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On Transcendental Entire Solutions of Systems of Complex Differential-Difference Equations

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Abstract By using the Nevanlinna value distribution theory, we will mainly investigate the form of entire solutions with finite order on a type of system of differential-difference equations and a type of differential-difference equations, two interesting results are obtained. And it extends some results concerning complex differential (difference) equations to the systems of differential-difference equations.

Keywords differential-difference equation; transcendental entire solutions; meromorphic functions

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1. Introduction

As we all know, the complex difference equation theory is an important topic in complex analysis. Some results can be found in [1], where Nevanlinna theory is an effective research tool. Let $\mathbb{C}$ be the complex plane and $f(z)$ be a meromorphic function on $\mathbb{C}$. For a meromorphic function $f(z)$, we assume that the reader is familiar with the standard notations and results, such as $m(r,f(z)), n(r,f(z)), N(r,f(z))$ and $T(r,f(z))$ denote the proximity function, the non-integrated counting function, the counting function and the characteristic function of $f(z)$, respectively. Especially, for the integrated counting function for distinct poles of $f(z)$ we use the notations $\overline{N}(r,f(z))$, the growth order of meromorphic function $f(z)$ is denoted by $\rho(f(z))$.

Let $h(z)$ be another meromorphic function. If $T(r,h(z)) = S(r,f)$, the meromorphic function $h(z)$ is said to be a small function of $f(z)$, where $S(r,f)$ is used to denote any quantity that satisfies $S(r,f) = o(T(r,f))$ as $r \to \infty$, possibly outside of a set of finite logarithmic measure in $\mathbb{R}^+$.

In the complex differential equation theory, Nevanlinna value distribution theory of meromorphic functions plays an important role, it has been extensively applied to resolve growth, value distribution [1], and solvability of meromorphic solutions of linear and nonlinear differential equations [2].

In 2004, Yang and Li [3] discussed the form of solutions of the following equation

$$f(z)^2 + (L(f))^2 = a(z),$$

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where n is a positive integer, $a(z), b_0(z), b_1(z), \ldots, b_n(z)$ are polynomials, and $b_n(z)$ is a nonzero constant, $a(z) \not\equiv 0$, $L(f) = \sum_{k=0}^{n} b_k(z) f^{(k)}(z)$. They obtained

Theorem 1.1 ([3]) *A transcendental meromorphic solution of the above equation must have the form*

$$f(z) = \frac{1}{2}(P(z)e^{R(z)} + Q(z)e^{-R(z)}),$$

where $P(z), Q(z), R(z)$ are polynomials, and $P(z)Q(z) = a(z)$.

Recently, some authors pay high attention to dealing with the existence or the growth of meromorphic solutions of difference equations and many results on meromorphic solutions of complex difference equations are rapidly obtained, such as [4–11] and so on.

In 2012, Liu et al. [10] considered Fermat type differential-difference equation

$$f'(z)^2 + f(z+c)^2 = 1, \tag{1.1}$$

and the following result is obtained.

Theorem 1.2 ([10]) *The transcendental entire solutions with finite order of the differential-difference equation (1.1) must satisfy $f(z) = \sin(z \pm iB)$, where B is a constant and $c = 2k\pi$ or $c = 2k\pi + \pi$, k is an integer.*

It is natural to ask what we can know about the solutions of the differential-difference equations of the following form

$$(w''(z) - w(z))^2 + w(z+c)^2 = 1. \tag{1.2}$$

Corresponding to the question, this paper first is devoted to considering the form of entire solutions of the above differential-difference equation.

For (1.2), we can obtain

Theorem 1.3 *Let $w(z)$ be transcendental entire solutions with $\rho(w(z)) < \infty$ of the differential-difference equation (1.2). Then $w(z)$ must satisfy*

$$w(z) = \frac{e^{\sqrt{2}z+b} + e^{-\sqrt{2}z-b}}{2} \quad or \quad w(z) = \frac{e^{-\sqrt{2}z+b} + e^{\sqrt{2}z-b}}{2},$$

where b is a constant, $c = \pm\frac{\frac{\pi i}{2}+2k\pi i}{\sqrt{2}}$, k is an integer.

We state some remarks below.

Remark 1.4 The following Example 1.5 shows that it satisfies Theorem 1.3.

Example 1.5 $w(z) = \frac{e^{\sqrt{2}z+\pi i}+e^{-\sqrt{2}z-\pi i}}{2}$ is a transcendental entire solution of the complex differential-difference equation of the form

$$(w''(z) - w(z))^2 + w(z - \frac{\pi i}{2\sqrt{2}})^2 = 1,$$

where $a = \sqrt{2}, b = \pi i, c = -\frac{\pi i}{2\sqrt{2}}$.

