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# $RL$-Topology and the Related Compactness

**Hongyan LI**[1,*], **Qinghua LI**[2]

1. *School of Mathematics and Information Science, Shandong Technology and Business University, Shandong* 264005*, P. R. China;*
2. *School of Mathematics and Information Science, Yantai University, Shandong* 264005*, P. R. China*

**Abstract** In this paper, the concept of $RL$-topology on $L$-fuzzy subset $A$ is defined, which makes the $L$-topology become special cases. The definition of $RL$-continuous map between $RL$-ts's is given on the topology. Furthermore, the definition of compactness is introduced by means of an inequality in $RL$-topology, and some properties of compactness are studied.

**Keywords** pseudo-complement; $RL$-topology; $RL$-continuous; $RL$-compactness

**MR(2010) Subject Classification** 54A40; 54D30; 03E72

## 1. Introduction

Chang [1] firstly introduced fuzzy set theory into topology. Afterward, many researchers have tried successfully to discuss various aspects of fuzzy topology, which are treated as a crisp subset of a powerset. For a more general case, in an $L$-topology, a lot of good results have been achieved [2–11].

Researching a topology or a fuzzy topology on a fuzzy subset is a pretty essential problem. The notion of $L$-topology on the fuzzy subset was first proposed in [12] and was applied to the study of the separation of axioms in the literature [13]. In [14], a fully stratified $L$-topological on a fuzzy subset was proposed and it was verified that the compactness and connectedness are absolute properties. At the same time, many more general fuzzy topologies on fuzzy sets have been studied [15–20].

The aim of this paper is to establish the $L$-topology on the $L$-fuzzy subset and to discuss its related properties. Therefore, the concept of $RL$-topology on the $L$-fuzzy subset $A$ is introduced, and the $L$-topology is its special cases. The concept of $RL$-continuous map between $RL$-ts's is given on the $RL$-topology, and the definition of compactness is introduced by means of an inequality. Some properties of compactness are studied. And the result that the compactness is preserved under $RL$-continuous map is confirmed.

## 2. Preliminaries

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* Corresponding author
E-mail address: lihongyan@sdtbu.edu.cn (Hongyan LI); liqhua-2002@163.com (Qinghua LI)

In this paper, $(L, \vee, \wedge, ')$ is a completely distributive DeMorgan algebra (i.e., completely distributive lattice with order-reversing involution) [6, 7]. The largest element and the smallest element in $L$ are denoted by $\top$ and $\bot$, respectively.

For a nonempty set $X$, the family of all $L$-sets on $X$ is denoted by $L^X$. $A \in L^X$ is called valuable if $A \nleq A'$, the family of all valuable $L$-sets on $X$ is denoted by $\mathcal{V}_X^L$, that is, $\mathcal{V}_X^L = \{A | A \nleq A', A \in L^X\}$. For $A \in \mathcal{V}_X^L$, we denote $\mathcal{F}_X^L(A) = \{G | G \leq A, G \in L^X\}$, which is called the powerset of the fuzzy set $A$.

**Definition 2.1** *Let $A \in \mathcal{V}_X^L$, $G \in F_X^L(A)$. $\langle_L^A G$ is called the pseudo-complement of $G$ relative to $A$, which is definend via* $\langle_L^A G = \begin{cases} A \wedge G', & G \neq A, \\ \bot_X, & G = A. \end{cases}$

**Proposition 2.2** *For all $A \in \mathcal{V}_X^L$, we have*

*(1)* $\langle_L^A G = A \Leftrightarrow G \leq A'$ *for all* $G \in F_X^L(A)$;

*(2)* $G \leq H \Rightarrow \langle_L^A H \leq \langle_L^A G$ *for all* $G, H \in \mathcal{F}_X^L(A)$;

*(3)* *For all* $\{G_j | j \in J\} \subseteq \mathcal{F}_X^L(A)$, $\langle_L^A \bigwedge_{j \in J} G_j = \bigvee_{j \in J} \langle_L^A G_j$;

*(4)* *For all* $\{G_j | j \in J\} \subseteq \mathcal{F}_X^L(A)$, $\langle_L^A \bigvee_{j \in J} G_j \leq \bigwedge_{j \in J} \langle_L^A G_j$. *The equation holds when* $\bigvee_{j \in J} G_j \neq A$.

