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Exact Boundary Controllability of Nodal Profile for Hyperbolic Systems

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Dedicated to the Memory of Professor L. C. HSU on the Occasion of His 100th Birthday

Abstract In this survey paper we first recall the known results on the exact boundary controllability of nodal profile, then give some suggestions on further studies on this subject.

Keywords exact boundary controllability; exact boundary controllability of nodal profile; exact boundary controllability of partial nodal profile; quasilinear hyperbolic system; classical solution; entropy solution; asymptotic stability

MR(2010) Subject Classification 35L50; 35L53; 35L65; 93B05; 93C20; 93D20

1. Introduction

The exact controllability is of great importance in both theory and applications. Thanks to Russell [1] and Lions [2, 3], a complete theory has been established for linear hyperbolic systems, in particular, for linear wave equations. There have also been some results for semilinear wave equations [4–6]. For quasilinear hyperbolic systems which have numerous applications in mechanics, physics and other applied sciences, a simple and efficient constructive method with modular structure has been suggested by the author of this paper and Rao Bopeng in recent years [7–11] to get the exact boundary controllability in one-space-dimensional case. Let me first recall briefly the related results as follows.

Consider the general 1-D first order quasilinear hyperbolic system

$$\frac{\partial u}{\partial t} + A(u)\frac{\partial u}{\partial x} = B(u), \tag{1.1}$$

where $u = (u_1, \ldots, u_n)^{\mathrm{T}}$ is the unknown vector function of (t, x), $A(u)$ is a given $n \times n$ matrix with C^1 elements $a_{ij}(u)$ $(i, j = 1, \ldots, n)$, and $B(u) = (b_1(u), \ldots, b_n(u))^{\mathrm{T}}$ is a given C^1 vector function of u, such that

$$B(0) = 0, \tag{1.2}$$

which means that $u = 0$ is an equilibrium of system (1.1).

By hyperbolicity, for any given u on the domain under consideration, $A(u)$ possesses n real eigenvalues $\lambda_1(u), \ldots, \lambda_n(u)$ and a complete set of left eigenvectors $l_i(u) = (l_{i1}(u), \ldots, l_{in}(u))$ $(i =$

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$1, \dots, n$):

$$l_i(u)A(u) = \lambda_i(u)l_i(u). \tag{1.3}$$

We suppose that all $\lambda_i(u)$ and $l_i(u)$ $(i = 1, \dots, n)$ have the same C^1 regularity as $A(u) = (a_{ij}(u))$.

Suppose that on the domain under consideration there are no zero eigenvalues:

$$\lambda_r(u) < 0 < \lambda_s(u), \quad r = 1, \dots, m; \ s = m+1, \dots, n. \tag{1.4}$$

Let

$$v_i = l_i(u)u, \quad i = 1, \dots, n. \tag{1.5}$$

v_i is called the diagonalizable variable corresponding to the i-th characteristic

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \lambda_i(u). \tag{1.6}$$

On the domain $\{(t, x) | t \geq 0, \ 0 \leq x \leq L\}$ we consider the forward mixed initial-boundary value problem for system (1.1) with the initial condition

$$t = 0: \ u = \varphi(x), \ 0 \leq x \leq L \tag{1.7}$$

and the following boundary conditions

$$x = 0: \ v_s = G_s(t, v_1, \dots, v_m) + H_s(t), \quad s = m+1, \dots, n, \tag{1.8}$$

$$x = L: \ v_r = G_r(t, v_{m+1}, \dots, v_n) + H_r(t), \quad r = 1, \dots, m, \tag{1.9}$$

where φ, G_i and H_i $(i = 1, \dots, n)$ are all C^1 functions with respect to their arguments and, without loss of generality, we assume that

$$G_i(t, 0, \dots, 0) \equiv 0, \quad i = 1, \dots, n. \tag{1.10}$$

We point out that (1.8) and (1.9) are the most general nonlinear boundary conditions to guarantee the well-posedness for the forward mixed problem [12]. In order to illustrate the characters of boundary conditions (1.8) and (1.9), we say that those characteristics which reach the corresponding boundary from the interior of the domain are called the coming characteristics, while, all other characteristics which enter the domain from the corresponding boundary are called the departing characteristics (see Figure 1).

