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Permanence and Global Attractivity of a Discrete Nicholson's Blowflies Model with Delay

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Abstract In this paper, we consider a discrete Nicholson's blowflies model with delay. By constructing suitable Lyapunov functional, a sufficient condition for the permanence and global attractivity of the system is obtained. An example together with its numerical simulation shows the feasibility of our main results.

Keywords discrete; Nicholson's blowflies model; delay; permanence; global attractivity

MR(2010) Subject Classification 34D23; 34K20; 92D25

1. Introduction

One of the most popular population models is the well known Nicholson's blowflies model

$$x'(t) = -\alpha x(t) + \beta x(t-\tau)e^{-\gamma x(t-\tau)}, \tag{1.1}$$

which was proposed by Gurney et al. [1]. Here $x(t)$ is the size of the population at time t, β is the maximum per capita daily egg production, $(1/\gamma)$ is the size at which the blowfly population reproduces at its maximum rate, α is the pair capita daily adult death rate and τ is the generation time. For more background of model (1.1), please see [2–4].

Though lots has been done for population models described by differential equations, it has been found that the dynamics of their discrete analogues is rather complex and richer than those of continuous ones. In addition, discrete time models can also provide efficient computational models of continuous time models for numerical simulations. It is reasonable to study discrete time models governed by difference equations. For similar work in this direction, we refer the reader to [5–11] and the references cited therein.

[12,13] studied the dynamic behavior of the following autonomous discrete differential equation

$$x(n+1) - x(n) = -\alpha x(n) + \beta x(n-\tau)e^{-\gamma x(n-\tau)}. \tag{1.2}$$

They obtained sufficient conditions for the global attractivity of all positive solutions about the positive equilibrium. The oscillation about the positive equilibrium was also discussed. Multiple stability results for autonomous model are well known, however there are only a few results for

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the global stability of non-autonomous equations. It has been found that the non-autonomous discrete systems can demonstrate quite rich and complicated dynamics. Recently, some scholars [14–16] paid attention to the non-autonomous discrete models, since such kind of model could be more appropriate. However, few papers have been published on the global attractivity of the Nicholson's blowflies model with discrete delay. This is the main motivation of this paper.

In this paper, we consider a discrete Nicholson's blowflies with delay:

$$x(n+1) - x(n) = -\alpha(n)x(n) + \beta(n)x(n-\tau)e^{-\gamma(n)x(n-\tau)}. \tag{1.3}$$

We consider system (1.3) with the following initial conditions:

$$x(\theta) = \varphi(\theta) > 0, \quad \theta \in N[-\tau, 0] = -\tau, -\tau+1, \ldots, 0. \tag{1.4}$$

Our main purpose is to establish a series of sufficient conditions for the boundedness, permanence and global attrativity of solutions of model (1.3) and (1.4) by developing the research method given in [6] for a class of discrete logistic equation. To the best of our knowledge, this is the first paper to study the global attractivity of a discrete Nicholson's blowflies model by developing some new analysis technique. Compared with some earlier works on the discrete Nicholson's blowflies models, our approach is novel. The result of this paper is completely new and improves some existing results in the literature.

2. Preliminaries

Throughout this paper, we assume that

(H_1) $\{\alpha(n)\}$, $\{\beta(n)\}$ and $\{\gamma(n)\}$ are all bounded nonnegative sequences such that

$$0 < \alpha^l < \alpha^u < 1, \quad 0 < \beta^l < \beta^u, \; 0 < \gamma^l < \gamma^u.$$

Here, for any bounded sequence $\{h(n)\}$, $h^u = \sup_{n\in N} h(n)$ and $h^l = \inf_{n\in N} h(n)$.

It is not difficult to see that solutions of (1.3) are all well defined for all $n \geq 0$ and satisfy $x(n) > 0$.