Compared with difference equation, we know that the system of difference equations is essentially different from single difference equation. The form of systems of difference equations is more complex and the research of them is more difficult. Thus, we must improve the method to solve different situations in the process of the theorem's proof.

A number of recent papers had discussed the existence or growth of some types of systems of complex difference equations, and obtained some results [12–15]. In the following, our additional aim is to investigate entire solutions with finite order of the system of differential-difference equations of the form

$$\begin{cases} (w_1''(z) - w_1(z))^2 + w_2(z+c)^2 = 1, \\ (w_2''(z) - w_2(z))^2 + w_1(z+c)^2 = 1, \end{cases} \tag{1.3}$$

here c is a nonzero constant.

The growth order of meromorphic solutions (w_1, w_2) of the system (1.3) is defined by

$$\rho = \rho(w_1, w_2) = \max\{\rho(w_1), \rho(w_2)\},$$

$$\rho(w_k) = \limsup_{r\to\infty} \frac{\log^+ T(r, w_k)}{\log r}, \quad k = 1, 2.$$

We obtain the main result as follows:

Theorem 1.6 *Let $(w_1(z), w_2(z))$ be transcendental entire solutions with $\rho(w_1, w_2) < \infty$ of the system of differential-difference equations (1.3). Then $(w_1(z), w_2(z))$ have the form of*

$$(w_1(z), w_2(z)) = (\frac{e^{\sqrt{2}z+b_1} + e^{-\sqrt{2}z-b_1}}{2}, \frac{e^{\sqrt{2}z+b_2} + e^{-\sqrt{2}z-b_2}}{2}),$$

or

$$(w_1(z), w_2(z)) = (\frac{e^{-\sqrt{2}z+b_1} + e^{\sqrt{2}z-b_1}}{2}, \frac{e^{-\sqrt{2}z+b_2} + e^{\sqrt{2}z-b_2}}{2}),$$

where b_1, b_2 are constants, $c = \pm\frac{\pi i+2k\pi i}{2\sqrt{2}}$, k is an integer.

Remark 1.7 The following Example 1.8 shows that it satisfies Theorem 1.6.

Example 1.8 $(w_1(z), w_2(z)) = (\frac{e^{\sqrt{2}z-\pi i}+e^{-\sqrt{2}z+\pi i}}{2}, \frac{e^{\sqrt{2}z+\pi i}+e^{-\sqrt{2}z-\pi i}}{2})$ is a transcendental entire solution of the system of complex differential-difference equations of the form

$$\begin{cases} (w_1''(z) - w_1(z))^2 + w_2(z - \frac{\pi i}{2\sqrt{2}})^2 = 1, \\ (w_2''(z) - w_2(z))^2 + w_1(z - \frac{\pi i}{2\sqrt{2}})^2 = 1, \end{cases}$$

where $a = \sqrt{2}, c = -\frac{\pi i}{2\sqrt{2}}, b_1 = -\pi i, b_2 = \pi i$.

2. Some lemmas

We will use the following Lemmas in our proofs of the above Theorems.

Lemma 2.1 ([1]) *Let $f_j(z)$ be meromorphic function. $f_k(z)$ $(k = 1, 2, \ldots, n-1)$ are nonconstant, satisfying $\sum_{j=1}^{n} f_j = 1$ and $n \geq 3$. If $f_n(z) \not\equiv 0$ and*

$$\sum_{j=1}^{n} N(r, \frac{1}{f_j}) + (n-1)\sum_{j=1}^{n} \overline{N}(r, f_j) < (\lambda + o(1))T(r, f_k),$$

where $\lambda < 1$ and $k = 1, 2, \ldots, n-1$, then $f_n \equiv 1$.

Lemma 2.2 ([1]) (Hadamard's factorization theorem) *Let f be an entire function of finite order $\rho(f)$ with zeros $\{z_1, z_2, \ldots\} \subset C\backslash\{0\}$ and a k-fold zero at the origin. Then*

$$f(z) = z^k P(z) e^{Q(z)},$$

where $P(z)$ is the canonical product of f formed with the non-null zeros of f, and $Q(z)$ is a polynomial of degree $\leq \rho(f)$.

3. Proof of Theorem 1.3

With the aid of the Lemmas above, first we give the proof of Theorem 1.3.

Proof Suppose that (1.2) has a transcendental entire solution $w(z)$ with $\rho(w(z)) < \infty$.

