from logical point of view and hence different filters correspond to different sets of provable formulas. Recall that

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Hájek introduced the axiom system of basic logic (BL, for short) for fuzzy propositional logic and then he defined the class of BL-algebras [5]. The MTL logic and Monoidal t-norm based on this logic were first introduced by Esteva and Godo [4]. This kind of logic is very interesting from many points of view. In fact, from the logic point of view, it can be regarded as a weak system of Fuzzy Logic. In connection with MTL logic, Esteva and Godo [4] introduced a new kind of algebra, namely, the MTL-algebra and they studied several basic properties of such algebra. By using fuzzy sets, Kim et al [7] studied the fuzzy structure of filters in MTL-algebras. As a continuation of the study of fuzzy filters of MTL-algebras [7], Jun et al. [6] gave some characterizations of fuzzy filters in MTL-algebras and he further investigated the properties of fuzzy filters in MTL-algebras. Recently, Zhang et al. [15] introduced the concepts of fuzzy ultra filters and fuzzy G-filters in MTL-algebras. They obtained some equivalent conditions for these filters.

In this paper, we first recall some basic definitions and results about MTL-algebras. Then in Section 3, we study the properties of interval valued $(\in, \in\vee q)$-fuzzy filters. Some types of interval valued$(\in, \in\vee q)$-fuzzy filters of MTL-algebras are considered in Section 4 and the relationships between these generalized fuzzy filters will be discussed.

2. Preliminaries

We call a lattice L residuated if $L = (L, \leq, \wedge, \vee, \odot, \rightarrow, 0, 1)$ contains the least element 0 and the largest element 1, and is endowed with two binary operations $\odot$ (called product) and $\rightarrow$ (called residuum) such that

(1) $\odot$ is associative, commutative and isotone;

(2) $\forall x \in L, x \odot 1 = x$;

(3) The Galois corresponding law holds, that is, $x \odot y \leq z \Leftrightarrow x \leq y \rightarrow z$, $\forall x, y, z \in L$.

Based on the results of Hájek [5], we know that the Axioms of MTL and the Formulas are provable in MTL. Moreover, Esteva and Godo [4] defined the MTL-algebras which are corresponding to the MTL-logic in the following way:

An MTL-algebra is a residuated lattice $L = (L, \leq, \wedge, \vee, \odot, \rightarrow, 1)$ satisfying the following pre-linearity equation:

$$(x \rightarrow y) \vee (y \rightarrow x) = 1.$$

In an MTL-algebra, the following properties hold:

(1) $x \rightarrow (y \vee z) = (x \rightarrow y) \vee (x \rightarrow z)$;

(2) $x \odot y \leq x \wedge y$;

(3) $x' = x''', x \leq x'', x' \odot x = 0$;

(4) If $x \vee x' = 1$, then $x \wedge x' = 0$,

where $x' = x \rightarrow 0$.

A non-empty subset A of an MTL-algebra L is called a filter of L if A satisfies the following conditions: (i) $\forall x, y \in L, x \odot y \in A$; (ii) $\forall x \in A, x \leq y \Rightarrow y \in A$. We know a non-empty subset A of L is a filter of L if and only if it satisfies: (i) $1 \in A$; (ii) $\forall x \in A, y \in L, x \rightarrow y \in A \Rightarrow y \in A$.

Now, we call a filter A of L a Boolean filter if for any $x \in L$, $x \vee x' \in A$. In general, a filter A of L is called a G-filter of L if for any $x, y \in L$, $x \odot x \to y \in A \Rightarrow x \to y \in A$ (see [11,15]).

A fuzzy set F in an MTL-algebra L is called a fuzzy filter of L if it satisfies the following conditions: (i) $\forall x, y \in L, F(x \odot y) \geq \min\{F(x), F(y)\}$; (ii) F is order-preserving, that is, for all $x, y \in L$, $x \leq y \Rightarrow F(x) \leq F(y)$ (see [6, 7, 10–12, 15]).

By an interval number $\tilde{a}$, we mean an interval $[a^-, a^+]$, where $0 \leq a^- \leq a^+ \leq 1$. Denote the set of all interval numbers by $D[0,1]$. Then the interval $[a, a]$ can be identified by the number $a \in [0,1]$.

Definition 2.1 *For the interval numbers* $\widetilde{a}_i = [a_i^-, a_i^+], \widetilde{b}_i = [b_i^-, b_i^+]$, $\widetilde{c}_i = [c_i^-, c_i^+] \in D[0,1]$, $i \in I$, *we define*

$$\mathrm{rmax}\{\widetilde{a}_i, \widetilde{b}_i, \widetilde{c}_i\} = [\max(a_i^-, b_i^-, c_i^-), \max(a_i^+, b_i^+, c_i^+)],$$

$$\mathrm{rmin}\{\widetilde{a}_i, \widetilde{b}_i, \widetilde{c}_i\} = [\min(a_i^-, b_i^-, c_i^-), \min(a_i^+, b_i^+, c_i^+)].$$

Definition 2.2 *For any interval numbers* $\widetilde{a}_1, \widetilde{a}_2$, *we define*

(1) $\widetilde{a}_1 \leq \widetilde{a}_2 \Longleftrightarrow a_1^- \leq a_2^-$ *and* $a_1^+ \leq a_2^+$;

(2) $\widetilde{a}_1 = \widetilde{a}_2 \Longleftrightarrow a_1^- = a_2^-$ *and* $a_1^+ = a_2^+$;

(3) $\widetilde{a}_1 < \widetilde{a}_2 \Longleftrightarrow \widetilde{a}_1 \leq \widetilde{a}_2$ *and* $\widetilde{a}_1 \neq \widetilde{a}_2$;

(4) $k\widetilde{a} = [ka^-, ka^+]$, *whenever* $0 \leq k \leq 1$.