Lemma 2.1 ([17]) *Assume that $A > 0$ and $y(0) > 0$, and further suppose that*

(1) $y(n+1) \leq Ay(n) + B(n)$, $n = 1, 2, \ldots$. Then for any integer $k \leq n$,

$$y(n) \leq A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1).$$

Especially, if $A < 1$ and B is bounded above with respect to M, then

$$\limsup_{n\to\infty} y(n) \leq \frac{M}{1-A}.$$

(2) $y(n+1) \geq Ay(n) + B(n)$, $n = 1, 2, \ldots$. Then for any integer $k \leq n$,

$$y(n) \geq A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1).$$

Especially, if $A < 1$ and B is bounded below with respect to m, then

$$\limsup_{n\to\infty} y(n) \geq \frac{m}{1-A}.$$

Lemma 2.2 *Let $x\{n\}$ be a solution of (1.3) with the initial condition (1.4). Then*

$$\limsup_{n\to\infty} x(n) \leq M, \quad \text{where} \quad M = \frac{\beta^u}{\alpha^l \gamma^l e}.$$

Proof Let $x\{n\}$ be an arbitrary solution of system (1.3). From system (1.3), it follows that

$$\begin{aligned} x(n+1) &= (1-\alpha(n))x(n) + \beta(n)x(n-\tau)e^{-\gamma(n)x(n-\tau)} \\ &\leq (1-\alpha^l)x(n) + \beta^u x(n-\tau)e^{-\gamma^l x(n-\tau)}. \end{aligned} \tag{2.1}$$

It follows from (2.1) and the fact that $\sup_{u\geq 0} ue^{-\gamma u} = \frac{1}{\gamma e}$, we have

$$x(n+1) \leq (1-\alpha^l)x(n) + \frac{\beta^u}{\gamma^l e}. \tag{2.2}$$

By applying Lemma 2.1 to (2.2), it immediately follows that

$$\limsup_{n\to\infty} x(n) \leq \frac{\beta^u}{\alpha^l \gamma^l e} \stackrel{\text{def}}{=} M.$$

This completes the proof of Lemma 2.2. □

3. Main results

Following we will state and prove the main results of this paper.

Theorem 3.1 *Assume that (H_1) holds; assume further that*

(H_2) $\beta^l > \alpha^u$ holds, then system (1.3) with the initial condition (1.4) is permanent. That is, there exist positive constants M, m, which are independent of the solution of the system, such that any positive solution $x(n)$ of system (1.3) and (1.4) satisfies

$$m \leq \liminf_{n\to\infty} x(n) \leq \limsup_{n\to\infty} x(n) \leq M.$$

Proof From Lemma 2.2, we obtain that

$$\limsup_{n\to\infty} x(n) \leq M = \frac{\beta^u}{\alpha^l \gamma^l e}.$$

Now we first prove that any positive solution $x(n)$ of system (1.3) and (1.4) satisfies

$$\liminf_{n\to\infty} x(n) > 0. \tag{3.1}$$

Suppose, for the sake of contradiction, $\liminf_{n\to\infty} x(n) = 0$. We define

$$t(n) = \max\{s : s \leq n,\ x(s) = \min_{0\leq\xi\leq n} x(\xi)\}.$$

Observe that $t(n) \to \infty$ as $n \to \infty$ and

$$\lim_{n\to\infty} x(t(n)) = 0. \tag{3.2}$$

However, $x(t(n)) = \min_{0\leq\xi\leq n} x(\xi)$, so $x(t(n+1)) - x(t(n)) \leq 0$, which implies that

$$0 \geq x(t(n+1)) - x(t(n)) = -\alpha(t(n))x(t(n)) + \beta(t(n))x(t(n)-\tau)e^{-\gamma(t(n))x(t(n)-\tau)}$$

$$\geq -\alpha^u x(t(n)) + \beta^l x(t(n)-\tau)e^{-\gamma^u x(t(n)-\tau)}.$$

Therefore,

$$0 = \lim_{n\to\infty} \alpha^u x(t(n)) \geq \lim_{n\to\infty} \beta^l x(t(n)-\tau)e^{-\gamma^u x(t(n)-\tau)}. \tag{3.3}$$

Hence, $\lim_{n\to\infty} x(t(n)-\tau) = 0$. This, together with (3.3) and the definition of $t(n)$, we have

$$\alpha^u \geq \liminf_{n\to\infty} \frac{\beta^l x(t(n)-\tau)}{x(t(n))} e^{-\gamma^u x(t(n)-\tau)} \geq \liminf_{n\to\infty} \beta^l e^{-\gamma^u x(t(n)-\tau)} = \beta^l,$$

which contradicts (H_2). Hence (3.1) holds.