**Definition 2.3** ([5, 6]) *An $L$-topological space (or $L$-space for short) is a pair $(X, \mathcal{T})$, where $\mathcal{T}$ is a subfamily of $L^X$ which contains three following requirements:*

*(1)* $\top_X, \bot_X \in \mathcal{T}$;

*(2)* $G \wedge H \in \mathcal{T}$ *for all* $G, H \in \mathcal{T}$;

*(3)* $\bigvee_{j \in J} G_j \in \mathcal{T}$ *for all* $G_j \in \mathcal{T}$, $j \in J$.

**Definition 2.4** ([21]) *Let $(X, \mathcal{T})$ be an $L$-space. $G \in L^X$ is called fuzzy compact if for every family $\mathcal{U} \subseteq T$, it follows that*

$$\bigwedge_{x \in X} \Big( G'(x) \wedge \bigwedge_{A \in \mathcal{U}} A(x) \Big) \leq \bigvee_{\mathcal{V} \in 2^{(\mathcal{U})}} \bigwedge_{x \in X} \Big( G'(x) \wedge \bigwedge_{A \in \mathcal{V}} A(x) \Big).$$

## 3. $RL$-topology and $RL$-cotopology

First, we will introduce the concept of $RL$-topology on an $L$-subset $A$ as follows.

**Definition 3.1** *Let $A \in \mathcal{V}_X^L$. A relative $L$-topology $\tau$ on an $L$-subset $A$, is a subfamily on $\mathcal{F}_X^L(A)$, that satisfies the following conditions:*

*(RL-O1)* $A \in \tau$ and $G \in \tau$ *for all* $G \leq A'$;

*(RL-O2)* $G \wedge H \in \tau$ *for all* $G, H \in \tau$;

*(RL-O3)* $\bigvee_{j \in J} G_j \in \tau$ *for all* $G_j \in \tau$, $j \in J$.

The pair $(A, \tau)$ is called a relative $L$-topological space on $A$ ($RL$-ts, in short). Each member of $\tau$ (that is, $G \in \tau$) is called an $RL$-open set and $H$ is called an $RL$-closed set if $\langle_L^A H \in \tau$.

When $A = \top_X$, it is easy to see that the relative $L$-topology on $A$ degenerates to $L$-topology

[5, 6].

Write $\langle_L^A\tau$ to represent the class of all $RL$-closed set, that is, $\langle_L^A\tau = \{H | \langle_L^A H \in \tau\}$. We have the following conclusions:

**Theorem 3.2** *Let $(A, \tau)$ be an RL-ts. Then the following three conclusions are true for $\langle_L^A\tau$.*

*(RL-C1)* $A \in \langle_L^A\tau$ *and* $H \in \langle_L^A\tau$ *for all* $H \leq A'$;

*(RL-C2)* $G \vee H \in \langle_L^A\tau$ *for all* $G, H \in \langle_L^A\tau$;

*(RL-C3)* $\bigwedge_{j\in J} G_j \in \langle_L^A\tau$ *for all* $G_j \in \langle_L^A\tau$, $j \in J$.

**Proof** (RL-C1) $A \in \langle_L^A\tau$ is obvious from $\perp_X \in \tau$; For all $H \leq A'$, we have that $H \neq A$ since $H \leq A'$ and $A \not\leq A'$. So $\langle_L^A H = H' \wedge A = A$ from $H \leq A' \Leftrightarrow A \leq H'$. Therefore $H \in \langle_L^A\tau$.

(RL-C2) Suppose that $G, H \in \langle_L^A\tau$. Then $\langle_L^A G$, $\langle_L^A H \in \tau$ according to the definition of $\langle_L^A\tau$.

If $G \vee H = A$, then $\langle_L^A(G \vee H) = \perp_X \in \tau$. So $G \vee H \in \langle_L^A\tau$. If $G \vee H \neq A$, then $G \neq A$ and $H \neq A$ from $G, H \in \mathcal{F}_X^L(A)$. So $\langle_L^A(G \vee H) = (G \vee H)' \wedge A = (G' \wedge H') \wedge A = (G' \wedge A) \wedge (H' \wedge A) = \langle_L^A G \wedge \langle_L^A H \in \tau$ by (RL-O2). Therefore $G \vee H \in \langle_L^A\tau$.