We easily observe that $(D[0,1], \leq, \vee, \wedge)$ forms a complete lattice with $0 = [0,0]$ as its least element and $1 = [1,1]$ as its greatest element.

By an interval valued fuzzy set F on X, we mean the set

$$F = \{(x, [\mu_F^-(x), \mu_F^+(x)]) \mid x \in X\},$$

where μ_F^- and μ_F^+ are fuzzy subsets of X such that $\mu_F^-(x) \leq \mu_F^+(x)$, for all $x \in X$. Now, putting $\widetilde{\mu_F}(x) = [\mu_F^-(x), \mu_F^+(x)]$, we see that $F = \{(x, \widetilde{\mu_F}(x)) \mid x \in X\}$, where $\widetilde{\mu_F} : X \to D[0,1]$.

3. Interval valued $(\in, \in \vee q)$-fuzzy filters

An interval valued fuzzy set $F = \{(x, \widetilde{\mu_F}(x)) | x \in X\}$ of an MTL-algebra L of the form

$$\widetilde{\mu_F}(y) = \begin{cases} \widetilde{t}(\neq [0,0]) & \text{if } y = x, \\ [0,0] & \text{if } y \neq x \end{cases}$$

is called the fuzzy interval value with support x and we denote the interval value $\widetilde{t}$ by $U(x; \widetilde{t})$. A fuzzy interval value $U(x; \widetilde{t})$ is called belonging to (resp., quasi-coincident with) an interval valued fuzzy set F, denoted by $U(x; \widetilde{t}) \in F$ (resp. $U(x; \widetilde{t})qF$) if $\widetilde{\mu_F}(x) \geq \widetilde{t}$ (resp., $\widetilde{\mu_F}(x) + \widetilde{t} > [1,1]$). If $U(x; \widetilde{t}) \in F$ or (resp., and) $U(x; \widetilde{t})qF$, then we write $U(x; \widetilde{t}) \in \vee q$(resp., $\in \wedge q$) F. The symbol $\overline{\in \vee q}$ means that $\in \vee q$ does not hold.

In what follows, L is always an MTL-algebra. Also we emphasize that every $\widetilde{\mu_F}(x) = [\mu_F^-(x), \mu_F^+(x)]$ satisfies the comparable condition and the following properties:

$$[\mu_F^-(x), \mu_F^+(x)] < [0.5, 0.5] \text{ or } [0.5, 0.5] \leq [\mu_F^-(x), \mu_F^+(x)], \quad \text{for all } x \in L.$$

Definition 3.1 *An interval valued fuzzy set F of L is called an interval valued fuzzy filter of L if the following conditions are satisfied for all $x, y \in L$:*

(F1) $\widetilde{\mu_F}(x \odot y) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y)\}$,

(FF1) $x \leq y \Rightarrow \widetilde{\mu_F}(x) \leq \widetilde{\mu_F}(y)$.

Now, let F be an interval valued fuzzy set. For every interval number $\widetilde{t} = [t^-, t^+]$, we call the set $F_{\widetilde{t}} = \{x \in L | \widetilde{\mu_F}(x) \geq \widetilde{t}\}$ the level subset of F.

The interval valued fuzzy filters can be characterized by their level filters. The following result can be easily verified.

Theorem 3.2 *An interval valued fuzzy set F of L is an interval valued fuzzy filter of L if and only if for any $[0,0] < \widetilde{t} \leq [1,1]$, $F_{\widetilde{t}} (\neq \emptyset)$ is a filter of L.*

Definition 3.3 *An interval valued fuzzy set F of L is said to be an interval valued $(\in, \in \vee q)$-fuzzy filter of L if the following conditions hold for all interval numbers $\widetilde{t}, \widetilde{r} \in D[0,1]$ and $x, y \in L$,*

(F2) $U(x; \widetilde{t}) \in F$ *and* $U(y; \widetilde{r}) \in F$ *imply* $U(x \odot y; \mathrm{rmin}\{\widetilde{t}, \widetilde{r}\}) \in \vee q F$,

(FF2) $U(x; \widetilde{r}) \in F$ *implies* $U(y; \widetilde{r}) \in \vee q F$ *with* $x \leq y$.

Example 3.4 Let $L = [0,1]$ and define a product $\odot$ and a residuum $\rightarrow$ on L as follows:

$$x \odot y = \begin{cases} x \wedge y & \text{if } x + y > \frac{1}{2}, \\ 0 & \text{otherwise}, \end{cases} \qquad x \rightarrow y = \begin{cases} 1 & \text{if } x \leq y, \\ \max\{1 - x, y\} & \text{otherwise}, \end{cases}$$

for all $x, y \in L$. Then L is an MTL-algebra.

We now define an interval valued fuzzy set F by

$$\widetilde{\mu_F}(x) = \begin{cases} \widetilde{\alpha} & \text{if } x \in (0.5, 1], \\ \widetilde{\beta} & \text{otherwise}, \end{cases}$$

where $\widetilde{\alpha} > [0.5, 0.5] > \widetilde{\beta}$. Then it is routine to verify that F is an interval valued $(\in, \in \vee q)$-fuzzy filter of L.

Theorem 3.5 *The above conditions (F2) and (FF2) in Definition 3.3 are equivalent to the following conditions:*

(F3) $\widetilde{\mu_F}(x \odot y) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y), [0.5, 0.5]\}$, *for all* $x, y \in L$,

(FF3) $\forall x, y \in L, x \leq y \Rightarrow \widetilde{\mu_F}(y) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), [0.5, 0.5]\}$.