Next we prove that there exists a positive constant m such that $\liminf_{n\to\infty} x(n) \geq m$. Define

$$\eta = \liminf_{n\to\infty} x(n), \quad h = \min\{g(\eta), g(M)\} \text{ where } g(x) = xe^{-\gamma^u x}.$$

This combining with system (1.3) and $0 < \alpha^u < 1$ leads to

$$\eta = \liminf_{n\to\infty} x(n) \geq \liminf_{n\to\infty} \big[(x(0) - \frac{\beta^l h}{\alpha^u})(\alpha^u - 1)^n + \frac{\beta^l h}{\alpha^u}\big] = \frac{\beta^l h}{\alpha^u}. \tag{3.4}$$

If $h = g(\eta)$, then $\eta \geq \frac{\beta^l}{\alpha^u}\eta e^{-\gamma^u \eta}$. So we have $\eta \geq \frac{1}{\gamma^u}\ln\frac{\beta^l}{\alpha^u}$. If $h = g(M)$, it follows from (3.4) that $\eta \geq \frac{\beta^l}{\alpha^u} M e^{-\alpha^u M}$. The above inequality leads to

$$\liminf_{n\to\infty} x(n) \geq \min\{\frac{1}{\gamma^u}\ln\frac{\beta^l}{\alpha^u}, \frac{\beta^l}{\alpha^u} M e^{-\alpha^u M}\} \overset{\text{def}}{=} m. \tag{3.5}$$

This completes the proof of Theorem 3.1. □

Theorem 3.2 *Assume that (H_1) and (H_2) hold; assume further that*

(H_3) $\gamma^l \cdot m > 1$ *and*

(H_4) $e^4\alpha^l(2-\alpha^u) > 2e^2(1-\alpha^l)\beta^u + 2e^2\tau\alpha^u\beta^u + (2\tau+1)\beta^u$ *hold, then system (1.3) with the initial condition (1.4) is globally attractive. That is, for any positive solutions $x(n)$ and $y(n)$ of system (1.3), we have* $\lim_{n\to\infty}(x(n)-y(n)) = 0$.

Proof For any solutions $x(n)$ and $y(n)$ of system (1.3), it follows from Theorem 3.1 that

$$m \leq \liminf_{n\to\infty} x(n) \leq \limsup_{n\to\infty} x(n) \leq M, \quad m \leq \liminf_{n\to\infty} y(n) \leq \limsup_{n\to\infty} y(n) \leq M.$$

For any positive constant $\varepsilon > 0$ small enough, there exists an integer n_0 such that for all $n \geq n_0$,

$$m \leq x(n), \quad y(n) \leq M. \tag{3.6}$$

Using the mean value theorem, we get

$$x(n)e^{-x(n)} - y(n)e^{-y(n)} = (1-\theta(n))e^{-\theta(n)}\big(x(n)-y(n)\big), \tag{3.7}$$

where $\theta(n)$ lies between $x(n)$ and $y(n)$. Let

$$W_1(n) = x(n) - y(n) + \sum_{s=n-\tau}^{n-1} \beta(s+\tau)\big(x(s)e^{-\gamma(s+\tau)x(s)} - y(s)e^{-\gamma(s+\tau)y(s)}\big).$$

Then, from system (1.3), we obtain

$$W_1(n+1) = \big(1-\alpha(n)\big)\big(x(n)-y(n)\big) + \beta(n)\big(x(n-\tau)e^{-\gamma(n)x(n-\tau)} - y(n-\tau)e^{-\gamma(n)y(n-\tau)}\big) +$$

$$\sum_{s=n+1-\tau}^{n} \beta(s+\tau)\big(x(s)e^{-\gamma(s+\tau)x(s)} - y(s)e^{-\gamma(s+\tau)y(s)}\big).$$

