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Path Cover in $K_{1,4}$-Free Graphs

Mingda LIU[1], **Xiaodong CHEN**[1,*], **Mingchu LI**[2]

1. *College of Science, Liaoning University of Technology, Liaoning* 121001*, P. R. China;*
2. *School of Softsware, Dalian University of Technology, Liaoning* 116024*, P. R. China*

Abstract For a graph G, a path cover is a set of vertex disjoint paths covering all the vertices of G, and a path cover number of G, denoted by $p(G)$, is the minimum number of paths in a path cover among all the path covers of G. In this paper, we prove that if G is a $K_{1,4}$-free graph of order n and $\sigma_{k+1}(G) \geq n-k$, then $p(G) \leq k$, where $\sigma_{k+1}(G) = \min\{\sum_{v\in S} \mathrm{d}(v) : S$ is an independent set of G with $|S| = k+1\}$.

Keywords path cover; path cover number; $K_{1,4}$-free graph; non-insertable vertex

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1. Introduction

In this paper, only finite and simple graphs are considered. Readers can refer to [1] for notation and terminology not defined here. A graph G is $K_{1,r}$-free, if G contains no induced subgraph isomorphic to $K_{1,r}$, where $r \geq 3$. Let $\alpha(G)$ denote the independent number of a graph G, i.e., the cardinality of a maximum independent set in G. For $S \subseteq V(G)$, $G[S]$ denotes the subgraph of G induced by S. For a vertex v of G, $N(v)$ denotes both the set of vertices adjacent to v and the induced subgraph $G[N(v)]$. Let $N_S(v)$ denote the set of all vertices in S adjacent to v and $d_S(v) = |N_S(v)|$. In particular, the degree of v is denoted by $d_G(v) = |N_G(v)|$ and briefly denoted by $d(v)$. We use $\delta(G)$ to denote the minimum degree of a graph G. For a subgraph H of a graph G, $G-H$ denotes the subgraph induced by $V(G)-V(H)$. We define $\sigma_{k+1}(G) = \min\{\sum_{v\in S} \mathrm{d}(v) : S$ is an independent set of G with $|S| = k+1\}$ if $k+1 \leq \alpha(G)$, otherwise, $\sigma_{k+1}(G) = +\infty$. For a graph G and $A, B \subseteq V(G)$, let $E(A,B) = \{uv \in E(G) : u \in A, v \in B\}$.

Given a positive orientation of a path P, $P[a,b]$ (or aPb) denotes a path from a to b along the positive orientation, and $P(a,b)$ denotes the path $P[a,b]-\{a,b\}$. For a path $P[a,b]$, if $x,y \in V(P)$, xPy denotes a subpath of $P[a,b]$ from x to y along the positive orientation, and yP^-x denotes the subpath from y to x along its negative orientation. For a graph G, a path cover of G is a spanning subgraph consisting of some vertex disjoint paths in G. For a graph G, the path cover number $p(G) = \min\{|\mathfrak{P}| : \mathfrak{P}$ is a path cover of $G\}$. If $\mathfrak{P}$ is a path cover of G with $|\mathfrak{P}| = p(G)$, then $\mathfrak{P}$ is called a minimum path cover of G.

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* Corresponding author
E-mail address: xiaodongchen74@126.com (Xiaodong CHEN)

Dirac [2] in 1952 showed that any graph G with order $n \geq 3$ and $\delta(G) \geq \frac{n}{2}$ is hamiltonian, and G contains a hamilton path if $\delta(G) \geq \frac{n-1}{2}$. Since then, there are a lot of results about the sufficient conditions for graphs to have a hamiton cycle (or path). It is well known that it is NP-hard to justify whether a graph contains a hamilton cycle (or path). As a result, the hamiltonicity of some special graphs, especially, $K_{1,r}$-free graphs are largely studied.

Theorem 1.1 ([3]) *Let G be a k-connected $K_{1,3}$-free graph of order n such that $k \geq 2$ and $\sigma_{k+1} \geq n-k$. Then G is hamiltonian.*

Theorem 1.2 ([4]) *For any k-connected $K_{1,4}$-free graph G of order $n \geq 3$, if $\sigma_{k+1}(G) \geq n+k$, then G is hamiltonian.*

Clearly, the upper bound of the path cover number of a given graph is a generalization of justifying if a graph contains a hamilton path. Thus, there are some results on the upper bound of the path cover number of general graphs as follows.