Proof (F2)$\Longrightarrow$(F3). Suppose that $x, y \in L$. Then we consider the following cases:

(a) $\mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y)\} < [0.5, 0.5]$,

(b) $\mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y)\} \geq [0.5, 0.5]$.

Case (a) Assume that $\widetilde{\mu_F}(x \odot y) < \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y), [0.5, 0.5]\}$. Then it implies that $\widetilde{\mu_F}(x \odot y) < \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y)\}$. Choose $\widetilde{t}$ such that $\widetilde{\mu_F}(x \odot y) < \widetilde{t} < \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y)\}$, we obtain $U(x; \widetilde{t}) \in F$ and $U(y; \widetilde{t}) \in F$ but $U(x \odot y; \widetilde{t}) \overline{\in \vee q} F$, which contradicts (F2).

Case (b) Assume that $\widetilde{\mu_F}(x \odot y) < [0.5, 0.5]$. Then $U(x; [0.5, 0.5]) \in F$ and $U(y; [0.5, 0.5]) \in F$, but $U(x \odot y; [0.5, 0.5]) \overline{\in \vee q} F$, which is clearly a contradiction. Hence (F3) holds.

(FF2)$\Longrightarrow$ (FF3). Suppose that $x, y \in L$. Then we consider the following two subcases:

subcase (i). $\widetilde{\mu_F}(x) < [0.5, 0.5]$;

subcase (ii). $\widetilde{\mu_F}(x) \geq [0.5, 0.5]$.

In subcase (i), we let $x \leq y$. Assume that $\widetilde{\mu_F}(x) = \widetilde{t} < [0.5, 0.5]$ and $\widetilde{\mu_F}(y) = \widetilde{r} < \widetilde{\mu_F}(x)$. Then we choose $\widetilde{s}$ such that $\widetilde{r} < \widetilde{s} < \widetilde{t}$ and $\widetilde{r} + \widetilde{s} < [1, 1]$. Thus $U(y; \widetilde{s}) \in F$, but $U(x; \widetilde{s}) \overline{\in \vee q} F$, which contradicts (FF2). Hence $\widetilde{\mu_F}(y) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), [0.5, 0.5]\}$.

In subcase (ii), we let $x \leq y$ and $\widetilde{\mu_F}(x) \geq [0.5, 0.5]$. If $\widetilde{\mu_F}(y) < \mathrm{rmin}\{\widetilde{\mu_F}(x), [0.5, 0.5]\}$, then $U(y; [0.5, 0.5]) \in F$, but $U(y; [0.5, 0.5]) \overline{\in \vee q} F$, which contradicts (FF2). Hence (FF3) holds.

(F3)$\Longrightarrow$(F2). Let $U(x; \widetilde{t}) \in F$ and $U(y; \widetilde{r}) \in F$. Then $\widetilde{\mu_F}(x) \geq \widetilde{t}$ and $\widetilde{\mu_F}(y) \geq \widetilde{r}$. Now, we have $\widetilde{\mu_F}(x \odot y) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y), [0.5, 0.5]\} \geq \mathrm{rmin}\{\widetilde{t}, \widetilde{r}, [0.5, 0.5]\}$.

If $\mathrm{rmin}\{\widetilde{t}, \widetilde{r}\} > [0.5, 0.5]$, then $\widetilde{\mu_F}(x \odot y) \geq [0.5, 0.5]$, which implies that $\widetilde{\mu_F}(x \odot y) + \mathrm{rmin}\{\widetilde{t}, \widetilde{r}\} > [1, 1]$. If $\mathrm{rmin}\{\widetilde{t}, \widetilde{r}\} \leq [0.5, 0.5]$, then $\widetilde{\mu_F}(x \odot y) \geq \mathrm{rmin}\{\widetilde{t}, \widetilde{r}\}$. Therefore, $U(x \odot y; \mathrm{rmin}\{\widetilde{t}, \widetilde{r}\}) \in \vee q F$.

(FF3)$\Longrightarrow$(FF2). Let $x \leq y$. Suppose that $U(x; \widetilde{t}) \in F$. Then $\widetilde{\mu_F}(x) \geq \widetilde{t}$, and hence we have $\widetilde{\mu_F}(y) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), [0.5, 0.5]\} \geq \mathrm{rmin}\{\widetilde{t}, [0.5, 0.5]\}$. This implies $\widetilde{\mu_F}(y) \geq \widetilde{t}$ or $\widetilde{\mu_F}(y) \geq [0.5, 0.5]$, according to $\widetilde{t} \leq [0.5, 0.5]$ or $\widetilde{t} > [0.5, 0.5]$. Therefore, $U(y; \widetilde{t}) \in \vee q F$.

By Definition 3.3 and Theorem 3.5, we immediately obtain the following corollary:

Corollary 3.6 *An interval valued fuzzy set F of L is an interval valued $(\in, \in \vee q)$-fuzzy filter of L if and only if the conditions (F3) and (FF3) in Theorem 3.5 hold.*

Theorem 3.7 *An interval valued fuzzy set F of L is an interval valued $(\in, \in \vee q)$-fuzzy filter of L if and only if the following conditions are satisfied:*

(F4) For all $x \in L$, $\widetilde{\mu_F}(1) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), [0.5, 0.5]\}$

(FF4) For all $x, y \in L$, $\widetilde{\mu_F}(y) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(x \to y), [0.5, 0.5]\}$.