So, we have

$$\begin{aligned}\Delta W_1(n) =& W_1(n+1) - W_1(n)\\ =& -\alpha(n)\big(x(n)-y(n)\big) + \beta(n+\tau)\big(x(n)e^{-\gamma(n+\tau)x(n)} - y(n)e^{-\gamma(n+\tau)y(n)}\big).\end{aligned}$$

Define $V_1(n) = W_1^2(n)$. Therefore

$$\begin{aligned}&\Delta V_1(n) = W_1^2(n+1) - W_1^2(n) = \Delta W_1(n)\big(W_1(n+1)+W_1(n)\big)\\
&\Big(-\alpha(n)\big(x(n)-y(n)\big) + \beta(n+\tau)\big(x(n)e^{-\gamma(n+\tau)x(n)} - y(n)e^{-\gamma(n+\tau)y(n)}\big)\Big)\cdot\\
&\quad\Big(\big(2-\alpha(n)\big)\big(x(n)-y(n)\big) + \beta(n+\tau)\big(x(n)e^{-\gamma(n+\tau)x(n)} - y(n)e^{-\gamma(n+\tau)y(n)}\big)+\\
&\quad 2\sum_{s=n-\tau}^{n-1} \beta(s+\tau)\big(x(s)e^{-\gamma(s+\tau)x(s)} - y(s)e^{-\gamma(s+\tau)y(s)}\big)\Big)\\
&= -\alpha(n)(2-\alpha(n))\big(x(n)-y(n)\big)^2+\\
&\quad 2(1-\alpha(n))\big(x(n)-y(n)\big)\Big(\frac{\beta(n+\tau)}{\gamma(n+\tau)}\big(\gamma(n+\tau)x(n)e^{-\gamma(n+\tau)x(n)} - \gamma(n+\tau)y(n)e^{-\gamma(n+\tau)y(n)}\big)\Big)-\\
&\quad 2\alpha(n)\big(x(n)-y(n)\big)\sum_{s=n-\tau}^{n-1}\frac{\beta(s+\tau)}{\gamma(s+\tau)}\big(\gamma(s+\tau)x(s)e^{-\gamma(s+\tau)x(s)} - \gamma(s+\tau)y(s)e^{-\gamma(s+\tau)y(s)}\big)+\\
&\quad \Big(\frac{\beta(n+\tau)}{\gamma(n+\tau)}\big(\gamma(n+\tau)x(n)e^{-\gamma(n+\tau)x(n)} - \gamma(n+\tau)y(n)e^{-\gamma(n+\tau)y(n)}\big)\Big)^2+\\
&\quad 2\frac{\beta(n+\tau)}{\gamma(n+\tau)}\big(\gamma(n+\tau)x(n)e^{-\gamma(n+\tau)x(n)} - \gamma(n+\tau)y(n)e^{-\gamma(n+\tau)y(n)}\big)\cdot\\
&\quad \Big(\sum_{s=n-\tau}^{n-1}\frac{\beta(s+\tau)}{\gamma(s+\tau)}\big(\gamma(s+\tau)x(s)e^{-\gamma(s+\tau)x(s)} - \gamma(s+\tau)y(s)e^{-\gamma(s+\tau)y(s)}\big)\Big).\end{aligned}\tag{3.8}$$