(RL-C3) Suppose that $G_j \in \langle_L^A\tau$, $j \in J$. Then $\langle_L^A G_j \in \tau$ for all $j \in J$. If $\bigwedge_{j\in J} G_j = A$, then $\langle_L^A(\bigwedge_{j\in J} G_j) = \perp_X \in \tau$. So $\bigwedge_{j\in J} G_j \in \langle_L^A\tau$. If $\bigwedge_{j\in J} G_j \neq A$, let $K = \{j | G_j \neq A, j \in J\}$. Then $K \neq \emptyset$. Thus $\langle_L^A(\bigwedge_{j\in J} G_j) = \langle_L^A(\bigwedge_{j\in K} G_j) = A \wedge (\bigwedge_{j\in K} G_j)' = \bigvee_{j\in K}(A \wedge G_j') = \bigvee_{j\in K} \langle_L^A G_j \in \tau$ by (RL-O3). Therefore $\bigwedge_{j\in J} G_j \in \langle_L^A\tau$. □

Given a function $f : \mathrm{X} \to \mathrm{Y}$. Let $G \in L^X$ and $H \in L^Y$. Here $f_L^{\rightarrow}(G)(y) = \bigvee\{G(x) | f(x) = y\}$ for all $y \in Y$ and $f_L^{\leftarrow}(H)(x) = \bigvee\{G(x) | f_L^{\rightarrow}(G) \leq H\} = H(f(x))$ for all $x \in X$ as defined by Zadeh.

**Definition 3.3** *Let $A \in \mathcal{V}_X^L$ and $B \in \mathcal{V}_Y^L$. The restriction of $f_L^{\rightarrow}$ on* A

$$\begin{aligned} f_L^{\rightarrow}|A: \quad & \mathcal{F}_X^L(A) \to L^Y \\ & G \in \mathcal{F}_X^L(A) \mapsto f_L^{\rightarrow}(G) \end{aligned}$$

*is called an RL-fuzzy mapping from $A$ to $B$ denoted $f_{L,A}^{\rightarrow} : A \to B$ if $f_L^{\rightarrow}(A) \leq B$. The inverse image of a fuzzy subset $H \in \mathcal{F}_Y^L(B)$ under $f_{L,A}^{\rightarrow}$ is defined by*

$$f_{L,A}^{\leftarrow}(H) = \bigvee\{G | f_L^{\rightarrow}(G) \leq H, G \in \mathcal{F}_X^L(A)\}.$$

Obviously, we have that $f_{L,A}^{\leftarrow}(H) = A \wedge f_L^{\leftarrow}(H)$.

**Definition 3.4** *Let $A \in \mathcal{V}_X^L$, $B \in \mathcal{V}_Y^L$ and $(A,\ \tau), (B,\ \delta)$ be two RL-ts's. An RL-fuzzy mapping $f_{L,A}^{\rightarrow} : A \to B$ is called a continuous map between RL-ts's if $f_{L,A}^{\leftarrow}(H) \in \langle_L^A\tau$ for all $H \in \langle_L^A\delta$.*

**Lemma 3.5** *Let $A \in \mathcal{V}_X^L$, $B \in \mathcal{V}_Y^L$, $f_{L,A}^{\rightarrow} : A \to B$ be a relative L-fuzzy mapping from $A$ to $B$ and $G \in \mathcal{F}_X^L(A)$. Then for any $\mathcal{P} \subseteq L^X$, it follows that*

$$\bigvee_{y\in Y}\Big(f_{L,A}^{\rightarrow}(G)(y) \wedge \bigwedge_{H\in\mathcal{P}} H(y)\Big) = \bigvee_{x\in X}\Big(G(x) \wedge \bigwedge_{H\in\mathcal{P}} f_{L,A}^{\leftarrow}(H)(x)\Big).$$

**Proof** This can be proved from the following two equations.

$$\begin{aligned}\bigvee_{y\in Y}\Big(f^{\rightarrow}_{L,A}(G)(y)\wedge\bigwedge_{H\in\mathcal{P}}H(y)\Big)&=\bigvee_{y\in Y}\Big(\Big(\bigvee_{f(x)=y}G(x)\Big)\wedge\bigwedge_{H\in\mathcal{P}}H(y)\Big)\\&=\bigvee_{y\in Y}\Big(\bigvee_{f(x)=y}\Big(G(x)\wedge\bigwedge_{H\in\mathcal{P}}H(f(x))\Big)\Big)\\&=\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{H\in\mathcal{P}}f^{\leftarrow}_{L}(H)(x)\Big),\\ \bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{H\in\mathcal{P}}f^{\leftarrow}_{L,A}(H)(x)\Big)&=\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{H\in\mathcal{P}}(f^{\leftarrow}_{L}(H)\wedge A)(x)\Big)\\&=\bigvee_{x\in X}\Big(G(x)\wedge A(x)\wedge\bigwedge_{H\in\mathcal{P}}f^{\leftarrow}_{L}(H)(x)\Big)\\&=\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{H\in\mathcal{P}}f^{\leftarrow}_{L}(H)(x)\Big).\quad\Box\end{aligned}$$

## 4. Compactness on $RL$-topology

In order to generalize the notion of compactness to $RL$-topology, an equivalent proposition about compactness in $L$-topology is recalled.