Proof Assume that F satisfies the conditions (F4) and (FF4). Let $x, y \in L$ be such that $x \leq y$. Then $x \to y = 1$, and thereby, $\widetilde{\mu_F}(y) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(1), [0.5, 0.5]\} = \mathrm{rmin}\{\widetilde{\mu_F}(x), [0.5, 0.5]\}$. This proves (FF3). Since $x \to (y \to (x \odot y)) = (x \odot y) \to (x \odot y) = 1$, by (F4) and (FF4),

$$\begin{aligned}
\widetilde{\mu_F}(x \odot y) &\geq \mathrm{rmin}\{\widetilde{\mu_F}(y), \widetilde{\mu_F}(y \to (x \odot y)), [0.5, 0.5]\} \\
&\geq \mathrm{rmin}\{\widetilde{\mu_F}(y), \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(x \to (y \to (x \odot y))), [0.5, 0.5]\} \\
&= \mathrm{rmin}\{\widetilde{\mu_F}(y), \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(1), [0.5, 0.5]\}, [0.5, 0.5]\} \\
&= \mathrm{rmin}\{\widetilde{\mu_F}(y), \widetilde{\mu_F}(x), [0.5, 0.5]\}.
\end{aligned}$$

Hence, (F3) is proved and F is an interval valued $(\in, \in \vee q)$-fuzzy filter of L.

Conversely, assume that F is an interval valued $(\in, \in \vee q)$-fuzzy filter of L. Since $x \leq 1$ for all $x \in L$, by (FF3), $\widetilde{\mu_F}(1) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), [0.5, 0.5]\}$, for all $x \in L$. Let $x, y \in L$. Since $x \leq (x \to y) \to y$, $x \odot (x \to y) \leq y$, by using Galois correspondence. Hence, $\widetilde{\mu_F}(y) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x \odot (x \to y)), [0.5, 0.5]\} \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(x \to y), [0.5, 0.5]\}$. This completes the proof. □

Theorem 3.8 *An interval valued fuzzy set F in L is an interval valued $(\in, \in \vee q)$-fuzzy filter of L if and only if the following condition is satisfied:*

$$\forall a, b, c \in L, a \leq b \to c \Rightarrow \widetilde{\mu_F}(c) \geq \text{rmin}\{\widetilde{\mu_F}(a), \widetilde{\mu_F}(b), [0.5, 0.5]\}.$$

Proof Straightforward. □

Theorem 3.9 *Let F be an interval valued $(\in, \in \vee q)$-fuzzy filter of L. Then the following conditions are equivalent:*

(i) $\forall x, y, z \in L, \widetilde{\mu_F}(x \to z) \geq \text{rmin}\{\widetilde{\mu_F}(x \to (y \to z)), \widetilde{\mu_F}(x \to y), [0.5, 0.5]\}$;

(ii) $\forall x, y \in L, \widetilde{\mu_F}(x \to y) \geq \text{rmin}\{\widetilde{\mu_F}(x \to (x \to y)), [0.5, 0.5]\}$;

(iii) $\forall x, y, z \in L, \widetilde{\mu_F}((x \to y) \to (x \to z)) \geq \text{rmin}\{\widetilde{\mu_F}(x \to (y \to z)), [0.5, 0.5]\}$.

Proof (i)$\Longrightarrow$ (ii). Suppose that F satisfies the condition (i). Then, by taking $z = y$ and $z = x$ in (i) and by (F4), we have

$$\begin{aligned}\widetilde{\mu_F}(x \to y) &\geq \text{rmin}\{\widetilde{\mu_F}(x \to (x \to y)), \widetilde{\mu_F}(x \to x), [0.5, 0.5]\}\\ &= \text{rmin}\{\widetilde{\mu_F}(x \to (x \to y)), \widetilde{\mu_F}(1), [0.5, 0.5]\}\\ &= \text{rmin}\{\widetilde{\mu_F}(x \to (x \to y)), [0.5, 0.5]\},\end{aligned}$$

for all $x, y, z \in L$.

(ii)$\Longrightarrow$ (iii). Suppose that F satisfies the condition (ii) and let $x, y, z \in L$. Since $x \to (y \to z) \leq x \to ((x \to y) \to (x \to z))$, it follows that

$$\begin{aligned}\widetilde{\mu_F}((x \to y) \to (x \to z)) &= \widetilde{\mu_F}(x \to ((x \to y) \to z))\\ &\geq \text{rmin}\{\widetilde{\mu_F}(x \to (x \to ((x \to y) \to z))), [0.5, 0.5]\}\\ &= \text{rmin}\{\widetilde{\mu_F}(x \to ((x \to y) \to (x \to z))), [0.5, 0.5]\}\\ &= \text{rmin}\{\widetilde{\mu_F}(x \to (y \to z)), [0.5, 0.5]\}.\end{aligned}$$

(iii)$\Longrightarrow$ (i). If F satisfies condition (iii), then $\widetilde{\mu_F}(x \to y) \geq \text{rmin}\{\widetilde{\mu_F}((x \to y) \to (x \to z)), \widetilde{\mu_F}(x \to y), [0.5, 0.5]\} \geq \text{rmin}\{\widetilde{\mu_F}(x \to (y \to z)), \widetilde{\mu_F}(x \to y), [0.5, 0.5]\}$.

Now, we characterize the interval valued $(\in, \in \vee q)$-fuzzy filters by using their level filters.