By applying (3.7) to (3.8), we have

$$\begin{aligned}\Delta V_1(n) =& -\alpha(n)(2-\alpha(n))\big(x(n)-y(n)\big)^2+\\
& 2(1-\alpha(n))\beta(n+\tau)\big(1-\theta(n)\gamma(n+\tau)\big)e^{-\theta(n)\gamma(n+\tau)}\big(x(n)-y(n)\big)^2-\\
& 2\alpha(n)\big(x(n)-y(n)\big)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(1-\theta(s)\gamma(s+\tau)\big)e^{-\theta(s)\gamma(s+\tau)}\big(x(s)-y(s)\big)+\\
& \Big(\beta(n+\tau)\big(1-\theta(n)\gamma(n+\tau)\big)e^{-\theta(n)\gamma(n+\tau)}\big(x(n)-y(n)\big)\Big)^2+\\
& 2\beta(n+\tau)\big(1-\theta(n)\gamma(n+\tau)\big)e^{-\theta(n)\gamma(n+\tau)}\big(x(n)-y(n)\big)\cdot\\
& \Big(\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(1-\theta(s)\gamma(s+\tau)\big)e^{-\theta(s)\gamma(s+\tau)}\big(x(s)-y(s)\big)\Big).\end{aligned}\tag{3.9}$$

Hence, applying the fact that $2ab \le a^2+b^2$ to (3.9), we obtain

$$\begin{aligned}\Delta V_1(n) \le& -\alpha(n)(2-\alpha(n))\big(x(n)-y(n)\big)^2+\\
& 2(1-\alpha(n))\beta(n+\tau)\big(1-\theta(n)\gamma(n+\tau)\big)e^{-\theta(n)\gamma(n+\tau)}\big(x(n)-y(n)\big)^2+\end{aligned}$$

$$\alpha(n)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(1-\theta(s)\gamma(s+\tau)\big)e^{-\theta(s)\gamma(s+\tau)}\big(x(n)-y(n)\big)^2+$$
$$\alpha(n)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(1-\theta(s)\gamma(s+\tau)\big)e^{-\theta(s)\gamma(s+\tau)}\big(x(s)-y(s)\big)^2+$$
$$\Big(\beta(n+\tau)\big(1-\theta(n)\gamma(n+\tau)\big)e^{-\theta(n)\gamma(n+\tau)}\big(x(n)-y(n)\big)\Big)^2+$$
$$\beta(n+\tau)\big(1-\theta(n)\gamma(n+\tau)\big)e^{-\theta(n)\gamma(n+\tau)}\cdot$$
$$\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(1-\theta(s)\gamma(s+\tau)\big)e^{-\theta(s)\gamma(s+\tau)}\big(x(n)-y(n)\big)^2+$$
$$\beta(n+\tau)\big(1-\theta(n)\gamma(n+\tau)\big)e^{-\theta(n)\gamma(n+\tau)}\cdot$$
$$\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(1-\theta(s)\gamma(s+\tau)\big)e^{-\theta(s)\gamma(s+\tau)}\big(x(s)-y(s)\big)^2. \tag{3.10}$$

According to (H_3), (3.6) and the fact that $\max_{x\in[1,+\infty]}(1-x)e^{-x}=\frac{1}{e^2}$, for $n\geq n_0$, we have

$$\begin{aligned}\Delta V_1(n)\leq &-\alpha(n)(2-\alpha(n))\big(x(n)-y(n)\big)^2+\frac{2}{e^2}(1-\alpha(n))\beta(n+\tau)\big(x(n)-y(n)\big)^2+\\ &\frac{1}{e^2}\alpha(n)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(x(n)-y(n)\big)^2+\frac{1}{e^2}\alpha(n)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(x(s)-y(s)\big)^2+\\ &\frac{1}{e^4}\Big(\beta(n+\tau)\big(x(n)-y(n)\big)\Big)^2+\frac{1}{e^4}\beta(n+\tau)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(x(n)-y(n)\big)^2+\\ &\frac{1}{e^4}\beta(n+\tau)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(x(s)-y(s)\big)^2.\end{aligned} \tag{3.11}$$

Let

$$V_2(n)=\sum_{u=n}^{n-1+\tau}\alpha(u)\sum_{s=u-\tau}^{n-1}\beta(s+\tau)\big(x(s)-y(s)\big)^2.$$