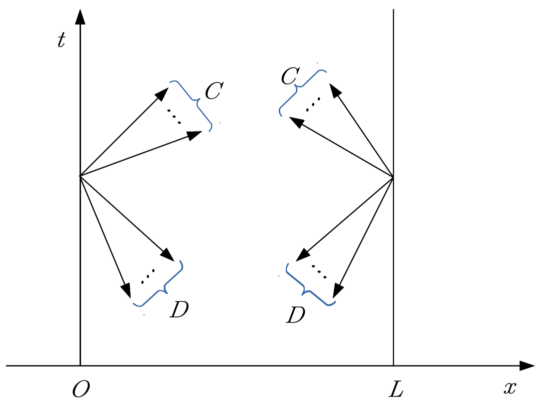

Figure 1 Coming characteristics and departing characteristics

Thus, it is easy to see from boundary conditions (1.8) and (1.9) that:

(1) The number of boundary conditions on $x = 0$ (resp., on $x = L$) is equal to the number of positive (resp., negative) eigenvalues, namely, to the number of coming characteristics on it.

(2) The boundary conditions on $x = 0$ (resp., on $x = L$) are written in the form that all the diagonalizable variables corresponding to the coming characteristics are explicitly expressed by other diagonalizable variables corresponding to the departing characteristics.

We always suppose that the conditions of C^1 compatibility are satisfied at the point $(t, x) = (0, 0)$ and $(0, L)$, respectively, then the forward mixed problem (1.1) and (1.7)–(1.9) admits at least a unique C^1 solution $u = u(t, x)$ locally in time.

For any given C^1 initial state $\varphi(x)$ $(0 \le x \le L)$ at $t = 0$ and any given C^1 final state at $t = T$:

$$t = T: \ u = \Phi(x), \ 0 \le x \le L, \tag{1.11}$$

we hope to choose suitable boundary controls $H_i(t)$ $(i = 1, \ldots, n)$ or a part of $H_i(t)$ $(i = 1, \ldots, n)$, such that the corresponding mixed initial-boundary value problem (1.1) and (1.7)–(1.9) admits a unique C^1 solution $u = u(t, x)$ on the domain $R(T) = \{(t, x) | 0 \le t \le T, \ 0 \le x \le L\}$, which satisfies exactly the final condition (1.11). If we can do so, then the exact boundary controllability is realized [8].

Since the hyperbolic wave has a finite speed of propagation, in order to get the exact boundary controllability, $T > 0$ should be suitably large.

Thus, in order to realize the exact boundary controllability, we need the C^1 solution $u = u(t, x)$ to the mixed problem on a domain $R(T) = \{(t, x) | 0 \le t \le T, \ 0 \le x \le L\}$, where $T > 0$ is a preassigned and possibly quite large number. This kind of C^1 solution, which is neither a local C^1 solution nor a global C^1 solution, is called a semi-global C^1 solution [8, 13]. In order to guarantee the existence of semi-global C^1 solution, some smallness hypotheses are needed. In other words, we should consider the solution in a neighbourhood of the equilibrium $u = 0$, and the obtained controllability is then called the local exact boundary controllability.

When all the boundary functions $H_i(t)$ $(i = 1, \ldots, n)$ acting on both ends $x = 0$ and $x = L$, respectively, are used to realize the exact boundary controllability, we get the two-sided exact boundary controllability, while, if, for instance, only the boundary functions $H_s(t)$ $(s = m + 1, \ldots, n)$ acting on one end $x = 0$ are used to realize the exact boundary controllability, we get the corresponding one-sided exact boundary controllability.

For fixing the idea, in what follows we consider only the one-sided exact boundary controllability. In this case we have the following result [8].

Proposition 1.1 *Suppose that the number of negative eigenvalues is not bigger than that of positive ones:*

$$m \le n - m, \ \text{i.e.,} \ n \ge 2m. \tag{1.12}$$

Suppose furthermore that boundary conditions (1.9) on $x = L$ can be equivalently rewritten in *a neighbourhood of $u = 0$ as*

$$x = L: \ v_{\bar{s}} = \bar{G}_{\bar{s}}(t, v_1, \ldots, v_m, v_{m+1}, \ldots, v_{n-m}) + \bar{H}_{\bar{s}}(t), \quad \bar{s} = n-m+1, \ldots, n, \tag{1.13}$$

where

$$\bar{G}_{\bar{s}}(t, 0, \ldots, 0) \equiv 0, \quad \bar{s} = n-m+1, \ldots, n. \tag{1.14}$$