Theorem 3.10 *Let F be an interval valued $(\in, \in \vee q)$-fuzzy filter of L. Then for all $[0, 0] < \widetilde{t} \leq [0.5, 0.5]$, $F_{\widetilde{t}}$ is an empty set or a filter of L. Conversely, if F is an interval valued fuzzy set of L such that $F_{\widetilde{t}}(\neq \emptyset)$ is a filter of L for all $[0, 0] < \widetilde{t} \leq [0.5, 0.5]$, then F is an interval valued $(\in, \in \vee q)$-fuzzy filter of L.*

Proof Let F be an interval valued $(\in, \in \vee q)$-fuzzy filter of L and $[0, 0] < \widetilde{t} \leq [0.5, 0.5]$. If $x, y \in F_{\widetilde{t}}$, then $\widetilde{\mu_F}(x) \geq \widetilde{t}$ and $\widetilde{\mu_F}(y) \geq \widetilde{t}$. Now we have $\widetilde{\mu_F}(x \odot y) \geq \text{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y), [0.5, 0.5]\}$ $\geq \text{rmin}\{\widetilde{t}, [0.5, 0.5]\} = \widetilde{t}$. This implies that $x \odot y \in F_{\widetilde{t}}$. Let $x, y \in L$ be such that $x \leq y$. If $x \in F_{\widetilde{t}}$, then by (FF3), $\widetilde{\mu_F}(y) \geq \text{rmin}\{\widetilde{\mu_F}(x), [0.5, 0.5]\} \geq \text{rmin}\{\widetilde{t}, [0.5, 0.5]\} = \widetilde{t}$. This implies that $y \in F_{\widetilde{t}}$ and hence, $F_{\widetilde{t}}$ is a filter of L.

Conversely, if F is an interval valued fuzzy set of L such that $F_{\widetilde{t}}(\neq \emptyset)$ is a filter of L, for all

$[0,0] < \widetilde{t} \leq [0.5, 0.5]$, then for every $x, y \in L$, we can write

$$\widetilde{\mu_F}(x) \geq \text{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y), [0.5, 0.5]\} = \widetilde{t_0},$$

$$\widetilde{\mu_F}(y) \geq \text{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y), [0.5, 0.5]\} = \widetilde{t_0}.$$

Hence, $x, y \in U(F; \widetilde{t_0})$, and so $x \odot y \in U(F; \widetilde{t_0})$. Thus, $\widetilde{\mu_F}(x \odot y) \geq \text{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y), [0.5, 0.5]\}$.

Also, let $x, y \in L$ be such that $x \leq y$. Now, by considering $\widetilde{\mu_F}(x) \geq \text{rmin}\{\widetilde{\mu_F}(x), [0.5, 0.5]\} = \widetilde{s_0}$, we have $x \in F_{\widetilde{s_0}}$ and so $y \in F_{\widetilde{s_0}}$. Thus, $\widetilde{\mu_F}(y) \geq \widetilde{s_0} = \text{rmin}\{\widetilde{\mu_F}(x), [0.5, 0.5]\}$. This proves that F is an interval valued $(\in, \in \vee q)$-fuzzy filter of L.

Naturally, a corresponding result of Theorem 3.10 can be similarly proved when $F_{\widetilde{t}}$ is a filter of L for all $[0.5, 0.5] < \widetilde{t} \leq [1, 1]$.

Theorem 3.11 *Let F be an interval valued fuzzy set of L. Then $F_{\widetilde{t}} (\neq \emptyset)$ is a filter of L for all $[0.5, 0.5] < \widetilde{t} \leq [1, 1]$ if and only if for all $x, y \in L$,*

(F5) $\text{rmax}\{\widetilde{\mu_F}(x \odot y), [0.5, 0.5]\} \geq \text{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y)\}$;

(FF5) $\text{rmax}\{\widetilde{\mu_F}(y), [0.5, 0.5]\} \geq \widetilde{\mu_F}(x)$ *if* $x \leq y$.

Remark 3.12 Let F be an interval valued fuzzy set of an MTL-algebra L and $J = \{\widetilde{t} | \widetilde{t} \in D[0, 1]$ and $F_{\widetilde{t}}$ is an empty set or a filter of $L\}$. In particular, if $J = D[0, 1]$, then F is an ordinary interval valued fuzzy filter of L (cf. Theorem 3.2); if $J = D[0, 0.5)$, then F is an interval valued $(\in, \in \vee q)$-fuzzy filter of L (cf. Theorem 3.10).

4. Interval valued $(\in, \in \vee q)$-fuzzy (Boolean, MV- and G-) filters

In this section, we consider some types of interval valued $(\in, \in \vee q)$-fuzzy filters of MTL-algebras and discuss the relationships between these generalized fuzzy filters.

Definition 4.1 *An interval valued $(\in, \in \vee q)$-fuzzy filter of L is said to be Boolean if it satisfies the following inequality:*

(F6) $\widetilde{\mu_F}(x \vee x') \geq [0.5, 0.5]$, *for all* $x \in L$.

Example 4.2 Let $L = \{0, a, b, 1\}$ be a chain with the following Cayley table:

$\odot$	0	a	b	1
0	0	0	0	0
a	0	a	a	a
b	0	a	a	b
1	0	a	b	1

$\to$	0	a	b	1
0	1	1	1	1
a	0	1	1	1
b	0	b	1	1
1	0	a	b	1

Define the " $\wedge$ " and " $\vee$ " operations on L as the "rmin" and "rmax", respectively. Then $(L, \wedge, \vee, \odot, \to, 0, 1)$ is an MTL-algebra. Define an interval valued fuzzy set F in L by $\widetilde{\mu_F}(0) = [0.2, 0.3], \widetilde{\mu_F}(1) = \widetilde{\mu_F}(b) = \widetilde{\mu_F}(a) = [0.7, 0.8]$. Then, one can easily verify that F is an interval valued $(\in, \in \vee q)$-fuzzy Boolean filter of L.

By using level Boolean filters of MTL-algebras, we can characterize the interval valued $(\in, \in \vee q)$-fuzzy Boolean filters.