**Theorem 4.1** ([21]) *Let $(X,\mathcal{T})$ be an $L$-space. Then $G\in L^X$ is fuzzy compact if and only if for every subfamily $\mathcal{P}\subseteq\mathcal{T}'$, it follows that*

$$\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{B\in\mathcal{P}}B(x)\Big)\geq\bigwedge_{\mathcal{F}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{B\in\mathcal{F}}B(x)\Big).$$

Accordingly, we get the following definition of the compactness of $RL$-topology.

**Definition 4.2** *Let $(A,\tau)$ be an RL-ts. $G\in\mathcal{F}^L_X(A)$ is called RL-compact with respect to $\tau$ if for every subfamily $\mathcal{P}\subseteq\langle^A_L\tau$, it follows that*

$$\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{H\in\mathcal{P}}H(x)\Big)\geq\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big).$$

**Theorem 4.3** *Let $(A,\tau)$ be an RL-ts. If $G_1,G_2\in\mathcal{F}^L_X(A)$ are RL-compact with respect to $\tau$, then $G_1\vee G_2$ is also RL-compact with respect to $\tau$.*

**Proof** Suppose that $\mathcal{P}\subseteq\langle^A_L\tau$. Since $G_1,G_2\in\mathcal{F}^L_X(A)$ are $RL$-compact with respect to $\tau$, we have that

$$\bigvee_{x\in X}\Big(G_1(x)\wedge\bigwedge_{H\in\mathcal{P}}H(x)\Big)\geq\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(G_1(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big),$$

$$\bigvee_{x\in X}\Big(G_2(x)\wedge\bigwedge_{H\in\mathcal{P}}H(x)\Big)\geq\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(G_2(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big).$$

So, we can get the following inequalities

$$
\begin{aligned}
&\bigvee_{x\in X}\Big((G_1\vee G_2)(x)\wedge\bigwedge_{H\in\mathcal{P}}H(x)\Big)\\
&=\bigvee_{x\in X}\Big((G_1(x)\vee G_2(x))\wedge\bigwedge_{H\in\mathcal{P}}H(x)\Big)\\
&=\bigvee_{x\in X}\Big(\Big(G_1(x)\wedge\bigwedge_{H\in\mathcal{P}}H(x)\Big)\vee\Big(G_2(x)\wedge\bigwedge_{H\in\mathcal{P}}H(x)\Big)\Big)\\
&=\geq\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(G_1(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big)\vee\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(G_2(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big)\\
&=\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\Big(\bigvee_{x\in X}\Big(G_1(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big)\vee\bigvee_{x\in X}\Big(G_2(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big)\Big)\\
&=\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(\Big(G_1(x)\vee G_2(x)\Big)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big)\\
&=\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(\Big(G_1\vee G_2\Big)(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big).
\end{aligned}
$$

Thus $G_1\vee G_2$ is $RL$-compact with respect to $\tau$ by Definition 4.2. □

**Theorem 4.4** *Let $(A,\tau)$ be an RL-ts. If $G_1\in\mathcal{F}_X^L(A)$ is RL-compact with respect to $\tau$ and $G_2\in\langle_L^A\tau$, then $G_1\wedge G_2$ is RL-compact with respect to $\tau$ as well.*

**Proof** Suppose that $\mathcal{P}\subseteq\langle_L^A\tau$. We have that $\mathcal{S}=\{G_2\}\cup\mathcal{P}\subseteq\langle_L^A\tau$ by $G_2\in\langle_L^A\tau$. Because $G_1\in\mathcal{F}_X^L(A)$ is $RL$-compact with respect to $\tau$, we know that

$$
\bigvee_{x\in X}\Big(G_1(x)\wedge\bigwedge_{H\in\mathcal{S}}H(x)\Big)\geq\bigwedge_{\mathcal{R}\in 2^{(\mathcal{S})}}\bigvee_{x\in X}\Big(G_1(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big).
$$