If

$$T > L\Big(\max_{r=1,\ldots,m} \frac{1}{|\lambda_r(0)|} + \max_{s=m+1,\ldots,n} \frac{1}{\lambda_s(0)}\Big), \tag{1.15}$$

then, for any given $H_r(t)$ $(r = 1, \ldots, m)$ with small $C^1[0, T]$ norm, satisfying the conditions of C^1 compatibility at the points $(t, x) = (0, L)$ and (T, L), respectively, there exist boundary controls $H_s(t)$ $(s = m+1, \ldots, n)$ at $x = 0$ with small $C^1[0, T]$ norm, such that the corresponding mixed initial-boundary value problem (1.1) and (1.7)–(1.9) admits a unique semi-global C^1 solution $u = u(t, x)$ with small C^1 norm on the domain $R(T) = \{(t, x) | 0 \le t \le T, \ 0 \le x \le L\}$, which verifies exactly the final condition (1.11) (Figure 2).

On the other hand, stimulated by some practical applications, Gugat et al. [14] proposed in 2010 another kind of exact boundary controllability, called the nodal profile control. Differently from the usual exact boundary controllability, this kind of controllability does not ask to exactly attain any given final state at a suitable time $t = T$ by means of boundary controls, instead it asks the state to exactly fit any given profile on a node after a suitable time $t = T$ by means of boundary controls. The author of this paper almost immediately understood the significance of this kind of controllability, called it the exact boundary controllability of nodal profile, and deeply studied it in the general situation [15].

More precisely, the exact boundary controllability of nodal profile on a boundary node $x = L$ can be defined as follows: For any given C^1 initial data $\varphi(x)$ and any given C^1 boundary functions $H_r(t)$ $(r = 1, \ldots, m)$ on $x = L$, satisfying the conditions of C^1 compatibility at the point $(t, x) = (0, L)$, for any given C^1 vector function $\bar{\bar{u}}$, if there exist $T > 0$ and C^1 boundary controls $H_s(t)$ $(s = m+1, \ldots, n)$ on $x = 0$, such that the C^1 solution $u = u(t, x)$ to the mixed initial-boundary value problem (1.1) and (1.7)–(1.9) fits exactly $\bar{\bar{u}}$ on $x = L$ for $t \ge T$, then we have the exact boundary controllability of nodal profile on the boundary node $x = L$.

In this definition, when $t \ge T$, the given value of solution $u = \bar{\bar{u}}$ on $x = L$ should satisfy the boundary conditions (1.9), in which $v_i = \bar{\bar{v}}_i(t) \stackrel{\text{def.}}{=} l_i(\bar{\bar{u}}(t))\bar{\bar{u}}(t)$ $(i = 1, \ldots, n)$. Hence, the requirement that the solution $u = u(t, x)$ fits exactly the given value $\bar{\bar{u}}(t)$ on $x = L$ for $t \ge T$ is equivalent to ask that v_s $(s = m+1, \ldots, n)$ fit exactly the given values $\bar{\bar{v}}_s(t)$ $(s = m+1, \ldots, n)$ on $x = L$ for $t \ge T$, then the value of v_r $(r = 1, \ldots, m)$ on $x = L$ for $t \ge T$ can be determined by the boundary conditions (1.9) as follows:

$$v_r = \bar{\bar{v}}_r(t) \stackrel{\text{def.}}{=} G_r(t, \bar{\bar{v}}_{m+1}(t), \ldots, \bar{\bar{v}}_n(t)) + H_r(t), \quad r = 1, \ldots, m. \tag{1.16}$$