We rewrite (1.2) as the following form

$$((w''(z) - w(z)) + iw(z+c))((w''(z) - w(z)) - iw(z+c)) = 1.$$

Then

$$(w''(z) - w(z)) + iw(z+c), (w''(z) - w(z)) - iw(z+c)$$

have no zeros. By Lemma 2.2, we can assume that

$$\begin{cases} (w''(z) - w(z)) + iw(z+c) = e^{p(z)}, \\ (w''(z) - w(z)) - iw(z+c) = e^{-p(z)}, \end{cases} \tag{3.1}$$

where $p(z)$ is a nonzero polynomial.

From (3.1), we get

$$w''(z) - w(z) = \frac{e^{p(z)} + e^{-p(z)}}{2}, \tag{3.2}$$

$$w(z+c) = \frac{e^{p(z)} - e^{-p(z)}}{2i}. \tag{3.3}$$

Combining (3.2) with (3.3), we have

$$\frac{i}{p''(z) - p'(z)^2 + 1} e^{p(z+c)+p(z)} + \frac{i}{p''(z) - p'(z)^2 + 1} e^{p(z)-p(z+c)} - \frac{p''(z) + p'(z)^2 - 1}{p''(z) - p'(z)^2 + 1} e^{2p(z)} = 1. \tag{3.4}$$

(i) If $-\frac{p''(z)+p'(z)^2-1}{p''(z)-p'(z)^2+1} e^{2p(z)} \equiv d_1$, we have $p(z)$ is also a constant, by (3.3), then we can get $w(z)$ is a constant, which is in contradiction with $w(z)$ being a transcendental entire function.

(ii) If both

$$\frac{i}{p''(z) - p'(z)^2 + 1} e^{p(z+c)+p(z)} \equiv d_2 \quad \text{and} \quad \frac{i}{p''(z) - p'(z)^2 + 1} e^{p(z)-p(z+c)} \equiv d_3$$

are constants, then $\frac{e^{p(z)+p(z+c)}}{e^{p(z)-p(z+c)}} \equiv d$, we obtain $e^{2p(z+c)} \equiv d$. Therefore, $p(z)$ is a constant, by (3.3), we also get a contradiction with $w(z)$ being a transcendental entire function.

Thus, we can get that $\frac{i}{p''(z)-p'(z)^2+1} e^{p(z)+p(z+c)}$ is a non-constant.

Combining the above cases (i) and (ii), we can obtain

$$-\frac{p''(z)+p'(z)^2-1}{p''(z)-p'(z)^2+1}e^{2p(z)},\quad \frac{i}{p''(z)-p'(z)^2+1}e^{p(z)+p(z+c)}$$

should be non-constants.

According to Lemma 2.1, we obtain

$$\frac{i}{p''(z)-p'(z)^2+1}e^{p(z)-p(z+c)}\equiv 1$$

which shows that $p''(z)-p'(z)^2+1$ has no zeros, $p(z)$ is a polynomial with $\deg p(z)=1$. Let $p(z)=az+b$, where $a\neq 0, b$ be constants. Thus, we get

$$\frac{i}{p''(z)-p'(z)^2+1}e^{p(z)-p(z+c)}=\frac{i}{-a^2+1}e^{-ac}\equiv 1. \tag{3.5}$$

From (3.5), we obtain

$$e^{-ac}=i(a^2-1), c=-\frac{\ln(i(a^2-1))+2k\pi i}{a}. \tag{3.6}$$

Since $e^{-ac}=i(a^2-1)$, from (3.2), we have

$$w''(z+c)-w(z+c)=\frac{e^{az+ac+b_1}+e^{-az-ac-b_1}}{2}=\frac{\frac{1}{i(a^2-1)}e^{az+b_1}+i(a^2-1)e^{-az-b_1}}{2}. \tag{3.7}$$

Again, by (3.3), we can obtain

$$w''(z+c)-w(z+c)=\frac{(a^2-1)e^{az+b_2}-(a^2-1)e^{-az-b_2}}{2i}. \tag{3.8}$$

Combining (3.7) and (3.8), we have

$$\frac{\frac{1}{i(a^2-1)}e^{az+b_1}+i(a^2-1)e^{-az-b_1}}{2}=\frac{(a^2-1)e^{az+b_2}-(a^2-1)e^{-az-b_2}}{2i},$$

i.e.,

$$\frac{1}{a^2-1}=a^2-1. \tag{3.9}$$

Since $a\neq 0$, from (3.9), we obtain $a=\pm\sqrt{2}$. If $a=\sqrt{2}$, then $c=-\frac{\frac{\pi i}{2}+2k\pi i}{\sqrt{2}}$ and

$$w(z)=\frac{e^{\sqrt{2}z+b}+e^{-\sqrt{2}z-b}}{2}.$$

If $a=-\sqrt{2}$, then $c=\frac{\frac{\pi i}{2}+2k\pi i}{\sqrt{2}}$ and $w(z)=\frac{e^{-\sqrt{2}z+b}+e^{\sqrt{2}z-b}}{2}$, where b is a constant.