Theorem 1.3 ([5]) *For a graph G of order n, the path cover number $p(G) \leq n - \sigma_2(G)$.*

Theorem 1.4 ([6]) *For a graph G with connectivity $k(G)$, if $\alpha(G) > k(G)$, then $p(G) \leq \alpha(G) - k(G)$, otherwise, $p(G) \leq \alpha(G)$.*

Inspired by the above results, there are some results about the path cover number of regular graphs [8–10]. In this paper, we give the following sufficient conditions for $K_{1,4}$-free graphs on the degree sum of vertices in an independent set with $k+1$ vertices.

Theorem 1.5 *For a positive integer k, if G is a $K_{1,4}$-free graph of order n and $\sigma_{k+1}(G) \geq n-k$, then $p(G) \leq k$.*

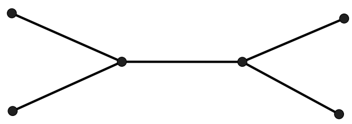

Figure 1 A $K_{1,4}$-free graph G with $\sigma_3(G) = n-3$.

Clearly, in Theorem 1.5, if $k=1$, then $\sigma_2(G) \geq n-1$, and G contains a hamilton path which confirms the conclusion Ore [7] proposed that if a graph G with $\sigma_2(G) \geq n-1$, then G contains a hamilton path. Figure 1 shows that the lower bound of $\sigma_{k+1}(G)$ in Theorem 1.5 is not best possible for $k=2$, since $\sigma_3(G) = n-3$ in Figure 1 and the path cover number is 2.

2. Proof of Theorem 1.5

Suppose that a graph G satisfies the assumption of Theorem 1.5 with $p(G) = t$, and to the contrary, $t > k$. Then $t \geq 2$. Let $\mathfrak{P} = \{P_1, P_2, \ldots, P_t\}$ be a minimum path cover of G. Assume any path P_i in $\mathfrak{P}$ is given a positive direction, and $P_i := u_{i1}u_{i2}\cdots u_{i|V(P_i)|}$, where $u_{i1}, u_{i2}, \ldots, u_{i|V(P_i)|}$ are all the vertices of P_i in order along its positive direction, $1 \leq i \leq t$. For a vertex v in P_i, if $uv, wv \in E(G)$ for two vertices u, w with $uw \in E(P_j)$ in some path $P_j \in \mathfrak{P} \setminus \{P_i\}$, then v is called an insertable vertex, and u, w are called a pair of acceptors of v in P_j. Clearly, for any vertex $x \in V(G)$, there is exact one path P_i in $\mathfrak{P}$ containing x, and in the following proof, we use x^- and x^+ to denote the predecessor and successor of x according to the orientation of P_i, respectively. By the definition of insertable vertex and the minimality of $|\mathfrak{P}|$, we can get Claims 1 and 2 as follows.

Claim 1 For each P_i in $\mathfrak{P}$, P_i contains a non-insertable vertex.

Proof To the contrary, suppose for some P_i, any vertex in P_i is an insertable vertex. If $|V(P_i)| = 1$, i.e., $P_i = u_{i1}$, then clearly, u_{i1} can be inserted between its one pair of acceptors in some path $P_j \in \mathfrak{P} \setminus \{P_i\}$, and then we can get a path cover consisting of $t-1$ paths, a contradiction. Suppose $|V(P_i)| \geq 2$, and assume u_{is} is the last vertex along the positive direction of P_i with the same one pair of acceptors u, w as u_{i1} in some path $P_j \in \mathfrak{P} \setminus \{P_i\}$, then all the vertices in $P_i[u_{i1}, u_{is}]$ can be inserted between u and w by the path $uu_{i1}P_iu_{is}w$. Similarly, any other vertex in P_i can be inserted between corresponding one pair of acceptors in some path in $\mathfrak{P} \setminus \{P_i\}$. Thus we can get a path cover of G consisting of $t-1$ paths, a contradiction. □