Then

$$\begin{aligned}\Delta V_2(n)&=V_2(n+1)-V_2(n)\\ &=\sum_{u=n+1}^{n+\tau}\alpha(u)\beta(n+\tau)\big(x(n)-y(n)\big)^2-\alpha(n)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(x(s)-y(s)\big)^2.\end{aligned} \tag{3.12}$$

Let

$$V_3(n)=\sum_{u=n}^{n-1+\tau}\beta(u+\tau)\sum_{s=u-\tau}^{n-1}\beta(s+\tau)\big(x(s)-y(s)\big)^2.$$

Then

$$\begin{aligned}\Delta V_3(n)&=V_3(n+1)-V_3(n)\\ &=\sum_{u=n+1}^{n+\tau}\beta(u+\tau)\beta(n+\tau)\big(x(n)-y(n)\big)^2-\beta(n+\tau)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(x(s)-y(s)\big)^2.\end{aligned} \tag{3.13}$$

Define

$$V(n) = V_1(n) + \frac{1}{e^2}V_2(n) + \frac{1}{e^4}V_3(n).$$

Then it follows from (3.12) and (3.13) that

$$\begin{aligned}\Delta V(n) \leq& -\alpha(n)(2-\alpha(n))\big(x(n)-y(n)\big)^2 + \frac{2}{e^2}(1-\alpha(n))\beta(n+\tau)\big(x(n)-y(n)\big)^2 +\\ &\frac{1}{e^2}\alpha(n)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(x(n)-y(n)\big)^2 + \frac{1}{e^2}\sum_{u=n+1}^{n+\tau}\alpha(u)\beta(n+\tau)\big(x(n)-y(n)\big)^2 +\\ &\frac{1}{e^4}\Big(\beta(n+\tau)\big(x(n)-y(n)\big)\Big)^2 + \frac{1}{e^4}\beta(n+\tau)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(x(n)-y(n)\big)^2 +\\ &\frac{1}{e^4}\sum_{u=n+1}^{n+\tau}\beta(u+\tau)\beta(n+\tau)\big(x(n)-y(n)\big)^2\\ =& -\alpha(n)(2-\alpha(n))\big(x(n)-y(n)\big)^2 + \frac{2}{e^2}(1-\alpha(n))\beta(n+\tau)\big(x(n)-y(n)\big)^2 +\\ &\frac{1}{e^2}\alpha(n)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(x(n)-y(n)\big)^2 + \frac{1}{e^2}\sum_{u=n+1}^{n+\tau}\alpha(u)\beta(n+\tau)\big(x(n)-y(n)\big)^2 +\\ &\frac{1}{e^4}\beta(n+\tau)\sum_{s=n-\tau}^{n-1}\beta(s+\tau)\big(x(n)-y(n)\big)^2 + \frac{1}{e^4}\sum_{u=n}^{n+\tau}\beta(u+\tau)\beta(n+\tau)\big(x(n)-y(n)\big)^2.\end{aligned} \tag{3.14}$$

From condition (H_4), we can choose a $\delta > 0$ small enough such that

$$\alpha^l(2-\alpha^u) - \big(\frac{2}{e^2}(1-\alpha^l)\beta^u + \frac{2\tau}{e^2}\alpha^u\beta^u + \frac{2\tau+1}{e^4}\beta^u\big) > \delta. \tag{3.15}$$

From (3.14) and (3.15), we obain

$$\begin{aligned}\Delta V(n) \leq& -\alpha^l(2-\alpha^u)\big(x(n)-y(n)\big)^2 + \frac{2}{e^2}(1-\alpha^l)\beta^u\big(x(n)-y(n)\big)^2 +\\ &\frac{1}{e^2}\alpha^u\beta^u\tau\big(x(n)-y(n)\big)^2 + \frac{1}{e^2}\alpha^u\beta^u\tau\big(x(n)-y(n)\big)^2 +\\ &\frac{1}{e^4}\beta^u\beta^u\tau\big(x(n)-y(n)\big)^2 + \frac{1}{e^4}\beta^u\beta^u(\tau+1)\big(x(n)-y(n)\big)^2\\ =& -\Big(\alpha^l(2-\alpha^u) - \big(\frac{2}{e^2}(1-\alpha^l)\beta^u + \frac{2\tau}{e^2}\alpha^u\beta^u + \frac{2\tau+1}{e^4}\beta^u\big)\Big)\big(x(n)-y(n)\big)^2\\ \leq& -\delta\big(x(n)-y(n)\big)^2.\end{aligned} \tag{3.16}$$