Thus, the proof of Theorem 1.3 is completed. □

4. Proof of Theorem 1.6

In the following, we give the proof of Theorem 1.6 further.

Proof Suppose that (1.3) has a transcendental entire solution $(w_1(z),w_2(z))$ with $\rho(w_1,w_2)<\infty$.

We rewrite (1.3) as the following form

$$\begin{cases} ((w_1''(z)-w_1(z))+iw_2(z+c))((w_1''(z)-w_1(z))-iw_2(z+c))=1,\\ ((w_2''(z)-w_2(z))+iw_1(z+c))((w_2''(z)-w_2(z))-iw_1(z+c))=1. \end{cases}$$

Then

$$(w_1''(z)-w_1(z))+iw_2(z+c),(w_1''(z)-w_1(z))-iw_2(z+c),$$

$$(w_2''(z)-w_2(z))+iw_1(z+c),(w_2''(z)-w_2(z))-iw_1(z+c)$$

have no zeros. By Lemma 2.2, we can assume that

$$\begin{cases} (w_1''(z)-w_1(z))+iw_2(z+c)=e^{p(z)},\\ (w_1''(z)-w_1(z))-iw_2(z+c)=e^{-p(z)},\\ (w_2''(z)-w_2(z))+iw_1(z+c)=e^{q(z)},\\ (w_2''(z)-w_2(z))-iw_1(z+c)=e^{-q(z)}, \end{cases} \tag{4.1}$$

where $p(z),q(z)$ are nonzero polynomials.

From(4.1), we get

$$w_1''(z)-w_1(z)=\frac{e^{p(z)}+e^{-p(z)}}{2}, \tag{4.2}$$

$$w_2(z+c)=\frac{e^{p(z)}-e^{-p(z)}}{2i}, \tag{4.3}$$

$$w_2''(z)-w_2(z)=\frac{e^{q(z)}+e^{-q(z)}}{2}, \tag{4.4}$$

$$w_1(z+c)=\frac{e^{q(z)}-e^{-q(z)}}{2i}. \tag{4.5}$$

Combining (4.2) with (4.5), (4.3) with (4.4), respectively, we have

$$\begin{cases} w_1''(z)-w_1(z)=\frac{e^{p(z)}+e^{-p(z)}}{2},\\ w_1(z+c)=\frac{e^{q(z)}-e^{-q(z)}}{2i}, \end{cases} \tag{4.6}$$

$$\begin{cases} w_2''(z)-w_2(z)=\frac{e^{q(z)}+e^{-q(z)}}{2},\\ w_2(z+c)=\frac{e^{p(z)}-e^{-p(z)}}{2i}. \end{cases} \tag{4.7}$$

By (4.6) and (4.7), we obtain

$$\frac{i}{q''(z)-q'(z)^2+1}e^{p(z+c)+q(z)}+\frac{i}{q''(z)-q'(z)^2+1}e^{q(z)-p(z+c)}-\frac{q''(z)+q'(z)^2-1}{q''(z)-q'(z)^2+1}e^{2q(z)}=1, \tag{4.8}$$

$$\frac{i}{p''(z)-p'(z)^2+1}e^{q(z+c)+p(z)}+\frac{i}{p''(z)-p'(z)^2+1}e^{p(z)-q(z+c)}-\frac{p''(z)+p'(z)^2-1}{p''(z)-p'(z)^2+1}e^{2p(z)}=1. \tag{4.9}$$

(i) If $-\frac{q''(z)+q'(z)^2-1}{q''(z)-q'(z)^2+1}e^{2q(z)}$ or $-\frac{p''(z)+p'(z)^2-1}{p''(z)-p'(z)^2+1}e^{2p(z)}$ is a constant, we have $q(z)$ or $p(z)$ is also a constant, by the second equation of (4.6) or (4.7), we can get $w_1(z)$ or $w_2(z)$ is a constant, a contradiction.