Theorem 4.3 *Let F be an interval valued $(\in, \in \vee q)$-fuzzy Boolean filter of L. Then for all $[0,0] < \widetilde{t} \leq [0.5, 0.5]$, $F_{\widetilde{t}}$ is an empty set or a Boolean filter of L. Conversely, if F is an interval valued fuzzy set of L such that $F_{\widetilde{t}}(\neq \emptyset)$ is a Boolean filter of L for all $[0,0] < \widetilde{t} \leq [0.5, 0.5]$, then F is an interval valued $(\in, \in \vee q)$-fuzzy Boolean filter of L.*

Proof Let F be an interval valued $(\in, \in \vee q)$-fuzzy Boolean filter of L and $[0,0] < \widetilde{t} \leq [0.5, 0.5]$. Then, by Theorem 3.10, $F_{\widetilde{t}}$ is known to be a filter of L. Since F is an interval valued $(\in, \in \vee q)$-fuzzy Boolean filter of L, $\widetilde{\mu_F}(x \vee x') \geq [0.5, 0.5] \geq \widetilde{t}$, and hence, $x \vee x' \in F_{\widetilde{t}}$. This shows that $F_{\widetilde{t}}$ is a Boolean filter of L.

Conversely, let F be an interval valued fuzzy set of L such that $U(F;t)(\neq \emptyset)$ is a Boolean filter of L for all $[0,0] < \widetilde{t} \leq [0.5, 0.5]$. Then by Theorem 3.10, F is an interval valued $(\in, \in \vee q)$-fuzzy filter of L. Since $F_{[0.5,0.5]}$ is a Boolean filter of L, for all $x \in L$, we have $x \vee x' \in F_{[0.5,0.5]}$, that is, $\widetilde{\mu_F}(x \vee x') \geq [0.5, 0.5]$. Therefore, F is an interval valued $(\in, \in \vee q)$-fuzzy Boolean filter of L. □

Naturally, a corresponding theorem of Theorem 4.3 can be similarly proved when $F_{\widetilde{t}}$ is a Boolean filter of L, for all $[0.5, 0.5] < \widetilde{t} \leq [1,1]$.

Theorem 4.4 *Let F be an interval valued fuzzy set of L. Then $F_{\widetilde{t}}(\neq \emptyset)$ is a Boolean filter of L for all $[0.5, 0.5] < \widetilde{t} \leq [1,1]$ if and only if for all $x, y \in L$,*

(F5) $\mathrm{rmax}\{\widetilde{\mu_F}(x \odot y), [0.5, 0.5]\} \geq \mathrm{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y)\}$;

(FF5) $\mathrm{rmax}\{\widetilde{\mu_F}(y), [0.5, 0.5]\} \geq \widetilde{\mu_F}(x)$ *if* $x \leq y$;

(F7) $\widetilde{\mu_F}(x \vee x') \geq \widetilde{t}$.

Lemma 4.5 *Every interval valued $(\in, \in \vee q)$-fuzzy Boolean filter F of L satisfies the following inequality:*

(F8) $\widetilde{\mu_F}(x \to z) \geq \mathrm{rmin}\{\widetilde{\mu_F}(x \to (z' \to y)), \widetilde{\mu_F}(y \to z), [0.5, 0.5]\}$, *for all* $x, y, z \in L$.

Proof In any MTL-algebra L, we have

$$y \to z \leq (z' \to y) \to (z' \to z) \leq (x \to (z' \to y)) \to (x \to (z' \to z)).$$

Hence, it follows from (FF3) that

$$\widetilde{\mu_F}((x \to (z' \to y)) \to (x \to (z' \to z))) \geq \mathrm{rmin}\{F(y \to z), [0.5, 0.5]\}.$$

Thus, from (FF4), we have

$$\begin{aligned}
&\widetilde{\mu_F}(x \to (z' \to z)) \\
&\quad \geq \mathrm{rmin}\{\widetilde{\mu_F}(x \to (z' \to y)), \widetilde{\mu_F}((x \to (z' \to y)) \to (x \to (z' \to z))), [0.5, 0.5]\} \\
&\quad \geq \mathrm{rmin}\{\widetilde{\mu_F}(x \to (z' \to y)), \widetilde{\mu_F}(y \to z), [0.5, 0.5]]\}.
\end{aligned}$$

Since $z'\vee z=((z'\to z)\to z)\wedge((z\to z')\to z')\leq(z'\to z)\to z$, we have

$$\widetilde{\mu_F}((z'\to z)\to z)\geq \mathrm{rmin}\{\widetilde{\mu_F}(z'\vee z),[0.5,0.5]\}=[0.5,0.5].$$

Since $x\to(z'\to z)\leq((z'\to z)\to z)\to(x\to z)$, it follows from (FF3) that

$$\widetilde{\mu_F}(((z'\to z)\to z)\to(x\to z))\geq \mathrm{rmin}\{\widetilde{\mu_F}(x\to(z'\to z)),[0.5,0.5]\}.$$

Thus,

$$\begin{aligned}\widetilde{\mu_F}(x\to z)&\geq \mathrm{rmin}\{\widetilde{\mu_F}((z'\to z)\to z),\widetilde{\mu_F}(((z'\to z)\to z)\to(x\to z)),[0.5,0.5]\}\\&\geq \mathrm{rmin}\{[0.5,0.5],\mathrm{rmin}\{\widetilde{\mu_F}(x\to(z'\to z)),[0.5,0.5]\},[0.5,0.5]\}\\&=\mathrm{rmin}\{\widetilde{\mu_F}(x\to(z'\to z)),[0.5,0.5]\}\\&\geq \mathrm{rmin}\{\widetilde{\mu_F}(x\to(z'\to y)),\widetilde{\mu_F}(y\to z),[0.5,0.5]\}.\end{aligned}$$

This completes the proof. □

The following lemmas are obvious and we omit the proofs.