Summating both sides of the above inequalities from $n_0+\tau$ to n, we have

$$\sum_{s=n_0+\tau}^{n}\big(V(s+1)-V(s)\big) \leq -\delta\sum_{s=n_0+\tau}^{n}\big(x(s)-y(s)\big)^2,$$

which implies

$$V(n+1) + \delta\sum_{s=n_0+\tau}^{n}\big(x(s)-y(s)\big)^2 \leq V(n_0+\tau),$$

that is

$$\sum_{s=n_0+\tau}^{n}\big(x(s)-y(s)\big)^2 \leq \frac{V(n_0+\tau)}{\delta}.$$

It follows from (3.6) that $V_i(n_0+\tau),\ i=1,2,3$ are all bounded. Hence

$$\sum_{s=n_0+\tau}^{n}\big(x(s)-y(s)\big)^2\le\frac{V(n_0+\tau)}{\delta}<+\infty,$$

which means that

$$\sum_{s=n_0+\tau}^{+\infty}\big(x(s)-y(s)\big)^2\le\frac{V(n_0+\tau)}{\delta}<+\infty.$$

This implies that $\lim_{n\to\infty}(x(n)-y(n))^2=0$, or $\lim_{n\to\infty}(x(n)-y(n))=0$. This completes the proof of Theorem 3.2. □

4. Example

The following example shows the feasibility of our main result.

Example 4.1 Consider the following equation

$$x(n+1)-x(n)=-\big(0.8+0.01\sin(\sqrt{2}n)\big)x(n)+\big(2.79+0.01\sin(\sqrt{3}n)\big)\cdot x(n-1)e^{-\big(0.72+0.01\sin n\big)x(n-1)}. \tag{4.1}$$

It is easy to calculate that $M\approx 1.8365$, $m\approx 1.424$, $\alpha^u=0.81$, $\alpha^l=0.79$, $\beta^u=2.8$, $\beta^l=2.78$, $\gamma^u=0.73$, $\gamma^l=0.71$, $\gamma^l m\approx 1.0111>1$, $e^4\alpha^l(2-\alpha^u)\approx 51.3277$, $2e^2(1-\alpha^l)\beta^u+2e^2\tau\alpha^u\beta^u+(2\tau+1)\beta^u\approx 50.6063$. Clearly, conditions (H_1)–(H_4) are satisfied. It follows from Theorem 3.2 that system (4.1) is globally attractive (see Figure 1).

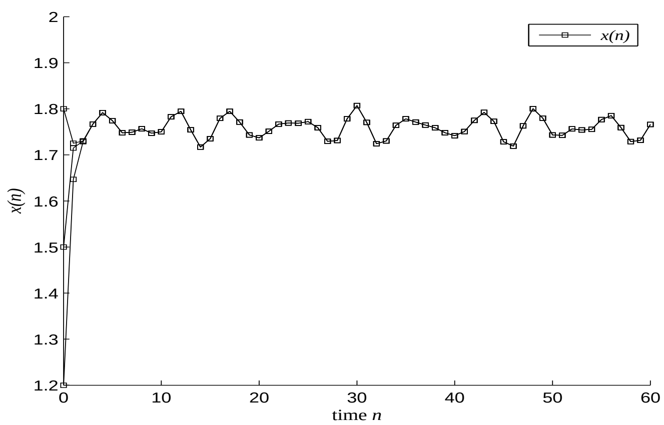

Figure 1 Dynamic behavior of the solution $x(n)$ of system (4.1) with the initial conditions $\varphi(\theta)=1.2,\ 1.5,$ and 1.8 for $\theta=-1,\ 0$, respectively.

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