And from the following two equations,

$$
\begin{aligned}
\bigvee_{x\in X}\Big(\Big(G_1\wedge G_2\Big)(x)\wedge\bigwedge_{H\in\mathcal{P}}H(x)\Big)&=\bigvee_{x\in X}\Big(\Big(G_1(x)\wedge G_2(x)\Big)\wedge\bigwedge_{H\in\mathcal{P}}H(x)\Big)\\
&=\bigvee_{x\in X}\Big(G_1(x)\wedge\Big(G_2(x)\wedge\bigwedge_{H\in\mathcal{P}}H(x)\Big)\Big)\\
&=\bigvee_{x\in X}\Big(G_1(x)\wedge\bigwedge_{H\in\mathcal{S}}H(x)\Big)
\end{aligned}
$$

and

$$
\begin{aligned}
&\bigwedge_{\mathcal{R}\in 2^{(\mathcal{S})}}\bigvee_{x\in X}\Big(G_1(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big)\\
&=\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(G_1(x)\wedge G_2(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big)\wedge\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(G_1(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big)\\
&=\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(G_1(x)\wedge G_2(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big)\\
&=\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(\Big(G_1\wedge G_2\Big)(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big),
\end{aligned}
$$

we can get the following inequality:

$$\bigvee_{x\in X}\Big(\Big(G_1\wedge G_2\Big)(x)\wedge\bigwedge_{H\in\mathcal{P}}H(x)\Big)\geq\bigwedge_{\mathcal{R}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(\Big(G_1\wedge G_2\Big)(x)\wedge\bigwedge_{H\in\mathcal{R}}H(x)\Big)$$

Thus $G_1\wedge G_2$ is $RL$-compact with respect to $\tau$ by Definition 4.2, the proof is completed. □

**Theorem 4.5** *Let $A\in\mathcal{V}_X^L$, $B\in\mathcal{V}_Y^L$, $(A,\ \tau),(B,\ \delta)$ be two $RL$-ts's and $f_{L,A}^{\rightarrow}:A\rightarrow B$ be a relative $L$-fuzzy continuous mapping from $A$ to $B$. If $G\in\mathcal{F}_X^L(A)$ is $RL$-compact with respect to $\tau$, then $f_{L,A}^{\rightarrow}(G)$ is also $RL$-compact with respect to $\delta$.*

**Proof** Suppose that $\mathcal{P}\subseteq\langle_L^A\delta$. Since $f_{L,A}^{\rightarrow}:A\rightarrow B$ is continuous, we have that $f_{L,A}^{\leftarrow}(H)\in\langle_L^A\tau$ for all $H\in\mathcal{P}$ by Definition 3.4. So $\mathcal{S}=\{f_{L,A}^{\leftarrow}(H)|H\in\mathcal{P}\}\subseteq\langle_L^A\tau$. Because $G\in\mathcal{F}_X^L(A)$ is $RL$-compact with respect to $\tau$, we have that

$$\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{D\in\mathcal{S}}D(x)\Big)\geq\bigwedge_{\mathcal{R}\in 2^{(\mathcal{S})}}\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{D\in\mathcal{R}}D(x)\Big),$$

and from Lemma 3.5, we can know that

$$\begin{aligned}\bigvee_{y\in Y}\Big(f_{L,A}^{\rightarrow}(G)(y)\wedge\bigwedge_{H\in\mathcal{P}}H(y)\Big)&=\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{H\in\mathcal{P}}f_{L,A}^{\leftarrow}(H)(x)\Big)\\&=\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{D\in\mathcal{S}}D(x)\Big)\end{aligned}$$

and

$$\begin{aligned}\bigwedge_{\mathcal{Q}\in 2^{(\mathcal{P})}}\bigvee_{y\in Y}\Big(f_{L,A}^{\rightarrow}(G)(y)\wedge\bigwedge_{H\in\mathcal{Q}}H(y)\Big)&=\bigwedge_{\mathcal{Q}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{H\in\mathcal{Q}}f_{L,A}^{\leftarrow}(H)(x)\Big)\\&=\bigwedge_{\mathcal{R}\in 2^{(\mathcal{S})}}\bigvee_{x\in X}\Big(G(x)\wedge\bigwedge_{D\in\mathcal{R}}D(x)\Big).\end{aligned}$$

Thus

$$\bigvee_{y\in Y}\Big(f_{L,A}^{\rightarrow}(G)(y)\wedge\bigwedge_{H\in\mathcal{P}}H(y)\Big)\geq\bigwedge_{\mathcal{Q}\in 2^{(\mathcal{P})}}\bigvee_{y\in Y}\Big(f_{L,A}^{\rightarrow}(G)(y)\wedge\bigwedge_{H\in\mathcal{Q}}H(y)\Big).$$

Therefore, $f_{L,A}^{\rightarrow}(G)$ is $RL$-compact with respect to $\delta$. □

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