We have the following

Proposition 1.2 ([15, 16]) *Let*

$$T > L \max_{s=m+1,\ldots,n} \frac{1}{\lambda_s(0)} \tag{1.17}$$

and $\bar{T}$ be an arbitrarily given number such that

$$\bar{T} > T. \tag{1.18}$$

For any given initial data $\varphi(x)$ with small C^1 norm $\|\varphi\|_{C^1[0,L]}$ and any given boundary functions $H_r(t)$ $(r = 1, \ldots, m)$ with small C^1 norms $\|H_r\|_{C^1[0,\bar{T}]}$ $(r = 1, \ldots, m)$, satisfying the conditions of C^1 compatibility at the point $(t, x) = (0, L)$, suppose that the given values $\bar{\bar{v}}_s(t)$ $(s = m+1, \ldots, n)$ on $x = L$ for $T \le t \le \bar{T}$ possess small C^1 norms $\|\bar{\bar{v}}_s\|_{C^1[T,\bar{T}]}$ $(s = m + 1, \ldots, n)$, then there exist boundary controls $H_s(t)$ $(s = m+1, \ldots, n)$ with small C^1 norms $\|H_s\|_{C^1[0,\bar{T}]}$ $(s = m+1, \ldots, n)$, such that the mixed initial-boundary value problem (1.1) and (1.7)–(1.9) admits a unique C^1 solution $u = u(t, x)$ with small C^1 norm on the domain $R(\bar{T}) = \{(t, x) | 0 \le t \le \bar{T},\ 0 \le x \le L\}$, which fits exactly the given values $v_s = \bar{\bar{v}}_s(t)$ $(s = m+1, \ldots, n)$, namely, the given value $u = \bar{\bar{u}}(t)$, on the boundary node $x = L$ for $T \le t \le \bar{T}$ (Figure 3).

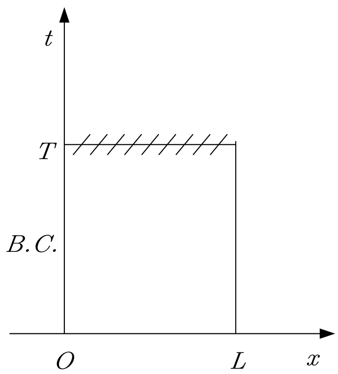

Figure 2 Exact boundary controllability

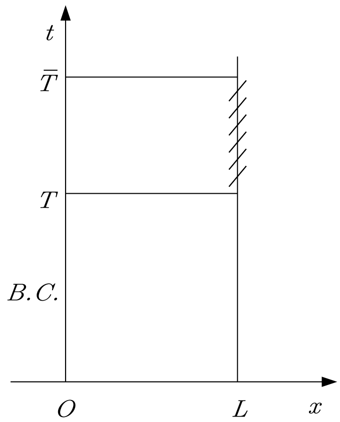

Figure 3 Exact boundary controllability of nodal profile

Both Propositions 1.1 and 1.2 can be proved by the constructive method with modular structure mentioned before [8, 16].

From the corresponding definitions and Propositions 1.1 and 1.2, it is easy to see that the exact boundary controllability of nodal profile can be regarded as a kind of weaker exact boundary controllability by the following reasons:

(1) The exact boundary controllability asks to fit n final states at time $t = T$, while the exact boundary controllability of nodal profile asks to fit $(n - m)$ boundary values on the end $x = L$ since time T, where $(n - m)$ is only the number of positive eigenvalues (namely, the number of departing characteristics on $x = L$), which is in general much less than n, the number of state variables.

(2) In order to realize the one-sided exact boundary controllability, the boundary controls should be acted on the end where the number of coming characteristics is bigger than or equal to the number of departing characteristics, moreover, the boundary conditions on the end without controls should satisfy certain solvability condition as shown in (1.13), however, there are no such restrictions for the exact boundary controllability of nodal profile.

(3) The controllability time (1.17) for the exact boundary controllability of nodal profile is essentially smaller than the controllability time (1.15) for the one-sided exact boundary controllability.

Both the exact boundary controllability and the exact boundary controllability of nodal profile can be also realized on a tree-like network with general topology. Since the interface conditions on any given multiple nodal of the network should be described according to the corresponding physical meanings, in order to study the exact boundary controllability and the exact boundary controllability of nodal profile on a network, we have to choose a suitable physical model, for instance, Saint-Venant system for the study.

On the other hand, both the exact boundary controllability and the exact boundary controllability of nodal profile can be also similarly studied for 1-D quasilinear wave equations

$$u_{tt} - (K(u, u_x))_x = F(u, u_x, u_t), \tag{1.19}$$

where u is a scalar unknown function of t and x, $K = K(u, v)$ is a given C^2 function with

$$K_v(u, v) > 0, \tag{1.20}$$

and $F = F(u, v, w)$ is a given C^1 function, satisfying

$$F(0, 0, 0) = 0. \tag{1.21}$$

All the obtained results on the exact boundary controllability and the exact boundary controllability of nodal profile for first order quasilinear hyperbolic systems (1.1) and 1-D quasilinear wave equations (1.19) can be found in monographs [8] and [16], respectively.