By Claim 1, for any path P_i in $\mathfrak{P}$, we denote by v_i the first non-insertable vertex in P_i. In the following proof, let $S = \{v_1, v_2, \ldots, v_t\}$, i.e., S consists of the first non-insertable vertex in each path of $\mathfrak{P}$. Since v_i is the first non-insertable vertex in P_i, any vertex in $P_i[u_{i1}, v_i)$ is an insertable vertex if $u_{i1} \neq v_i$. By the proof of Claim 1, any vertex in $P_i[u_{i1}, v_i)$ can be inserted between corresponding one pair of acceptors in some path $P_j \in \mathfrak{P} \setminus \{P_i\}$.

Claim 2 Let P_i and P_j be two distinct paths in $\mathfrak{P}$ and let $p = |V(P_i)|, q = |V(P_j)|$. For any vertex $u \in P_i[u_{i1}, v_i]$ and any vertex $v \in P_j[u_{j1}, v_j]$, $1 \leq i, j \leq t$, the following properties hold.

(a) $uv \notin E(G)$;

(b) If $t \geq 3$, then u, v have no common pair of acceptors in $\mathfrak{P} \setminus \{P_i, P_j\}$;

(c) Assume $t \geq 3, P_r \in \mathfrak{P} \setminus \{P_i, P_j\}$. Then for any vertex $x \in V(P_r)$, if $ux \in E(G)$, then $x^-v, x^+v \notin E(G)$; By symmetry, if $vx \in E(G)$, then $x^-u, x^+u \notin E(G)$;

(d) For any vertex $x \in P_i(v_i, u_{ip}]$, if $ux \in E(G)$, then $x^-v \notin E(G)$; By symmetry, if $x \in P_j(v_j, u_{jq}]$ and $vx \in E(G)$, then $x^-u \notin E(G)$;

(e) For any vertex x in $P_r \cup P_i(v_i, u_{ip}) \cup P_j(v_j, u_{jq}), x^-x^+ \notin E(G)$ if $ux, xv \in E(G)$, where $P_r \in \mathfrak{P} \setminus \{P_i, P_j\}$.

Proof We prove (a), (b), (c), (d), (e) by contradiction, respectively. In the following proof, to the contrary, assume $u = u_{is} \in P_i[u_{i1}, v_i], v = u_{jm} \in P_j[u_{j1}, v_j]$ are the pair of vertices with the minimum subscript sum $s+m$ which are not satisfying (a), (b), (c), (d), (e), respectively.

(a) To the contrary, suppose $uv \in E(G)$. Clearly, $u_{i1}u_{j1} \notin E(G)$, i.e., $u \neq u_{i1}$ or $v \neq u_{j1}$, otherwise, there exists a path cover consisting of $t-1$ paths, a contradiction. By the minimality of the subscript sum of u, v, $E(P_i[u_{i1}, u), P_j[u_{j1}, v)) = \emptyset$. It follows that there is no vertex in $P_i[u_{i1}, u]$ has a pair of acceptors v^- and v; Similarly, there is no vertex in $P_j[u_{j1}, v]$ has a pair of acceptors u^- and u. We replace $P_i[u, u_{ip}] \cup P_j[v, u_{jq}]$ by $P_{ij} := u_{ip}P_i^-uvP_ju_{jq}$. Then we insert every vertex in $P_i[u_{i1}, u) \cup P_j[u_{j1}, v)$ between its corresponding one pair of acceptors in some path in $(\mathfrak{P}\setminus\{P_i,P_j\})\cup\{P_{ij}\}$. Then we can get a path cover consisting of $t-1$ paths, a contradiction.