Lemma 4.6 *If an interval valued $(\in,\in\vee q)$-fuzzy filter F of L satisfies the following inequality:*

(F9) $\widetilde{\mu_F}(x)\geq \mathrm{rmin}\{\widetilde{\mu_F}((x\to y)\to x),[0.5,0.5]\}$, *for all $x,y\in L$, then the filter is Boolean.*

Lemma 4.7 *If F is an interval valued $(\in,\in\vee q)$-fuzzy filter of L satisfying condition (F8), then F also satisfies condition (F9).*

By combining Lemmas 4.5, 4.6 and 4.7, we obtain a characterization theorem of an interval valued $(\in,\in\vee q)$-fuzzy Boolean filters.

Theorem 4.8 *Let F be an interval valued $(\in,\in\vee q)$-fuzzy filter of L. Then the following conditions are equivalent:*

(i) *F is Boolean;*

(ii) $\forall x,y,z\in L,\widetilde{\mu_F}(x\to z)\geq \mathrm{rmin}\{\widetilde{\mu_F}(x\to(z'\to y)),\widetilde{\mu_F}(y\to z),[0.5,0.5]\}$;

(iii) $\forall x,y\in L,\widetilde{\mu_F}(x)\geq \mathrm{rmin}\{\widetilde{\mu_F}((x\to y)\to x),[0.5,0.5]\}$.

Corollary 4.9 *Every interval valued $(\in,\in\vee q)$-fuzzy Boolean filter F of L satisfies the following inequality:*

(F10) $\forall x,y,z\in L,\widetilde{\mu_F}(x\to z)\geq \mathrm{rmin}\{\widetilde{\mu_F}(x\to(y\to z)),\widetilde{\mu_F}(x\to y),[0.5,0.5]\}$.

Definition 4.10 *An interval valued $(\in,\in\vee q)$-fuzzy filter F in L is called an interval valued $(\in,\in\vee q)$-fuzzy MV-filter of L if it satisfies the following inequality:*

(F11) $\forall x,y\in L,\widetilde{\mu_F}(((y\to x)\to x)\to y)\geq \mathrm{rmin}\{F(x\to y),[0.5,0.5]\}$.

Theorem 4.11 *In an MV-algebra, every interval valued $(\in,\in\vee q)$-fuzzy filter is an interval valued $(\in,\in\vee q)$-fuzzy MV-filter.*

Proof Let F be an interval valued $(\in,\in\vee q)$-fuzzy filter of an MV-algebra L. Since $x\to y\leq((x\to y)\to y)\to y=((y\to x)\to x)\to y$, we have $\widetilde{\mu_F}(((y\to x)\to x)\to y)\geq\widetilde{\mu_F}(x\to y)$, and so F is an interval valued $(\in,\in\vee q)$-fuzzy MV-filter of L.

Theorem 4.12 *Every $(\in, \in \vee q)$-fuzzy Boolean filter is an interval valued $(\in, \in \vee q)$-fuzzy MV-filter.*

Proof Let F be an $(\in, \in \vee q)$-fuzzy Boolean filter of L. Since $y \leq ((y \to x) \to x) \to y$, $(((y \to x) \to x) \to x \leq y \to x$. This implies that $x \to y \leq ((y \to x) \to x) \to ((y \to x) \to y)$ $= (y \to x) \to x) \to (((y \to x) \to x) \to y) \leq ((((y \to x) \to x) \to y) \to x) \to (((y \to x) \to x) \to x) \to y)$. It follows from Theorem 4.8 (iii) and (FF3) that $\widetilde{\mu_F}(((y \to x) \to x) \to y)$ $\geq \text{rmin}\{\widetilde{\mu_F}((((((y \to x) \to x) \to y) \to x) \to (((y \to x) \to x) \to y)), [0.5, 0.5]\} \geq \text{rmin}\{\widetilde{\mu_F}(x \to y), [0.5, 0.5]\}$. Therefore, F is an interval valued $(\in, \in \vee q)$-fuzzy filter MV-filter of L.

We now introduce the concept of $(\in, \in \vee q)$-fuzzy G-filters of an MTL-algebra.

Definition 4.13 *An interval valued $(\in, \in \vee q)$-fuzzy filter of an MTL-algebra L is called an interval valued $(\in, \in \vee q)$-fuzzy G-filter of L if it satisfies the following inequality:*

(F12) $\forall x, y \in L, \widetilde{\mu_F}(x \to y) \geq \text{rmin}\{\widetilde{\mu_F}(x \odot x \to y), [0.5, 0.5]\}$.

Example 4.14 Let $L = [0, 1]$ and define a product $\odot$ and a residuum $\to$ on L as follows:

$\to$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	d	1	b	b	d	1
b	0	a	1	a	d	1
c	d	1	1	1	d	1
d	a	1	1	1	1	1
1	0	a	b	c	d	1

$\odot$	0	a	b	c	d	1
0	0	0	0	0	0	0
a	0	a	c	c	0	a
b	0	c	b	c	d	b
c	0	c	c	c	0	c
d	0	0	d	0	0	d
1	0	a	b	c	d	1

Then $L(\wedge, \vee, \odot, \to, 0, 1)$ is clearly an MTL-algebra.

Define an interval valued fuzzy set F in L by

$$\widetilde{\mu_F}(x) = \begin{cases} \widetilde{\alpha} & \text{if } x \in \{1, a\}, \\ \widetilde{\beta} & \text{otherwise,} \end{cases}$$

where $\widetilde{\alpha} \geq [0.5, 0.5] \geq \widetilde{\beta}$.