2. Further studies on the exact boundary controllability of nodal profile

The exact boundary controllability of nodal profile for hyperbolic systems can be further studied in the following directions.

(A) In Proposition 1.2, the exact boundary controllability of nodal profile is considered in the case that the nodal profile is given on a finite time interval $[T, \bar{T}]$, where $\bar{T} > 0$ is an arbitrarily given and possibly quite large number. This requirement is suitable and convenient for many practical applications, however, it restricts the possibility to study the asymptotic behaviour of the solution as well as the corresponding boundary controls to the exact boundary controllability of nodal profile. Therefore, it is worthwhile to consider the case that the nodal profile is given on a semi-bounded time interval $[T, +\infty)$ with certain asymptotic properties as $t \to +\infty$. The most important situations in practice can be as follows:

(i) The nodal profile given on $[T, +\infty)$ tends to a limit with polynomial decaying property as $t \to +\infty$;

(ii) The nodal profile given on $[T, +\infty)$ tends to a limit with exponential decaying property as $t \to +\infty$;

(iii) The nodal profile given on $[T, +\infty)$ is time-periodic with a period $T_0 > 0$.

For the exact boundary controllability of nodal profile on a single space interval $[0, L]$, when the nodal profile is given on the semi-bounded time interval $[T,+\infty)$, the constructive method with modular structure previously used in the finite time interval case $[T,\bar{T}]$ still works, however, after having changed the roles of t and x, the corresponding problem then reduces to get the semi-global C^1 solution along the x-direction from 0 to L for a leftward or rightward mixed problem on a semi-bounded initial axis from T to $+\infty$, which can be realized under the assumption that the C^1 norm of the nodal profile on the time interval $[T,+\infty)$ is small enough, moreover, it can be proved that the solution keeps the corresponding property as mentioned in (i)–(iii), respectively [17].

For the exact boundary controllability of nodal profile on a tree-like network, when the nodal profile is given on the semi-bounded time interval $[T,+\infty)$, according to the constructive method with modular structure, we have to not only change the roles of t and x and solve a corresponding leftward or rightward mixed problem on certain individual x-intervals, but also solve the forward mixed problem on each subnetwork composed of other x-intervals [16]. Thus, we should prove the global existence of piecewise C^1 solutions on these subnetworks, respectively, moreover, the asymptotic behaviours of these global piecewise C^1 solutions should be carefully considered. These requirements cause certain new problems in the study of the global existence of C^1 or piecewise C^1 solutions and of the corresponding asymptotic behaviours, in which the time-periodic case is more challenging and difficult [18].

The corresponding study should also be done for 1-D quasilinear wave equations (1.19) (see [19]).

(B) We know that the exact boundary controllability can not be realized in general on a network with loops [20], however, we do have the exact boundary controllability of nodal profile for Saint-Venant system on some special networks with loops [21]. In order to get a more general theory on it, we have to give a unified and effective method to treat the exact boundary controllability of nodal profile on a network with loops. The basic idea is to cut the network with loops under consideration in a suitable way such that the network becomes some tree-like networks (i.e., without loops), then one can sufficiently apply the result on the exact boundary controllability of nodal profile on a tree-like network to achieve the exact boundary controllability of nodal profile for the original network with loops. Hopefully, this method can be applied to much more networks with loops [22].

(C) A careful analysis shows that the constructive method with modular structure, originally proposed for classical solutions, can be essentially applied, under certain hypotheses, to entropy solutions to quasilinear hyperbolic systems of conservation (or balance) laws, therefore, the exact boundary controllability has been generalized from classical or piecewise classical solutions of quasilinear hyperbolic systems to entropy solutions of quasilinear hyperbolic systems of conservation (or balance) laws [23–25]. It is then significant to similarly consider the exact boundary controllability of nodal profile for entropy solutions to quasilinear hyperbolic systems of conservation (or balance) laws on a single space interval or on a tree-like network for the case that the nodal profile is given on a bounded time interval $[T,\bar{T}]$ or on a semi-bounded time interval $[T,+\infty)$ with some asymptotic properties [26].

(D) A new concept, namely, the exact boundary controllability of partial nodal profile, has been introduced [27], in which only partial information of nodal profile is given. In this way it is possible to reduce the number of boundary controls and the controllability time, increase the number of charged nodes on which the nodal profiles are given, and a node can be simultaneously with a control and with partial nodal profile. Deeply studying the corresponding problems mentioned before for this new concept will be significant and interesting.

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