(b) To the contrary, u and v have a common pair of acceptors $u_{rg},u_{r(g+1)}$ in $P_r\in\mathfrak{P}\setminus\{P_i,P_j\}, 1\le g<|V(P_r)|$. Let $P_{ir}:=u_{ip}P_i^-uu_{rg}P_r^-u_{r1}, P_{jr}:=u_{rf}P_r^-u_{r(g+1)}vP_ju_{jq}$, where $f=|V(P_r)|$. Then by the minimality of subscript sum of u and v, no pair of vertices $u_{ih}\in P_i[u_{i1},u), u_{jl}\in P_j[u_{j1},v)$ have common pair of acceptors in any path of $\mathfrak{P}\setminus\{P_i,P_j\}$, $1\le h<s, 1\le l<m$. By (a), no vertex in $P_i[u_{i1},u)$ has a pair of acceptors in $P_j[u_{j1},v)$. Likewise, no vertex in $P_j[u_{j1},v)$ has a pair of acceptors in $P_i[u_{i1},u)$. Then we insert each vertex in $P_i[u_{i1},u)\cup P_j[u_{j1},v)$ into corresponding pair of acceptors in $(\mathfrak{P}\setminus\{P_i,P_j\})\cup\{P_{ir},P_{jr}\}$ as the operation in the proof of Claim 1, and replace $P_i[u,u_{ip}]\cup P_r\cup P_j[v,u_{jq}]$ by $P_{ir}\cup P_{jr}$. Clearly, by the above two operations, we can get a path cover with $t-1$ paths, a contradiction.

(c) Suppose $ux\in E(G)$, and to the contrary, $vx^-\in E(G)$. By the minimality of the subscript sum of u,v, there is no vertex in $P_i[u_{i1},u)$ adjacent to x, which implies no vertex in $P_i[u_{i1},u)$ has a pair of acceptors x^-,x in P_r. Likewise, there is no vertex in $P_j[u_{j1},v)$ adjacent to x^-, and then no vertex in $P_j[u_{j1},v)$ has a pair of acceptors x^-,x in P_r. By (a), no vertex in $P_i[u_{i1},u)$ has a pair of acceptors in $P_j[u_{j1},v)$, and no vertex in $P_j[u_{j1},v)$ has a pair of acceptors in $P_i[u_{i1},u)$. We replace $P_r\cup P_i[u,u_{ip}]\cup P_j[v,u_{jq}]$ by $P_{ir}:=u_{ip}P_i^-uxP_ru_{rl}$ and $P_{rj}:=u_{r1}P_rx^-vP_ju_{jq}$, where $l=|V(P_r)|$; By (b) and the proof of Claim 1, we insert every vertex in $P_i[u_{i1},u)\cup P_j[u_{j1},v)$ between corresponding one pair of acceptors in some path in $(\mathfrak{P}\setminus\{P_i,P_j,P_r\})\cup\{P_{ir},P_{rj}\}$. Then we can get a path cover consisting of $t-1$ paths, a contradiction. Thus $x^-v\notin E(G)$. Similarly, $x^+v\notin E(G)$. By symmetry, for any vertex x in $V(G)-V(P_i\cup P_j)$, $x^-u,x^+u\notin E(G)$ if $vx\in E(G)$.

(d) Suppose $x\in P_i(v_i,u_{ip}]$, and to the contrary, $x^-v\in E(G)$. By the minimality of the subscript sum of u,v, there is no vertex in $P_j[u_{j1},v)$ adjacent to x^-, which implies no vertex in $P_j[u_{j1},v)$ has a pair of acceptors x^-,x. By (a), no vertex in $P_i[u_{i1},u]$ has a pair of acceptors in $P_j[u_{j1},v]$, and no vertex in $P_j[u_{j1},v]$ has a pair of acceptors in $P_i[u_{i1},u]$. Then we replace $P_i[u,u_{ip}]$ and $P_j[v,u_{jq}]$ by $P_{ij}:=u_{ip}P_i^-xuP_ix^-vP_ju_{jq}$; We insert each vertex in $P_i[u_{i1},u)\cup P_j[u_{j1},v)$ between corresponding one pair of acceptors in some path in $(\mathfrak{P}\setminus\{P_i,P_j\})\cup\{P_{ij}\}$. Then we can get a path cover consisting of $t-1$ paths, a contradiction. By symmetry, if $x\in P_j(v_j,u_{jq}]$ and $xv\in E(G)$, then $x^-u\notin E(G)$.