(ii) If both

$$\frac{i}{q''(z)-q'(z)^2+1}e^{p(z+c)+q(z)} \equiv \hat{d}_3 \quad \text{and} \quad \frac{i}{q''(z)-q'(z)^2+1}e^{q(z)-p(z+c)} \equiv \hat{d}_4$$

are constants, then $\frac{e^{q(z)+p(z+c)}}{e^{q(z)-p(z+c)}} \equiv d$, we obtain $e^{2p(z+c)} \equiv d$. Therefore, $p(z)$ is a constant, similarly, we can get a contradiction with $w_2(z)$ being a transcendental entire function.

Thus, we can assume that $\frac{i}{q''(z)-q'(z)^2+1}e^{q(z)+p(z+c)}$ is a non-constant.

Similarly, we can also assume that $\frac{i}{p''(z)-p'(z)^2+1}e^{q(z+c)+p(z)}$ is a non-constant.

Combining the above cases (i) and (ii), we can have

$$-\frac{q''(z)+q'(z)^2-1}{q''(z)-q'(z)^2+1}e^{2q(z)}, \quad -\frac{p''(z)+p'(z)^2-1}{p''(z)-p'(z)^2+1}e^{2p(z)},$$

$$\frac{i}{q''(z)-q'(z)^2+1}e^{q(z)+p(z+c)}, \quad \frac{i}{p''(z)-p'(z)^2+1}e^{q(z+c)+p(z)}$$

should be non-constants.

According to Lemma 2.1, we obtain

$$\frac{i}{q''(z)-q'(z)^2+1}e^{q(z)-p(z+c)} \equiv 1, \frac{i}{p''(z)-p'(z)^2+1}e^{p(z)-q(z+c)} \equiv 1$$

which shows that both $q''(z)-q'(z)^2+1$ and $p''(z)-p'(z)^2+1$ have no zeros, $p(z)$ and $q(z)$ are polynomials with $\deg p(z)=1, \deg q(z)=1$. Let $p(z)=az+b_1, q(z)=az+b_2$, where $a \neq 0, b_1, b_2$ be constants. Thus, we get

$$\frac{i}{q''(z)-q'(z)^2+1}e^{q(z)-p(z+c)} = \frac{i}{-a^2+1}e^{b_2-b_1-ac} = 1, \tag{4.10}$$

$$\frac{i}{p''(z)-p'(z)^2+1}e^{p(z)-q(z+c)} = \frac{i}{-a^2+1}e^{b_1-b_2-ac} = 1. \tag{4.11}$$

From (4.10) and (4.11) we obtain

$$e^{b_2-b_1-ac} = i(a^2-1), e^{b_1-b_2-ac} = i(a^2-1). \tag{4.12}$$

Hence, $e^{-2ac} = -(a^2-1)^2, c = -\frac{\ln(-(a^2-1)^2)+2k\pi i}{2a}$.

From the first equation of (4.6), we get

$$w_1''(z+c)-w_1(z+c) = \frac{e^{az+ac+b_1}+e^{-az-ac-b_1}}{2} = \frac{\frac{1}{i(a^2-1)}e^{az+b_2}+i(a^2-1)e^{-az-b_2}}{2}. \tag{4.13}$$

By the second equation of (4.6), we immediately have

$$w_1''(z+c)-w_1(z+c) = \frac{(a^2-1)e^{az+b_2}-(a^2-1)e^{-az-b_2}}{2i}. \tag{4.14}$$

Combining (4.13) and (4.14), we have

$$\frac{\frac{1}{i(a^2-1)}e^{az+b_2}+i(a^2-1)e^{-az-b_2}}{2} = \frac{(a^2-1)e^{az+b_2}-(a^2-1)e^{-az-b_2}}{2i}.$$

i.e.,

$$\frac{1}{a^2-1} = a^2-1. \tag{4.15}$$

Since $a \neq 0$, from (4.15), we obtain

$$a = \pm\sqrt{2}.$$

If $a = \sqrt{2}$, then $c = -\frac{\pi i + 2k\pi i}{2\sqrt{2}}$ and

$$(w_1(z), w_2(z)) = (\frac{e^{\sqrt{2}z+b_1} + e^{-\sqrt{2}z-b_1}}{2}, \frac{e^{\sqrt{2}z+b_2} + e^{-\sqrt{2}z-b_2}}{2}).$$

If $a = -\sqrt{2}$, then $c = \frac{\pi i + 2k\pi i}{2\sqrt{2}}$ and

$$(w_1(z), w_2(z)) = (\frac{e^{-\sqrt{2}z+b_1} + e^{\sqrt{2}z-b_1}}{2}, \frac{e^{-\sqrt{2}z+b_2} + e^{\sqrt{2}z-b_2}}{2}),$$

where b_1, b_2 are constants.

Thus, the proof of Theorem 1.6 is completed. □

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