Now, it is routine to verify that F is an interval valued $(\in, \in \vee q)$-fuzzy G-filter of L.

By using level G-filters of MTL-algebras, we can characterize the interval valued $(\in, \in \vee q)$-fuzzy G-filters as follows:

Theorem 4.15 *Let F be an interval valued $(\in, \in \vee q)$-fuzzy G-filter of L. Then for all $[0, 0] < \widetilde{t} \leq [0.5, 0.5]$, $F_{\widetilde{t}}$ is an empty set or a G-filter of L. Conversely, if F is an interval valued fuzzy set of L such that $F_{\widetilde{t}} (\neq \emptyset)$ is a G-filter of L for all $[0, 0] < \widetilde{t} \leq [0.5, 0.5]$, then F is an interval valued $(\in, \in \vee q)$-fuzzy G-filter of L.*

Proof The proof is similar to the proof of Theorem 4.3.

Naturally, a corresponding result of Theorem 4.15 can be similarly obtained when $F_{\widetilde{t}}$ is a G-filter of L for all $[0.5, 0.5] < \widetilde{t} \leq [1, 1]$.

Theorem 4.16 *Let F be an interval valued fuzzy set of L. Then $F_{\widetilde{t}}(\neq \emptyset)$ is a G-filter of L for all $[0.5, 0.5] < \widetilde{t} \leq [1, 1]$ if and only if for all $x, y \in L$,*

(F5) $\text{rmax}\{\widetilde{\mu_F}(x \odot y), [0.5, 0.5]\} \geq \text{rmin}\{\widetilde{\mu_F}(x), \widetilde{\mu_F}(y)\}$;

(FF5) $\text{rmax}\{\widetilde{\mu_F}(y), [0.5, 0.5]\} \geq \widetilde{\mu_F}(x)$ *if* $x \leq y$;

(F13) $\text{rmax}\{\widetilde{\mu_F}(x \to y), [0.5, 0.5]\} \geq \widetilde{\mu_F}(x \odot x \to y)$.

Theorem 4.17 *An interval valued fuzzy set in L is an interval valued $(\in, \in \vee q)$-fuzzy Boolean filter if and only if it is both an interval valued $(\in, \in \vee q)$-fuzzy G-filter and an interval valued $(\in, \in \vee q)$-fuzzy MV-filter of L.*

Proof Suppose that F is an interval valued $(\in, \in \vee q)$-fuzzy Boolean filter of L. Then, by Theorem 4.12, F is an interval valued $(\in, \in \vee q)$-fuzzy MV-filter of L. Now, for any $x, y \in L$, by Theorem 3.7(F4) and Corollary 4.9(F10), we have

$$\begin{aligned}
\widetilde{\mu_F}(x \to y) &\geq \text{rmin}\{\widetilde{\mu_F}(x \to (x \to y)), \widetilde{\mu_F}(x \to x), [0.5, 0.5]\} \\
&= \text{rmin}\{\widetilde{\mu_F}(x \to (x \to y)), \widetilde{\mu_F}(1), [0.5, 0.5]\} \\
&= \text{rmin}\{\widetilde{\mu_F}(x \to (x \to y)), [0.5, 0.5]\} \\
&= \text{rmin}\{\widetilde{\mu_F}(x \odot x \to y), [0.5, 0.5]\}.
\end{aligned}$$

Thus, F is an interval valued $(\in, \in \vee q)$-fuzzy G-filter of L.

Conversely, let F be an interval valued $(\in, \in \vee q)$-fuzzy G-filter and interval valued $(\in, \in \vee q)$-fuzzy MV-filter of L. Since $(x \to y) \to x \leq ((x \to y) \odot (x \to y)) \to y$, for $x, y \in L$, it follows from (FF3) and (F12) that

$$\begin{aligned}
\widetilde{\mu_F}((x \to y) \to y) &\geq \text{rmin}\{\widetilde{\mu_F}((x \to y) \odot (x \to y) \to y), [0.5, 0.5]\} \\
&\geq \text{rmin}\{\widetilde{\mu_F}((x \to y) \to x), [0.5, 0.5]\}. \qquad (*)
\end{aligned}$$

And, since $(x \to y) \to y \leq (x \to (x \to y)) \to (x \to y)$, we obtain

$$((x \to (x \to y)) \to (x \to y)) \to x \leq ((x \to y) \to y) \to x.$$

Hence, by (FF3) and (F11), we have

$$\begin{aligned}
\widetilde{\mu_F}(((x \to y) \to y) \to x) &\geq \text{rmin}\{\widetilde{\mu_F}(((x \to (x \to y)) \to (x \to y)) \to x), [0.5, 0.5]\} \\
&\geq \text{rmin}\{\widetilde{\mu_F}((x \to y) \to x), [0.5, 0.5]\}. \qquad (**)
\end{aligned}$$

Consequently, by $(*)$, $(**)$ and Theorem 3.7 (FF4), we deduce that

$$\begin{aligned}
\widetilde{\mu_F}(x) &\geq \text{rmin}\{\widetilde{\mu_F}((x \to y) \to y), \widetilde{\mu_F}(((x \to y) \to y) \to x), [0.5, 0.5]\} \\
&\geq \text{rmin}\{\widetilde{\mu_F}((x \to y) \to x), \widetilde{\mu_F}((x \to y) \to x), [0.5, 0.5]\} \\
&= \text{rmin}\{\widetilde{\mu_F}((x \to y) \to x), [0.5, 0.5]\}.
\end{aligned}$$

Therefore, by Theorem 4.8(iii), F is an interval valued $(\in, \in \vee q)$-fuzzy Boolean filter of L. This completes the proof. □

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