(e) Suppose $x\in P_i(v_i,u_{ip}], ux,vx\in E(G)$, and to the contrary, $x^-x^+\in E(G)$. By the choice of u,v, there is no vertex in $P_j[u_{j1},v)$ adjacent to x, which implies no vertex in $P_j[u_{j1},v)$ has a pair of acceptors x^-,x or x,x^+. By (a), no vertex in $P_i[u_{i1},u)$ has a pair of acceptors in $P_j[u_{j1},v)$, and no vertex in $P_j[u_{j1},v)$ has a pair of acceptors in $P_i[u_{i1},u)$. Then we replace $P_i[u,u_{ip}]$ and $P_j[v,u_{jq}]$ by $P_{ij}:=u_{ip}P_i^-x^+x^-P_i^-uxvP_ju_{jq}$; We insert each vertex in $P_i[u_{i1},u)\cup P_j[u_{j1},v)$ between corresponding one pair of acceptors in $(\mathfrak{P}\setminus\{P_i,P_j\})\cup\{P_{ij}\}$. Then we can get a path cover consisting of $t-1$ paths, a contradiction. Similarly, if $x\in P_j(v_j,u_{iq}]$, and $ux,vx\in E(G)$, then $x^-x^+\notin E(G)$.

Suppose $t\ge 3, x\in V(P_r)$, $P_r\in\mathfrak{P}\setminus\{P_i,P_j\}, xv,xu\in E(G)$, and to the contrary, $x^-x^+\in E(G)$. By the choice of u,v, there is no vertex in $P_i[u_{i1},u)\cup P_j[u_{j1},v)$ adjacent to x, which implies no vertex in $P_i[u_{i1},u)\cup P_j[u_{j1},v)$ has a pair of acceptors x^-,x, or x^+,x. By (a), no vertex in $P_i[u_{i1},u)$ has a pair of acceptors in $P_j[u_{j1},v)$, and no vertex in $P_j[u_{j1},v)$ has a pair of acceptors in $P_i[u_{i1},u)$. We replace $P_r\cup P_i[u,u_{ip}]\cup P_j[v,u_{jq}]$ by $P'_r:=u_{rl}P_r^-x^+x^-P_r^-u_{r1}$ and $P_{ij}:=u_{ip}P_i^-uxvP_ju_{jq}$, where $l=|V(P_r)|$; We insert each vertex in $P_i[u_{i1},u)\cup P_j[u_{j1},v)$ between corresponding one pair of acceptors in some path in $(\mathfrak{P}\setminus\{P_i,P_j,P_r\})\cup\{P'_r,P_{ij}\}$. Then we can get a path cover consisting of $t-1$ paths, a contradiction. □

Recall that $S=\{v_1,v_2,\dots,v_t\}$ is the vertex set consisting of the first non-insertable vertex of each path in $\mathfrak{P}$. By Claim 2(a), we can get the following results.

Claim 3 S is an independent set of G.

Claim 4 For any path $P_i\in\mathfrak{P}$, and any vertex $u\in P_i[u_{i1},v_i]$, $N_S(u)\subseteq\{v_i\}$, $1\le i\le t$.

Claim 5 For any path $P_i=P_i[u_{i1},u_{ip}]$, $N_S(u_{ip})\subseteq\{v_i\}$, where, $p=|V(P_i)|, 1\le i\le t$.

Proof Suppose to the contrary, $u_{ip}v_j\in E(G), v_j\in S-\{v_i\}$. We replace $P_i[u_{i1},u_{ip}]\cup P_j[v_j,u_{jq}]$ by $P_{ij}:=u_{i1}P_iu_{ip}v_jP_ju_{jq}$, where $q=|V(P_j)|$. Then we insert each vertex in $P_j[u_{j1},v_j)$ between corresponding one pair of acceptors of some path in $(\mathfrak{P}\setminus\{P_i,P_j\})\cup\{P_{ij}\}$. Then we can get a path cover consisting of $t-1$ paths, a contradiction. □

Claim 6 For any path $P_i\in\mathfrak{P}$ and any vertex $u\in V(P_i)$, $d_S(u)\le 2$, and if $d_S(u)=2$, then $v_i\in N_S(u), 1\le i\le t$.

Proof To the contrary, suppose there exists some vertex $u\in V(P_i)$ with $d_S(u)\ge 3$. By Claim 4 and Claim 5, $u\in P_i(v_i,u_{ip})$, where $p=|V(P_i)|$. Thus u^- and u^+ exist. Assume $v_j,v_m\in N_S(u)-\{v_i\}$, where $1\le j,m\le t$ and $j\ne m$. By the definition of non-insertable vertex, $v_ju^-,v_ju^+,v_mu^-,v_mu^+\notin E(G)$. By Claim 2(e), $u^-u^+\notin E(G)$. It follows that $G[u,u^-,u^+,v_j,v_m]$ $=K_{1,4}$, a contradiction. Thus $d_S(u)\le 2$. By the previous proof, if $d_S(u)=2$, and $v_i\notin N_S(u)$, then we can get a contradiction. Thus $v_i\in N_S(u)$. □

Claim 7 For any path $P_i\in\mathfrak{P}$, let $z_1,z_2,\dots,z_m$ be all the vertices in order along the positive direction of P_i with $N_S(z_j)=\emptyset, 1\le i\le t, 1\le j\le m$. If $m\ge 2$, then for any $j\in[1,m-1]$, any segment $P_i(z_j,z_{j+1})$ contains at most one vertex u with $d_S(u)=2$, and $u=z_{j+1}^-$ if $d_S(u)=2$.

Proof By Claim 4, $d_S(u)\le 1$ for any vertex $u\in P_i[u_{i1},v_i]$. By Claim 3, $d_S(v_i)=0$ and then $\{z_1,z_2,\dots,z_m\}\ne\emptyset$. Suppose for some segment $P_i(z_j,z_{j+1})$, u is the first vertex in $P_i(z_j,z_{j+1})$ with $d_S(u)=2$. Then by Claim 6, assume $N_S(u)=\{v_i,v_h\}$. In order to get $u=z_{j+1}^-$, it suffices to prove $P_i(u,z_{j+1})=\emptyset$. To the contrary, suppose $P_i(u,z_{j+1})\ne\emptyset$ and $v=u^+$. Since $v\in P_i(z_j,z_{j+1})$, $N_S(v)\ne\emptyset$. Since $v_iu,v_hu\in E(G)$, $vv_i\notin E(G)$ by Claim 2(d). By Claim 6, $d_S(v)=1$. Suppose $N_S(v)=\{v_s\}$, $v_s\in S-\{v_i\}$. Clearly, $v^-v_s\notin E(G)$, i.e., $uv_s\notin E(G)$, otherwise, v_s is an insertable vertex, a contradiction. Thus $v_s\ne v_h$. Since $v\notin V(P_s\cup P_h)$, vv_s, $uv_h\in E(G)$, i.e., $v^-v_h\in E(G)$, we can get a contradiction to Claim 2(c). □

Claim 8 If $N_S(u)\ne\emptyset$ for any vertex u in $P_i(v_i,u_{ip}]$, then $N_S(u)=\{v_i\}$ for each path $P_i\in\mathfrak{P}$, where $p=|V(P_i)|, 1\le i\le t$.

Proof Since $N_S(u_{ip}) \neq \emptyset$, $N_S(u_{ip}) = \{v_i\}$ by Claim 5. Then by $N_S(u) \neq \emptyset$ and Claim 2(d), for any vertex $u \in P_i(v_i, u_{ip})$, $N_S(u) = \{v_i\}$. $\square$

By Claims 7 and 8, we can obtain the upper bound of $\sum_{v\in S} d_{P_i}(v)$ for any path $P_i \in \mathfrak{P}$, as follows.

Claim 9 For any path $P \in \mathfrak{P}$, $\sum_{v\in S} d_P(v) = \sum_{u\in V(P)} d_S(u) \leq |V(P)| - 1$.

Now, let us complete Theorem 1.5. Clearly, $\sum_{v\in S} d(v) = \sum_{P\in\mathfrak{P}} \sum_{v\in S} d_P(v)$, and then by Claim 9, $\sum_{v\in S} d(v) \leq \sum_{P\in\mathfrak{P}}(|V(P)| - 1) = n - t$. It follows that $\sigma_{k+1} \leq \sigma_t \leq n - t < n - k$ by $k < t$, which contradicts $\sigma_{k+1}(G) \geq n - k$. Thus Theorem 1.5 holds. $\square$

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